

Human Reasoning and Computational Logic

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Problem 4.1

Given the following program \mathcal{P} :

$$\begin{aligned} p(X) &\leftarrow q(X) \wedge \neg r(X) \wedge s(X). \\ q(a) &\leftarrow \top. \\ r(a) &\leftarrow \perp. \end{aligned}$$

1. Determine ground \mathcal{P} , $g\mathcal{P}$, the set of facts, the set of assumptions and the set of definitions for defined atoms in \mathcal{P} .
2. Define the completion of \mathcal{P} , $c\mathcal{P}$, and the weak completion of \mathcal{P} , $wc\mathcal{P}$.
3. Find all the two-valued and the three-valued interpretations of \mathcal{P} .
 - (a) Which two-valued interpretations are models of \mathcal{P} , of $c\mathcal{P}$ and of $wc\mathcal{P}$?
 - (b) Which three-valued interpretations I are models of \mathcal{P} , of $c\mathcal{P}$ and of $wc\mathcal{P}$ under (three-valued) Łukasiewicz logic?
 - (c) Are there models under Łukasiewicz logic, which are not models under other three-valued logics? If yes, which ones?

Problem 4.2

1. Which of the following equivalences hold under Łukasiewicz logic? Motivate your answer.
 - (a) Equivalence: $F \leftrightarrow G \equiv (F \leftarrow G) \wedge (G \leftarrow F)$
 - (b) Implication: $F \rightarrow G \equiv \neg F \vee G$
 - (c) Syllogism: $(F \rightarrow G) \wedge (G \rightarrow H) \equiv F \rightarrow H$
 - (d) Excluded Middle: $F \vee \neg F \equiv \top$
 - (e) Contradiction: $F \vee \neg F \equiv \perp$
2. Which of the equivalences above hold under Fitting logic but do not hold under Łukasiewicz logic? Motivate your answer.

Problem 4.3

Show that the model intersection property, $\bigcap\{I \mid I \models \mathcal{P}\} \models \mathcal{P}$, does not hold under Fitting logic.