Problem 3.1

Given the following program \( \mathcal{P} \):

\[
\begin{align*}
p(X) & \leftarrow q(X) \land \neg r(X) \land s(X). \\
q(a) & \leftarrow \top. \\
r(a) & \leftarrow \bot.
\end{align*}
\]

1. Determine ground \( \mathcal{P}, g\mathcal{P} \), the set of facts, the set of assumptions and the set of definitions for defined atoms in \( \mathcal{P} \).

2. Define the completion of \( \mathcal{P} \), \( c\mathcal{P} \), and the weak completion of \( \mathcal{P} \), \( wc\mathcal{P} \).

3. Find all the two-valued and the three-valued interpretations of \( \mathcal{P} \).
   (a) Which two-valued interpretations are models of \( \mathcal{P} \), of \( c\mathcal{P} \) and of \( wc\mathcal{P} \)?
   (b) Which three-valued interpretations \( I \) are models of \( \mathcal{P} \), of \( c\mathcal{P} \) and of \( wc\mathcal{P} \) under (three-valued) Łukasiewicz logic?
   (c) Are there models under Łukasiewicz logic, which are not models under other three-valued logics? If yes, which ones?

Problem 3.2

1. Which of the following equivalences hold under Łukasiewicz logic? Motivate your answer.
   (a) Equivalence: \( F \leftrightarrow G \equiv (F \leftarrow G) \land (G \leftarrow F) \)
   (b) Implication: \( F \rightarrow G \equiv \neg F \lor G \)
   (c) Syllogism: \( (F \rightarrow G) \land (G \rightarrow H) \equiv F \rightarrow H \)
   (d) Excluded Middle: \( F \lor \neg F \equiv \top \)
   (e) Contradiction: \( F \lor \neg F \equiv \bot \)

2. Which of the equivalences above hold under Fitting logic but do not hold under Łukasiewicz logic? Motivate your answer.

Problem 3.3

Show that the model intersection property, \( \cap \{ I \mid I \models \mathcal{P} \} \models \mathcal{P} \), does not hold under Fitting logic.