

Complexity Theory

Exercise 8: Polynomial Hierarchy

19th December 2023

Exercise 8.1. Show that Cook-reducibility is transitive. In other words, show that if **A** is Cook-reducible to **B** and **B** is Cook-reducible to **C**, then **A** is Cook-reducible to **C**.

Exercise 8.2. Show that there exists an oracle **C** such that $\text{NP}^{\text{C}} \neq \text{coNP}^{\text{C}}$.

Hint:

BAKER-CUILL-ROJALSKY THEOREM for coNP instead of P .

What kind of Turing machines exist for languages in coNP ? Use the answer to adapt the proof of the

Exercise 8.3. Show $\text{NP}^{\text{SAT}} \subseteq \Sigma_2\text{P}$.

Exercise 8.4. Show the following result: *If there is any k such that $\Sigma_k^{\text{P}} = \Sigma_{k+1}^{\text{P}}$ then $\Sigma_j^{\text{P}} = \Pi_j^{\text{P}} = \Sigma_k^{\text{P}}$ for all $j > k$, and therefore $\text{PH} = \Sigma_k^{\text{P}}$.*

Exercise 8.5. Show that $\text{PH} \subseteq \text{PSPACE}$.

Exercise 8.6. Let **A** be a language and let **F** be a finite set with $\mathbf{A} \cap \mathbf{F} = \emptyset$. Show that $\text{P}^{\mathbf{A}} = \text{P}^{\mathbf{A} \cup \mathbf{F}}$ and $\text{NP}^{\mathbf{A}} = \text{NP}^{\mathbf{A} \cup \mathbf{F}}$.