Problem 2.1
In the lectures, $\approx_E$ was defined to be the least congruence relation generated by $E$. What does it mean?

Problem 2.2
Consider the set of clauses
\[
F = \{ [p(f(Y)), q(Y), r(b)], [\neg p(b)], [\neg q(a)], [\neg r(a)] \}
\]
and the equational system
\[
E = \{ (\forall X) f(X) \approx X, a \approx b \}.
\]
Show by paramodulation, resolution and factoring that $F \cup E \cup E_\approx$ is unsatisfiable. Also give the mgu $\theta$ used in every step.

Problem 2.3
Let $R$ be a term rewriting system and let $s$ and $t$ be terms. Prove that:

1. $s \rightarrow_R t$ implies $s \approx_{E_R} t$.
2. $s \leftrightarrow^*_R t$ implies $s \approx_{E_R} t$.

Problem 2.4
A non terminating term rewriting system can be confluent. True or false? Prove it.

Problem 2.5
Prove that a term rewriting system $R$ is Church-Rosser if and only if it is confluent.
Problem 2.6

Consider the following term rewriting system:

\[ f(f(X,Y),Z) \rightarrow f(X,f(Y,Z)); \]
\[ f(X,1) \rightarrow X. \]

1. Is it terminating? Justify your answer.

2. Compute all the critical pairs, and show how you got them.

3. Can you orientate the critical pairs, i.e., add a rule \( s \rightarrow t \) or \( t \rightarrow s \) for each critical pair \( \langle s, t \rangle \), such that termination is preserved? (If it is possible, do it . . . )

Note: When executing the completion algorithm you have to go on trying to build critical pairs with the iteratively added rules.

Problem 2.7

Let \( \mathcal{R} \) be a term rewriting system and \( >/2 \) a termination ordering. If for all rules \( l \rightarrow r \in \mathcal{R} \) the relation \( l > r \) holds, then \( \mathcal{R} \) is terminating.

Problem 2.8

Consider the term rewriting system

\[ \mathcal{R} = \{ f(g(X)) \rightarrow g(X), \] \[ g(h(X)) \rightarrow g(X) \} \]

Show that \( \mathcal{R} \) is canonical.