Exercise 10.1. Denote with $\text{add}: \{0, 1\}^{2n} \to \{0, 1\}^{n+1}$ the function that takes two binary $n$-bit numbers $x$ and $y$ and returns their $n + 1$-bit sum. Show that $\text{add}$ can be computed with size $O(n)$ circuits.

Exercise 10.2. Define the function $\text{maj}_n: \{0, 1\}^n \to \{0, 1\}^n$ by

$$\text{maj}_n(x_1, \ldots, x_n) := \begin{cases} 0 & \text{if } \sum x_i < n/2 \\ 1 & \text{if } \sum x_i \geq n/2. \end{cases}$$

Devised a circuit to compute $\text{maj}_3$ and test it on the example input 101 and 010.

Exercise 10.3. Show $\text{NC}^1 \subseteq L$.

Exercise 10.4. Show that every Boolean function with $n$ variables can be computed with a circuit of size $O(n \cdot 2^n)$.

Exercise 10.5. Show that every language $L \subseteq \{1^n \mid n \in \mathbb{N}\}$ is contained in $\text{P/poly}$. Conclude that $\text{P/poly}$ contains undecidable languages.

Exercise 10.6. Find a decidable language in $\text{P/poly}$ that is not contained in $\text{P}$.

Hint: Take a language over $\{0, 1\}$ that is 2ExpTime-hard and consider its unary encoding.