PROBLEM SOLVING AND SEARCH
IN ARTIFICIAL INTELLIGENCE

Lecture 5 Answer-Set Programming Motivation and Introduction

* slides adapted from Torsten Schaub [Gebser et al.(2012)]

Sarah Gaggl

Dresden, 6th May 2016
Agenda

1. Introduction
2. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
3. Local Search, Stochastic Hill Climbing, Simulated Annealing
4. Tabu Search
5. Answer-set Programming (ASP)
6. Constraint Satisfaction (CSP)
7. Structural Decomposition Techniques (Tree/Hypertree Decompositions)
8. Evolutionary Algorithms/ Genetic Algorithms
Outline

1 Motivation
   – Declarative Problem Solving
   – ASP in a Nutshell
   – ASP Paradigm

2 Introduction
   – Syntax
   – Semantics
   – Examples
   – Language Constructs
   – Modeling
Informatics

“What is the problem?” versus “How to solve the problem?”

Problem

Solution

Computer

Output
Informatics

“What is the problem?” versus “How to solve the problem?”
Traditional programming

“What is the problem?” versus “How to solve the problem?”
Traditional programming

“What is the problem?” versus “How to solve the problem?”
Declarative problem solving

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Declarative problem solving

“What is the problem?” versus “How to solve the problem?”

Problem Representation Solution
Modeling Interpreting Solving

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PSSAI
slide 9 of 141
Declarative problem solving

- What is the problem?
- How to solve the problem?

Problem Representation Solution Output

Modeling Solving Interpreting
Answer Set Programming
in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
Answer Set Programming in a Nutshell

- ASP is an approach to **declarative problem solving**, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
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• ASP allows for solving all search problems in \( NP \) (and \( NP^{NP} \)) in a uniform way
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- ASP is versatile as reflected by the ASP solver clasp, winning
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- **ASP** allows for solving all search problems in $NP$ (and $NP^{NP}$) in a uniform way

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- **ASP** embraces many emerging application areas
Answer Set Programming

in a Hazelnutshell

• ASP is an approach to declarative problem solving, combining
  – a rich yet simple modeling language
  – with high-performance solving capacities
  tailored to Knowledge Representation and Reasoning
Answer Set Programming
in a Hazelnutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
tailored to Knowledge Representation and Reasoning

\[
\text{ASP} = \text{DB}+\text{LP}+\text{KR}+\text{SAT}
\]
KR’s shift of paradigm

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query
KR’s shift of paradigm

Theorem Proving based approach (eg. Prolog)
1. Provide a representation of the problem
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Model Generation based approach (eg. SATisfiability testing)
1. Provide a representation of the problem
2. A solution is given by a model of the representation
<table>
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<tr>
<th>Representation</th>
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LP-style playing with blocks

**Prolog program**

```prolog
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

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LP-style playing with blocks

### Prolog program

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### Prolog queries

```prolog
?- above(a,c).
true.
```
LP-style playing with blocks

Prolog program

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Prolog queries

?- above(a, c).
true.

?- above(c, a).
no.
LP-style playing with blocks

**Prolog program**

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on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

**Prolog queries (testing entailment)**

```
?- above(a,c).
true.

?- above(c,a).
neg.
```
LP-style playing with blocks

Shuffled Prolog program

on(a,b).
on(b,c).

above(X,Y) :- above(X,Z), on(Z,Y).
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LP-style playing with blocks

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on(a,b).
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Prolog queries

?- above(a,c).
LP-style playing with blocks

Shuffled Prolog program

\begin{verbatim}
on(a,b).
on(b,c).

above(X,Y) :- above(X,Z), on(Z,Y).
above(X,Y) :- on(X,Y).
\end{verbatim}

Prolog queries (answered via fixed execution)

?- above(a,c).

Fatal Error: local stack overflow.
SAT-style playing with blocks

**Formula**

\[
\begin{align*}
on(a, b) \\
\land on(b, c) \\
\land (on(X, Y) \rightarrow above(X, Y)) \\
\land (on(X, Z) \land above(Z, Y) \rightarrow above(X, Y))
\end{align*}
\]
SAT-style playing with blocks

**Formula**

\[
on(a, b) \land on(b, c) \land (on(X, Y) \rightarrow above(X, Y)) \land (on(X, Z) \land above(Z, Y) \rightarrow above(X, Y))\]

**Herbrand model**

\[
\{ on(a, b), on(b, c), on(a, c), on(b, b), above(a, b), above(b, c), above(a, c), above(b, b), above(c, b) \}\]
Formula

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on(a, b) \\
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Herbrand model (among 426!)

\[
\{ \quad on(a, b), \quad on(b, c), \quad on(a, c), \quad on(b, b), \\
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\]
### Formula

\[
\begin{align*}
& \text{on}(a, b) \\
& \land \quad \text{on}(b, c) \\
& \land \quad (\text{on}(X, Y) \rightarrow \text{above}(X, Y)) \\
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Answer Set Programming (ASP)
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ASP-style playing with blocks

Logic program

on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).

Stable Herbrand model

{on(a,b), on(b,c), above(b,c), above(a,b), above(a,c)}
ASP-style playing with blocks

Logic program

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on(a, b). \\
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ASP-style playing with blocks

Logic program

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\]

Stable Herbrand model (and no others)

\{ on(a, b), on(b, c), above(b, c), above(a, b), above(a, c) \}
ASP-style playing with blocks

**Logic program**

\[
\begin{align*}
on (a, b) . \\
on (b, c) . \\
above (X, Y) &:= above (Z, Y), on (X, Z) . \\
above (X, Y) &:= on (X, Y) .
\end{align*}
\]

**Stable Herbrand model (and no others)**

\{ on(a, b), on(b, c), above(b, c), above(a, b), above(a, c) \}
## ASP versus LP

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<th>Prolog</th>
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<tr>
<td>Model generation</td>
<td>Query orientation</td>
</tr>
<tr>
<td>Bottom-up</td>
<td>Top-down</td>
</tr>
<tr>
<td>Modeling language</td>
<td>Programming language</td>
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### Rule-based format

<table>
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<tr>
<th>Instantiation</th>
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<tr>
<td>Flat terms</td>
<td>Nested terms</td>
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(Turing +) $NP^{NP}$

Turing
### ASP versus SAT

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<td><strong>Constructive Logic</strong></td>
<td>Classical Logic</td>
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<td>Closed (and open)</td>
<td>Open world reasoning</td>
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<td>world reasoning</td>
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<tr>
<td><strong>Modeling language</strong></td>
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<td><strong>Complex reasoning modes</strong></td>
<td>Satisfiability testing</td>
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ASP solving

Problem

Modeling

Logic Program

Grounder

Solving

Solver

Solution

Interpreting

Stable Models
SAT solving

- Problem
  - Formula (CNF)
    - Programming
  - Solver
    - Solving
    - Classical Models
  - Solution
    - Interpreting
Rooting ASP solving

- Problem
- Modeling
- Logic Program
- Grounder
- Solving
- Solver
- Solution
- Interpreting
- Stable Models
Rooting ASP solving

1. Problem
2. Logic Program (LP)
3. Grounder
4. Solver
5. Solution
6. Stable Models (DB+KR+LP)

Modeling (KR) → Logic Program (LP) → Grounder → Solver → Stable Models (DB+KR+LP)

Interpreting (DB+Solving+SAT)
Two sides of a coin

- **ASP as High-level Language**
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules

- **ASP as Low-level Language**
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation
What is ASP good for?

- Combinatorial search problems in the realm of $P$, $NP$, and $NP^{NP}$ (some with substantial amount of data), like
  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
  - Model Checking
  - Product Configuration
  - Robotics
  - System Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more
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What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - Including: data, frame axioms, exceptions, defaults, closures, etc
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   - Declarative Problem Solving
   - ASP in a Nutshell
   - ASP Paradigm

2 Introduction
   - Syntax
   - Semantics
   - Examples
   - Language Constructs
   - Modeling
Problem solving in ASP: Syntax

Problem → Logic Program → Stable Models
Modeling

Solution
Interpreting
Solving
Normal logic programs

- A (normal) logic program over a set $\mathcal{A}$ of atoms is a finite set of rules.
- A (normal) rule, $r$, is of the form

$$a_0 \leftarrow a_1, \ldots, a_m, \text{not } a_{m+1}, \ldots, \text{not } a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$. 

Notation:
- $\text{head}(r) = a_0$
- $\text{body}(r) = \{a_1, \ldots, a_m, \text{not } a_{m+1}, \ldots, \text{not } a_n\}$
- $\text{body}(r) + = \{a_1, \ldots, a_m\}$
- $\text{body}(r) - = \{a_{m+1}, \ldots, a_n\}$
Normal logic programs

- A (normal) logic program over a set \( A \) of atoms is a finite set of rules.
- A (normal) rule, \( r \), is of the form
  \[
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  \]
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- Notation

  \[
  \begin{align*}
  \text{head}(r) &= a_0 \\
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  \text{body}(r)^+ &= \{a_1, \ldots, a_m\} \\
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  \end{align*}
  \]
Normal logic programs

- A (normal) logic program over a set $\mathcal{A}$ of atoms is a finite set of rules
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$$\text{body}(r)^- = \{a_{m+1}, \ldots, a_n\}$$

- A program is called positive if $\text{body}(r)^- = \emptyset$ for all its rules
We sometimes use the following notation interchangeably in order to stress the respective view:

<table>
<thead>
<tr>
<th>Source Code</th>
<th>Logic Program Formula</th>
<th>Default Negation</th>
<th>Classical Negation</th>
</tr>
</thead>
<tbody>
<tr>
<td>true, false</td>
<td>if, and, or, iff, not</td>
<td>not</td>
<td>¬</td>
</tr>
<tr>
<td>←, →, ;</td>
<td>⊥, ⊤</td>
<td>⊥, ⊤</td>
<td>¬, ¬</td>
</tr>
</tbody>
</table>
Problem solving in ASP: Semantics

Problem

Logic Program

Modeling

Solving

Solution

Interpreting

Stable Models

TU Dresden, 6th May 2016
Formal Definition

Stable models of positive programs

- A set of atoms $X$ is **closed under** a positive program $P$ iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - $X$ corresponds to a model of $P$ (seen as a formula)
Formal Definition

Stable models of positive programs

- A set of atoms $X$ is closed under a positive program $P$ iff for any $r \in P$, $\text{head}(r) \in X$ whenever $\text{body}(r)^+ \subseteq X$.
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- The smallest set of atoms which is closed under a positive program $P$ is denoted by $Cn(P)$.
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of $P$ (ditto).
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- The **smallest** set of atoms which is closed under a positive program $P$ is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of $P$ (ditto)

- The set $Cn(P)$ of atoms is the **stable model** of a positive program $P$
Some “logical” remarks

- Positive rules are also referred to as **definite clauses**
  - Definite clauses are disjunctions with **exactly one** positive atom:
    \[ a_0 \lor \neg a_1 \lor \cdots \lor \neg a_m \]
  - A set of definite clauses has a (unique) smallest model
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• **Horn clauses** are clauses with **at most** one positive atom
  – Every definite clause is a Horn clause but not vice versa
  – Non-definite Horn clauses can be regarded as integrity constraints
  – A set of Horn clauses has a smallest model or none
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• This smallest model is the intended semantics of such sets of clauses
  – Given a positive program \( P \), \( Cn(P) \) corresponds to the smallest model of the set of definite clauses corresponding to \( P \)
Basic idea

Consider the logical formula $\Phi$ and its three (classical) models:

$\{p, q\}, \{q, r\},$ and $\{p, q, r\}$
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$$\Phi \equiv q \land (q \land \neg r \rightarrow p)$$
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$p \leftrightarrow 1$
$q \leftrightarrow 1$
$r \leftrightarrow 0$
Basic idea

Consider the logical formula $\Phi$ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

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Formula $\Phi$ has one stable model, often called answer set:

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Informally, a set $X$ of atoms is a stable model of a logic program $P$ if

- $X$ is a (classical) model of $P$ and
- if all atoms in $X$ are justified by some rule in $P$

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))
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Formal Definition

Stable model of normal programs

- The Gelfond-Lifschitz Reduct [Gelfond and Lifschitz (1991)], $P^X$, of a program $P$ relative to a set $X$ of atoms is defined by

$$P^X = \{ \text{head}(r) \leftarrow \text{body}(r)^+ \mid r \in P \text{ and } \text{body}(r)^- \cap X = \emptyset \}$$
Formal Definition

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- A set $X$ of atoms is a **stable model** of a program $P$, if $Cn(P^X) = X$

- Note: $Cn(P^X)$ is the $\subseteq$—smallest (classical) model of $P^X$
- Note: Every atom in $X$ is justified by an “applying rule from $P$”
A closer look at $P^X$

- In other words, given a set $X$ of atoms from $P$, $P^X$ is obtained from $P$ by deleting:
  1. each rule having $\text{not } a$ in its body with $a \in X$ and then
  2. all negative atoms of the form $\text{not } a$ in the bodies of the remaining rules
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- Note: Only negative body literals are evaluated w.r.t. $X$
A first example

\[ P = \{ p \leftarrow p, \ q \leftarrow \text{not } p \} \]
A first example

\[ P = \{p \leftarrow p, \ q \leftarrow \text{not } p\} \]

<table>
<thead>
<tr>
<th>(X)</th>
<th>(\text{Cn}(P^X))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\emptyset)</td>
<td></td>
</tr>
<tr>
<td>(\{p})</td>
<td></td>
</tr>
<tr>
<td>(\{q})</td>
<td></td>
</tr>
<tr>
<td>(\{p, q})</td>
<td></td>
</tr>
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</table>
**A first example**

$P = \{ p \leftarrow p, \ q \leftarrow not\ p \}$

<table>
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<tr>
<th>$X$</th>
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<tbody>
<tr>
<td>$\emptyset$</td>
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<tr>
<td></td>
<td>$q \leftarrow$</td>
<td></td>
</tr>
<tr>
<td>${p}$</td>
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<td>$\emptyset$</td>
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<td></td>
<td>$q \leftarrow$</td>
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</tr>
<tr>
<td></td>
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<td></td>
</tr>
<tr>
<td>${p, q}$</td>
<td>$p \leftarrow p$</td>
<td>$\emptyset$</td>
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A first example

\[ P = \{ p \leftarrow p, \ q \leftarrow \text{not } p \} \]

<table>
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<tbody>
<tr>
<td>( \emptyset )</td>
<td>( p \leftarrow p ) \qquad q \leftarrow )</td>
<td>{q} \quad \xmark</td>
</tr>
<tr>
<td>{p}</td>
<td>( p \leftarrow p ) \qquad q \leftarrow )</td>
<td>\emptyset</td>
</tr>
<tr>
<td>{q}</td>
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<tr>
<td></td>
<td>(q \leftarrow)</td>
<td>(\times)</td>
</tr>
<tr>
<td>({p})</td>
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<tr>
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TU Dresden, 6th May 2016
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Some properties

- A logic program may have zero, one, or multiple stable models!
Some properties

- A logic program may have zero, one, or multiple stable models!
- If $X$ is a stable model of a logic program $P$, then $X$ is a model of $P$ (seen as a formula)
- If $X$ and $Y$ are stable models of a normal program $P$, then $X \not\subset Y$
Programs with Variables

Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$
Programs with Variables

Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of variable-free terms (also called Herbrand universe)
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$ (also called alphabet or Herbrand base)
Programs with Variables

Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- **Ground Instances** of $r \in P$: Set of variable-free rules obtained by replacing all variables in $r$ by elements from $\mathcal{T}$:

  $$\text{ground}(r) = \{r\theta \mid \theta : \text{var}(r) \rightarrow \mathcal{T}, \text{var}(r\theta) = \emptyset\}$$

  where $\text{var}(r)$ stands for the set of all variables occurring in $r$; $\theta$ is a (ground) substitution
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- Ground Instantiation of $P$:  
  $$
ground(P) = \bigcup_{r \in P} \text{ground}(r)$$
An example

\[ P = \{ \text{r}(a, b) \leftarrow, \text{r}(b, c) \leftarrow, t(X, Y) \leftarrow r(X, Y) \} \]
\[ \mathcal{T} = \{a, b, c\} \]
\[ \mathcal{A} = \{ \text{r}(a, a), \text{r}(a, b), \text{r}(a, c), \text{r}(b, a), \text{r}(b, b), \text{r}(b, c), \text{r}(c, a), \text{r}(c, b), \text{r}(c, c), t(a, a), t(a, b), t(a, c), t(b, a), t(b, b), t(b, c), t(c, a), t(c, b), t(c, c) \} \]
An example

\[ P = \{ r(a, b) \leftarrow, \ r(b, c) \leftarrow, \ t(X, Y) \leftarrow r(X, Y) \} \]

\[ T = \{ a, b, c \} \]

\[ A = \{ r(a, a), r(a, b), r(a, c), r(b, a), r(b, b), r(b, c), r(c, a), r(c, b), r(c, c), \]
\[ t(a, a), t(a, b), t(a, c), t(b, a), t(b, b), t(b, c), t(c, a), t(c, b), t(c, c) \} \]

\[ \text{ground}(P) = \begin{cases} 
\{ r(a, b) \leftarrow, \\
r(b, c) \leftarrow, \\
t(a, a) \leftarrow r(a, a), t(b, a) \leftarrow r(b, a), t(c, a) \leftarrow r(c, a), \\
t(a, b) \leftarrow r(a, b), t(b, b) \leftarrow r(b, b), t(c, b) \leftarrow r(c, b), \\
t(a, c) \leftarrow r(a, c), t(b, c) \leftarrow r(b, c), t(c, c) \leftarrow r(c, c) \} 
\end{cases} \]
An example

\[ P = \{ r(a, b) \leftarrow, r(b, c) \leftarrow, t(X, Y) \leftarrow r(X, Y) \} \]
\[ \mathcal{T} = \{a, b, c\} \]
\[ \mathcal{A} = \left\{ r(a, a), r(a, b), r(a, c), r(b, a), r(b, b), r(b, c), r(c, a), r(c, b), r(c, c), t(a, a), t(a, b), t(a, c), t(b, a), t(b, b), t(b, c), t(c, a), t(c, b), t(c, c) \right\} \]

\[ \text{ground}(P) = \left\{ \begin{array}{c}
  r(a, b) \leftarrow, \\
  r(b, c) \leftarrow, \\
  t(a, b) \leftarrow, \\
  t(b, c) \leftarrow
\end{array} \right\} \]

- Intelligent Grounding aims at reducing the ground instantiation
Stable models of programs with Variables

Let $P$ be a normal logic program with variables
Stable models of programs with Variables

Let $P$ be a normal logic program with variables

- A set $X$ of (ground) atoms is a stable model of $P$, if $Cn(\text{ground}(P)^X) = X$
Problem solving in ASP: Extended Syntax

- Problem
- Logic Program
- Stable Models

Modeling → Logic Program → Solving → Stable Models

Solution

Interpreting
Language Constructs

- **Variables (over the Herbrand Universe)**
  - \( p(X) :- q(X) \) over constants \{ a, b, c \} stands for \( p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \)

- **Conditional Literals**
  - \( p :- q(X) : r(X) \) given \( r(a), r(b), r(c) \) stands for \( p :- q(a), q(b), q(c) \)

- **Disjunction**
  - \( p(X) | q(X) :- r(X) \)

- **Integrity Constraints**
  - \( :- q(X), p(X) \)

- **Choice**
  - \( 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \)

- **Aggregates**
  - \( s(Y) :- r(Y), 2 \#count \{ p(X,Y) : q(X) \} 7 \)
  - also: \#sum, \#avg, \#min, \#max, \#even, \#odd
Language Constructs

• Variables (over the Herbrand Universe)
  – \( p(X) :- q(X) \) over constants \( \{a, b, c\} \) stands for
    \[
    p(a) :- q(a), \quad p(b) :- q(b), \quad p(c) :- q(c)
    \]

  • Conditional Literals
    \( p :- q(X) : r(X) \) given \( r(a), r(b), r(c) \) stands for
    \[
    p :- q(a), q(b), q(c)
    \]

  • Disjunction
    \( p(X) | q(X) :- r(X) \)

• Integrity Constraints
  \( :- q(X), p(X) \)

• Choice
  \( 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \)

• Aggregates
  \( s(Y) :- r(Y), \#\text{count} \{ p(X,Y) : q(X) \} \)
  – also: \#\text{sum}, \#\text{avg}, \#\text{min}, \#\text{max}, \#\text{even}, \#\text{odd}
Language Constructs

- **Conditional Literals**
  - \( p :\quad q(X) : r(X) \quad \text{given} \quad r(a), r(b), r(c) \) stands for
  \( p :\quad q(a), q(b), q(c) \)
Language Constructs

- **Disjunction**
  
  \[
  \neg p(X) \lor q(X) :\neg r(X)
  \]
Language Constructs

- Variables (over the Herbrand Universe)
  - \( p(X) :- q(X) \) over constants \( \{ a, b, c \} \) stands for \( p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \)

- Conditional Literals
  - \( p :- q(X) : r(X) \) given \( r(a), r(b), r(c) \) stands for \( p :- q(a), q(b), q(c) \)

- Disjunction
  - \( p(X) | q(X) :- r(X) \)

- Integrity Constraints
  - :\( \neg \) q(X), p(X)

- Choice
  - \( 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \)

- Aggregates
  - \( s(Y) :- r(Y), \#count \{ p(X,Y) : q(X) \} \)
  - also: \#sum, \#avg, \#min, \#max, \#even, \#odd
Language Constructs

- **Variables** (over the Herbrand Universe)
  \[ p(X) :- q(X) \]
  over constants \{ a, b, c \} stands for
  \[ p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \]

- **Conditional Literals**
  \[ p :- q(X) : r(X) \]
given \[ r(a), r(b), r(c) \] stands for
  \[ p :- q(a), q(b), q(c) \]

- **Disjunction**
  \[ p(X) \lor q(X) :- r(X) \]

- **Integrity Constraints**
  \[ :- q(X), p(X) \]

- **Choice**
  \[ 2 \{ p(X, Y) : q(X) \} 7 :- r(Y) \]

- **Aggregates**
  \[ s(Y) :- r(Y), \#count \{ p(X, Y) : q(X) \} \]
  also: \#sum, \#avg, \#min, \#max, \#even, \#odd
Language Constructs

- **Variables (over the Herbrand Universe)**
  
  \[ p(X) :- q(X) \]
  
  stands for
  
  \[ p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \]

- **Conditional Literals**
  
  \[ p :- q(X) : r(X) \]
  
  given
  
  \[ r(a), r(b), r(c) \]
  
  stands for
  
  \[ p :- q(a), q(b), q(c) \]

- **Disjunction**
  
  \[ p(X) | q(X) :- r(X) \]

- **Integrity Constraints**
  
  \[ :- q(X), p(X) \]

- **Choice**
  
  \[ 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \]

- **Aggregates**
  
  - \[ s(Y) :- r(Y), 2 \text{#count} \{ p(X,Y) : q(X) \} 7 \]
  
  - also: \#sum, \#avg, \#min, \#max, \#even, \#odd

TU Dresden, 6th May 2016
Language Constructs

- **Variables** (over the Herbrand Universe)
  
  - $p(X) :- q(X)$ \textit{over constants \{a, b, c\} stands for}
  
  $p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)$

- **Conditional Literals**
  
  - $p :- q(X) : r(X)$ \textit{given \{r(a), r(b), r(c)\} stands for}$
  
  $p :- q(a), q(b), q(c)$

- **Integrity Constraints**
  
  - $:- q(X), p(X)$

- **Choice**
  
  - $2 \{ p(X, Y) : q(X) \} 7 :- r(Y)$

- **Aggregates**
  
  - $s(Y) :- r(Y), 2 \#count \{ p(X, Y) : q(X) \} 7$
  
  - \textbf{also:} \#sum, \#avg, \#min, \#max, \#even, \#odd
Modeling

• For solving a problem class $\mathbf{C}$ for a problem instance $\mathbf{I}$, encode

1. the problem instance $\mathbf{I}$ as a set $P_I$ of facts and
2. the problem class $\mathbf{C}$ as a set $P_C$ of rules

such that the solutions to $\mathbf{C}$ for $\mathbf{I}$ can be (polynomially) extracted from the stable models of $P_I \cup P_C$
Modeling

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such that the solutions to $\mathcal{C}$ for $\mathcal{I}$ can be (polynomially) extracted from the stable models of $P_I \cup P_C$

• $P_I$ is (still) called problem instance
• $P_C$ is often called the problem encoding
Modeling

• For solving a problem class $\mathbf{C}$ for a problem instance $\mathbf{I}$, encode
  1. the problem instance $\mathbf{I}$ as a set $P_I$ of facts and
  2. the problem class $\mathbf{C}$ as a set $P_C$ of rules

such that the solutions to $\mathbf{C}$ for $\mathbf{I}$ can be (polynomially) extracted from the stable models of $P_I \cup P_C$

• $P_I$ is (still) called problem instance
• $P_C$ is often called the problem encoding

• An encoding $P_C$ is uniform, if it can be used to solve all its problem instances
  That is, $P_C$ encodes the solutions to $\mathbf{C}$ for any set $P_I$ of facts
Example 3-Colorability

- Vertices are represented with predicates node(X);
- Edges are represented with predicates edge(X, Y).

Question: Is there a valid assignment of three colors for an input graph $G$ such that no two adjacent vertices have the same color?
Graph coloring

\texttt{node(1..6).}
Graph coloring

node(1..6).

edge(1, 2). edge(1, 3). edge(1, 4).
edge(2, 4). edge(2, 5). edge(2, 6).
edge(3, 1). edge(3, 4). edge(3, 5).
edge(4, 1). edge(4, 2).
edge(5, 3). edge(5, 4). edge(5, 6).
edge(6, 2). edge(6, 3). edge(6, 5).
Graph coloring

node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).
Graph coloring

node(1..6).

edge(1, 2). edge(1, 3). edge(1, 4).
edge(2, 4). edge(2, 5). edge(2, 6).
edge(3, 1). edge(3, 4). edge(3, 5).
edge(4, 1). edge(4, 2).
edge(5, 3). edge(5, 4). edge(5, 6).
edge(6, 2). edge(6, 3). edge(6, 5).

col(r). col(b). col(g).
Graph coloring

node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 { color(X,C) : col(C) } 1 :- node(X).
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
Graph coloring

node(1..6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

Problem encoding

\{ color(X,C) : col(C) \} 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
Graph coloring

node(1..6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

1 { color(X,C) : col(C) } 1 :- node(X).
:- edge(X,Y), color(X,C), color(Y,C).
color.lp

node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
ASP solving process

Modeling

Problem

Logic Program

Grounder

Solver

Stable Models

Solution

Interpreting

Solving
Graph coloring: Grounding

$ gringo --text color.lp
Graph coloring: Grounding

$ gringo --text color.lp

node(1). node(2). node(3). node(4). node(5). node(6).

edge(1,2). edge(1,3). edge(1,4). edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5). edge(4,1). edge(4,2). edge(5,3).
edge(5,4). edge(5,6). edge(6,2). edge(6,3). edge(5,6).

col(r). col(b). col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

:- color(1, r), color(2, r). :- color(2, g), color(5, g). ... :- color(6, r), color(2, r).
:- color(1, b), color(2, b). :- color(2, r), color(6, r). :- color(6, g), color(2, g).
:- color(1, g), color(2, g). :- color(2, b), color(6, b). :- color(6, r), color(3, r).
:- color(1, r), color(3, r). :- color(2, b), color(6, g). :- color(6, b), color(3, b).
:- color(1, b), color(3, b). :- color(3, r), color(1, r). :- color(6, b), color(3, b).
:- color(1, g), color(3, g). :- color(3, b), color(1, b). :- color(6, g), color(3, g).
:- color(1, r), color(4, r). :- color(3, g), color(1, g). :- color(6, r), color(5, r).
:- color(1, b), color(4, b). :- color(3, r), color(4, r). :- color(6, b), color(5, b).
:- color(1, g), color(4, g). :- color(3, b), color(4, b). :- color(6, g), color(5, g).
:- color(2, r), color(4, r). :- color(3, g), color(4, g).
:- color(2, b), color(4, b). :- color(3, r), color(5, r).
:- color(2, g), color(4, g). :- color(3, b), color(5, b).
:- color(2, r), color(5, r). :- color(3, g), color(5, g).
:- color(2, b), color(5, b). :- color(4, g), color(1, r).

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Graph coloring: Solving

$ gringo color.lp | clasp 0
Graph coloring: Solving

$ \text{gringo color.lp | clasp 0}$

classp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models : 6
Time : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time : 0.000s
Problem solving in ASP: Reasoning Modes

Problem

Modeling

Logic Program

Solving

Solution

Interpreting

Stable Models
Reasoning Modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization

- and combinations of them

† without solution recording
‡ without solution enumeration
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