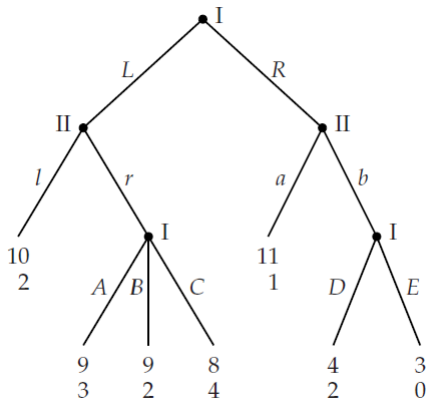
Algorithmic Game Theory
Problems 3

15.05.2023

Problem 1. *Consider the following game tree. Note that the top payoffs at a leaf are for player I and bottom payoffs are for player II.*



(a) *What is the number of strategies of player I and of player II?*

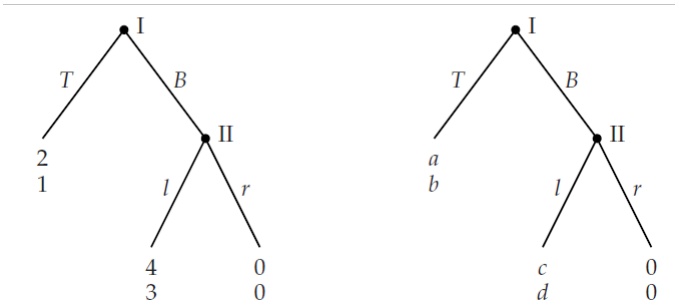
(b) *How many reduced strategies do they have? Recall that a reduced strategy in a game tree of a player specifies a move for every decision node of that player, expect for those moves that are unreachable due to an earlier move.*

(c) *Give the reduced strategic form of the game.*

(d) *What are the equilibria of the game in reduced strategies?*

(e) *What are the subgame-perfect equilibria of the game?*

Problem 2. *Consider the following game trees.*



(a) *Find all equilibria for the game tree on the left. Which of these are subgame-perfect?*

(b) *In the game tree on the right, the payoffs a, b, c, d are positive real numbers. For each of the following statements (i), (ii), (iii), decide if it is true or false, justifying your answer with an argument or counterexample.*

(i) *the game always has a subgame-perfect equilibrium (SPE);*

(ii) *the payoff to player II in any SPE is always at least as high as her payoff in any equilibrium;*

(iii) *the payoff to player I in any SPE is always at least as high as his payoff in any equilibrium.*

Problem 3. *In 1981, Robert Rosenthal introduced the so-called* ***centipede game****. The game in extensive form can be described as follows*

- *At stage 1, player I chooses between move* $\boldsymbol{R}$ *and* $\boldsymbol{D}$*.*
  - *If she chooses* $\boldsymbol{D}$*, player 1 gets 1 and player 2 gets 0;*
  - *If she chooses* $\boldsymbol{R}$*, the game goes to round 2.*
- *At stage 2, player 2 chooses between* $\boldsymbol{r}$ *and* $\boldsymbol{d}$*.*
  - *If he chooses* $\boldsymbol{d}$*, player 1 gets 0 and player 2 gets 2;*
  - *If he chooses* $\boldsymbol{r}$*, the game moves to round 3.*
- *At stage 3, player 1 chooses between* $\boldsymbol{R}$ *and* $\boldsymbol{D}$*.*
  - *If she chooses* $\boldsymbol{D}$*, player 1 gets 3 and player 2 gets 1;*
  - *If she chooses* $\boldsymbol{R}$*, the games moves to round 4.*
- *At stage 4, player chooses between* $\boldsymbol{r}$ *and* $\boldsymbol{d}$*.*
  - *If he chooses* $\boldsymbol{d}$*, player 1 gets 2 and player 2 gets 4.*
  - *If he chooses* $\boldsymbol{r}$*, both players get 3.*

Do the following

- *Draw the tree representation of the game.*
- *Apply backward induction and find its outcome.*
- *Give the pure strategies of both players and the payoff matrix of the normal form of the game.*
- *Find all (sub-game perfect) Nash equilibria.*

After its introduction, the ***centipede game*** *has been extensively studied in experiments. What do you think: Were the people taking part in the experiments more inclined to play the Nash equilibrium or to cooperate?*

Problem 4. *Argue that in any sequential game, the backward induction strategy profile is in fact a Nash equilibrium.*