TU Dresden, Fakultät Informatik
Stephan Mennicke, Sergei Obiedkov Winter Term 2025/2026

Complexity Theory
Exercise 3: Time Complexity
3 November 2025

Exercise 3.1. Show that P is closed under concatenation and Kleene star.

Exercise 3.2. A language $\mathbf{L} \in \mathrm{P}$ is complete for P under polynomial-time reductions if $\mathbf{L}' \leq_p \mathbf{L}$ for every $\mathbf{L}' \in \mathrm{P}$. Show that every language in P except $\emptyset$ and Σ^* is complete for P under polynomial-time reductions.

Exercise 3.3. Consider the problem **CLIQUE**:

Input:	An undirected graph G and some $k \in \mathbb{N}$
Question:	Does there exists a clique in G of size at least k?

For an undirected graph $G = (V, E)$ (i.e., with symmetric $E \subseteq V \times V$), a *clique* in G of size $k \in \mathbb{N}$ is a subset of nodes $C \subseteq V$ with $|C| = k$ and $C \times C \subseteq E$.

Suppose **CLIQUE** can be solved in time $T(n)$ for some $T\colon \mathbb{N} \to \mathbb{N}$ with $T(n) \geq n$ for all $n \in \mathbb{N}$. Furthermore, show that then the optimisation problem

Input:	An undirected graph G
Compute:	A clique in G of maximal size

can be computed in time $\mathcal{O}(n \cdot T(n))$. You can assume that T is monotone.

Exercise 3.4. Show that if **SAT** $\in \mathrm{P}$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given satisfiable propositional formula.

Exercise 3.5. Convince yourself of the following fact: If $T : \mathbb{N} \to \mathbb{N}$ is a polynomial function, then there is a polynomial time-bounded TM $\mathcal{M}$ computing the binary representation of $T(n)$, given an input $\mathbf{1}^n$ (that is n repetitions of the symbol $\mathbf{1}$).

Exercise 3.6. We consider propositional formulae φ in *conjunctive normal form* (CNF). We say that φ is a 2CNF-formula, written $\varphi \in$ 2CNF, if φ is a formula in CNF and every clause consists of at most two literals. A formula is in CNF_2 if every variable occurs at most twice in φ.

a) Show that **2SAT** $:= \{\langle\varphi\rangle \mid \varphi \in \text{2CNF is satisfiable}\}$ is in P.

b) Show that $\mathbf{CNF}_2 := \{\langle\varphi\rangle \mid \varphi \in \mathsf{CNF}_{\mathbf{2}} \text{ is satisfiable}\}$ is in P.