

Knowledge Graphs

Exercise Sheet 1: Getting to Know Graphs

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2026-10-21

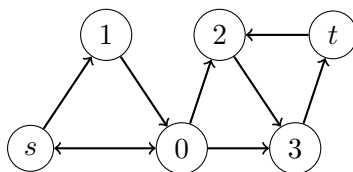
Winter Term 2026/2027

Exercise 1

Show that the number of vertices of odd degree is even in every simple graph.

Exercise 2

Consider the following graph:



- Is the graph connected?
- Find all directed paths from vertex s to vertex t of length at most 6.
- Is the graph strongly connected? If not, which edge(s) would need to be added to make it strongly connected?
- Find the strongly connected components (i.e., the maximal induced subgraphs that are strongly connected) of the graph.

Exercise 3

A *bipartite graph* is a simple graph $G = \langle V, E \rangle$, where V can be partitioned into two sets X, Y (i.e., $X \cup Y = V$, and $X \cap Y = \emptyset$), such that every edge $\{a, b\} \in E$ coincides with both X and Y , i.e., $\{a, b\} \cap X \neq \emptyset$ and $\{a, b\} \cap Y \neq \emptyset$.

Show that the following are equivalent:

- $G = \langle V, E \rangle$ is bipartite.
- G is *2-colourable*, i.e., there is a map $c : V \rightarrow \{0, 1\}$ such that no two adjacent vertices a, b have the same colour, i.e., $c(a) \neq c(b)$ for all $\{a, b\} \in E$.
- G does not contain a cycle $v_1 \xrightarrow{e_1} v_2 \xrightarrow{e_2} \dots \xrightarrow{e_{n-1}} v_n \xrightarrow{e_n} v_1$ of odd length.

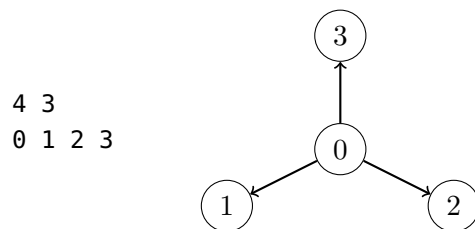
Exercise 4

Write a program that reads a directed graph from a file in the format of Exercise 0.2 and prints out the graph in METIS graph format:

$$\begin{array}{cccccc}
 n & m & & & & \\
 v_1 & n_1^1 & n_1^2 & \cdots & n_1^{d_1} & \\
 v_2 & n_2^1 & n_2^2 & \cdots & n_2^{d_2} & \\
 \vdots & \vdots & \vdots & \cdots & \vdots & \\
 v_n & n_n^1 & n_n^2 & \cdots & n_n^{d_n} &
 \end{array}$$

The first line consists of two integers n and m , separated by a space, where n is the number of vertices, and m is the total number of edges. Each of the following lines specifies the neighbours $n_i^1, n_i^2, \dots, n_i^{d_i}$ of vertex v_i .

As an example, the directed star S_3 would be encoded as:



Exercise 5

A triangle in a directed graph is a simple directed path $v_1 \xrightarrow{e_1} v_2 \xrightarrow{e_2} v_3 \xrightarrow{e_3} v_1$.

Write a program that reads a directed graph G from a file in the format of Exercise 1.4 and prints out the number of triangles in G . How does the runtime of your program scale with the size of the input graph?