Exercise 3

SAT Solving

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Exercise 3.1 (Conflict/Falsified Clauses)

Recall the following definition from the lecture (p.31 on the second set of slides).
Let F be a formula in CNF and $\tau \in 2^{\mathrm{Var}(F)}$ a partial assignment.

a. *Assignment τ* satisfies *F if F_τ is empty.*

b. *Assignment τ* falsifies *F if F_τ contains the empty clause.*
   Then, we also call F_τ a conflict. *If we have a conflict (i.e., $\emptyset \in F_\tau$),*
   there is a clause $C \in F$ where $C_\tau = \emptyset$. We call such a clause C falsified clause.

F_τ is the formula under the truth assignment (or reduct) as discussed in Exercise 2.5.

Consider the following formula:

$$F =< [x, z], [\neg z], [\neg x, \neg y, u], \neg x, \neg u, \neg w] >$$

Each of following assignments[1] yields a conflict. Give a falsified clause for F_{τ_i}:

a. $\tau_1 = \{\neg x, \neg z\}$

b. $\tau_2 = \{z\}$

c. $\tau_3 = \{x, u, \neg w\}$

Exercise 3.2 (Another DPLL Example)

Given the following formula in CNF:

$$< [x, z], [y, \neg z], [x, \neg y, u], [\neg y, \neg u], [u, v], [\neg x, \neg v], [\neg u, w], [\neg x, \neg u, \neg w] >$$

a. Construct the full DPLL-search tree (without applying unit propagation).

b. Give at each total assignment a falsified clause
   (thereby showing that the formula is unsatisfiable).

Exercise 3.3 (Connection between DPLL and Resolution)

Consider the DPLL-search tree and falsified clauses at the leaves that you constructed from the previous example. Resolve each of these falsified clauses from bottom up. In other words, if you obtained the conflict after a decision literal ℓ and have C_ℓ and $C_{\neg\ell}$, then you resolve C_ℓ and $C_{\neg\ell}$. After doing this with all leaves, you up one decision and resolve the previously obtained resolvents. *For example, you may obtain the falsified clauses $[x, z]$ and $[y, \neg z]$ from the assignment $\{\neg x, \neg y, \neg z\}$ and $\{\neg x, \neg y, z\}$, respectively. After resolving these you obtain $[x, y]$ then you proceed along the tree from the leaves to the root.* Explain what you obtained.

[1] We give the assignments as sets of literals.