

Knowledge Graphs

Lecture 1: Introduction

Markus Krötzsch

Knowledge-Based Systems

TU Dresden, 14 Oct 2025

More recent versions of this slide deck might be available.
For the most current version of this course, see
https://iccl.inf.tu-dresden.de/web/Knowledge_Graphs/en

Introduction and Organisation

Course Tutors



Markus Krötzsch
Lectures



Maximilian Marx
Exercises

Organisation

- **Lectures**

Tuesday, DS 2 (9:20–10:50), BAR 106

Videos are available online:

<https://youtube.com/channel/UCCvDWNsR8YlQrB1tSj9Xqsw>

(see playlist “Knowledge Graphs”)

- **Exercise Sessions**

Wednesday, DS 4 (13:00–14:30), N63 A001

- **Web Page**

[https://iccl.inf.tu-dresden.de/web/Knowledge_Graphs_\(WS2025/26\)](https://iccl.inf.tu-dresden.de/web/Knowledge_Graphs_(WS2025/26))

- **Lecture Notes**

Slides of current and past lectures will be online.

- **Modules:** INF-25-MA-FTK-KM, INF-B-510, INF-B-520, INF-BAS2, INF-VERT2, INF-BAS6, INF-VERT6, INF-E-3, INF-PM-FOR, INF-PM-ANW, MCL-KR, MCL-TCSL, CMS-COR-KM

How exercises work

Exercise classes are **interactive** events:

- Tasks are published online
- Students solve the tasks before the date of the class
- In the meeting, students ask questions and present their solutions
- The tutor answers questions and helps with problems

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Corollary: You must do your homework to benefit from the classes.

How the examination works

The details of the examination depend on your module:

- Most modules: oral examination, sometimes in combination with other subjects
- For students of Computational Modelling and Simulation in module CMS-LM-COR:
 - written (60min) if more than 10 examinees
 - oral (20min) otherwise

All examinations are “closed book” (no auxiliary materials allowed)

How to be prepared for examinations

Here are some hints that can help you to succeed in the examination:

- Follow the course closely during the semester
- Do all your homework
- When watching videos or attending classes: take hand-written notes
- Be prepared to reproduce most of the material that is on the slides when asked
- Be prepared to solve tasks that are like those in the exercises
- Expect that you will have to write answers on paper (also during oral exams)

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Main conclusion: You need to be able to produce knowledge and solutions **actively** rather than merely understanding them **passively**.

Goals and Prerequisites

Goals

- Introduce basic notions of **graph-based knowledge representation(s)**
- Study important **graph data management approaches** and **query languages** (RDF, SPARQL)
- Learn about relevant **methods, tools, and datasets**
- Discuss aspects of **ontological modelling and quality assurance**
- Get to know some **methods for analysing networks and graphs**

(Non-)Prerequisites

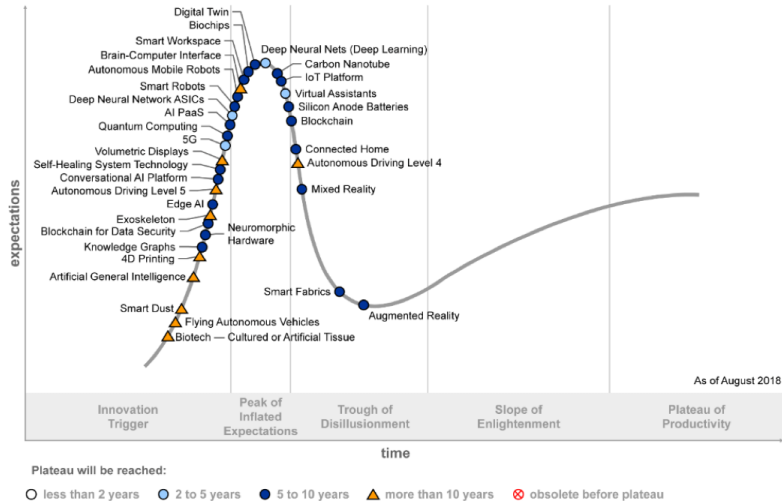
- No particular prior courses needed
- Basic programming skills are assumed; practical experience beyond basic courses will be helpful
- Interesting optional synergies: databases, machine learning, social networks, graph theory

Course Outline

- **Resource Description Framework (RDF)**
Underlying graph model; URIs; syntax
- **SPARQL**
Query features; syntax and semantics; expressive power and complexity
- **Wikidata**
Data model; applications; aspects of modelling; query answering
- **Datalog and rules**
Datalog; negation; aggregation; existential rules
- **Ontologies for knowledge graphs**
OWL, ontology-based data access
- **RDF constraint languages**
SHACL & ShEX; syntax and semantics; complexity and implementation

Motivation

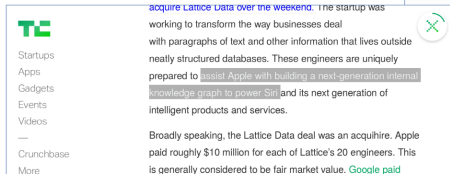
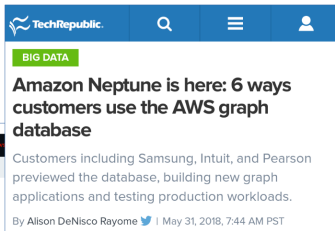
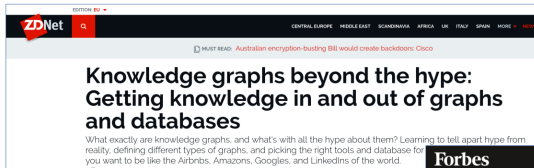
The Hype



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Knowledge Graphs Everywhere



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Knowledge Graphs in the Age of Large Language Models?

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Let's ask ChatGPT (version "mini4o")!

Where was Markus Krötzsch born?

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I don't have direct access to external sources to provide citations, but you can find information about Markus Krötzsch and his background on academic websites, his personal or professional pages, or in interviews. Searching for him on platforms like Google Scholar or his university's website may yield the details you need. If you want, I can help guide you on how to

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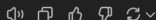
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Searched 6 sites ▾

Markus Krötzsch was born in Dresden, Germany. He is a professor at the Technical University of Dresden, where he leads the chair for Knowledge-Based Systems. His research has significantly impacted fields like semantic technologies and ontology languages. Krötzsch also played a key role in the development of Wikidata and the Semantic MediaWiki

COMPLAGIC CENTER

NEWS






Knowledge Graphs in the Age of Large Language Models?

Ok, let's check Google.

Google

Where was Markus Krötzsch born?

×



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 Wikipedia
https://de.wikipedia.org/wiki/Markus_Krötzsch · [Translate this page](#) · [More](#)



Markus Krötzsch


Markus Krötzsch (* 6. Mai 1981 in **Rodewisch**) ist ein deutscher Informatiker. Markus Krötzsch 2014. Krötzsch studierte nach dem Abitur in Auerbach im ...

Missing: **born?** | Show results with: **born?**

 TU Dresden
https://tu-dresden.de/informatik/die-fakultaet/news/Markus_Krötzsch_became_the_new_Professor_for_... · [More](#)

Markus Krötzsch became the new Professor for " ...

1 Aug 2016 — **He was born in Vogtland**, had studied Computer Science in Dresden and did his PhD at the Institute of Technology in Karlsruhe (KIT) about ...

 Wikidata ID
https://www.wikidata.org/wiki/Markus_Krötzsch · [More](#)

Markus Krötzsch

Markus Krötzsch. German computer scientist. Markus Kroetzsch. In more languages ... date of birth. **6 May 1981**. 1 reference. reference URL · <http://www.dfg.de/> ...

The Knowledge Graph at Google

More popular entities get richer information displays, fed directly from Google's KG:

Google

Where was Olaf Scholz born?

×

All

Images

Maps

News

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Web

Books


⋮ More

Tools

Olaf Scholz / Place of birth

Osnabrück, Germany

Scholz was born on 14 June 1958, in Osnabrück, Lower Saxony, and grew up in Hamburg's Rahlstedt district. His parents worked in the textile industry.




Wikipedia


https://en.wikipedia.org/wiki/Olaf_Scholz

Olaf Scholz - Wikipedia


People also search for




Oldenbu




Osnabri




Münster




Bielefeld



Bremen




Hanover



Ibbenbü

Feedback



Osnabrück

City in Germany ⌵

Osnabrück is a city in northwest Germany. The Town Hall is where the 1648 Peace of Westphalia was negotiated, bringing the 30 Years' War to an end. It sits on the market square, along with gabled houses and St. Marien, a 13th-century Gothic church. The Felix Nussbaum House shows a large collection of works by the local surrealist painter. To the south, the grounds of Osnabrück Castle are a venue for summer concerts. — Google

Population: 164,748 (2019) Eurostat

Area: 119.8 km²

Elevation: 64 m

District: Urban district

Postal codes: 49074–49090

More about Osnabrück →

Feedback

What is a Knowledge Graph?

What is a Knowledge Graph?

The original “Knowledge Graph” (Google, 2012):

The screenshot shows the Google Inside Search interface. At the top is the "Google Inside Search" header. Below it is a navigation bar with links: Home, How Search Works, Tips & Tricks, **Features**, Search Stories, Playground, Blog, and Help. The main content area features a dark background with a network of nodes and connecting lines, representing the Knowledge Graph. Nodes include portraits of historical figures like Leonardo da Vinci and Michelangelo, as well as various icons like a Canadian flag and a globe. A large blue circle highlights a portrait of Leonardo da Vinci. To the right, a search result for "Leonardo da Vinci" is displayed, showing a portrait and a list of facts: "Leonardo di ser Piero da Vinci was an Italian Renaissance polymath: painter, sculptor, architect, musician, scientist, mathematician, engineer, inventor, anatomist, geologist, cartographer, botanist, and writer. wikipedia". Below this, a list of related items is shown: "Born: April 15, 1452, Anchiano", "Died: May 2, 1519, Clos Lucé", "Buried: Chateau d'Amboise", "Parents: Caterina da Vinci, Piero da Vinci", and "Structures: Volpam Band Da Vinci Project". A blue button with a right arrow is visible on the left side of the main content area. The text "The Knowledge Graph" is prominently displayed in the center, with the subtitle "Learn more about one of the key breakthroughs behind the future of search." below it. To the right, the text "See it in action" is displayed, with the subtitle "Discover answers to questions you never thought to ask, and explore collections and lists." below it.

Google Inside Search

Home How Search Works Tips & Tricks **Features** Search Stories Playground Blog Help

The Knowledge Graph

Learn more about one of the key breakthroughs behind the future of search.

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(c) Google. All rights reserved.

Many knowledge graphs, many technologies

There are a number of widely used publicly available knowledge graphs:



... and a variety of technologies for working with them:



So what is a Knowledge Graph?

A first attempt at a definition:

A Knowledge Graph is a knowledge base that is a graph.

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So what is a knowledge base?

- “A knowledge base is a technology used to store complex **structured and unstructured information** used by a computer system. [...] [It] represents **facts about the world**” – Wikipedia (26 Oct 2020, id 983269427)
- “A **collection of knowledge** expressed using some **formal knowledge representation language**.” – Free Online Dictionary of Computing, 15 Oct 2018
- 1. a **store of information or data** that is available to draw on.
 2. the underlying **set of facts, assumptions, and rules** which a computer system has available to solve a problem.– Lexico (Oxford University Press/Dictionary.com), 26 Oct 2020

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So what is a graph?

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So what is a graph?

- “a **collection of points and lines** connecting some (possibly empty) subset of them”
– Wolfram MathWorld, 26 Oct 2020
- “a **collection of vertices and edges** that join pairs of vertices” – Merriam-Webster, 26 Oct 2020
- “a structure amounting to a **set of objects** in which some pairs of the objects are **in some sense ‘related’**.” – Wikipedia (26 Oct 2020, id 984093316)

(we'll have more to say about mathematical graphs later)

So what is a Knowledge Graph?

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In summary:

So what is a Knowledge Graph?

A first attempt at a definition:

A Knowledge Graph is a knowledge base that is a graph.

In summary:

- a collection of facts, rules, or other forms of knowledge
- that express some kind of relationships or connections

↪ a paradigm rather than a specific class of things

What is special about Knowledge Graphs?

A second attempt at a definition:

A Knowledge Graph is a data set that is:

- **structured** (in the form of a specific data structure)
- **normalised** (consisting of small units, such as vertices and edges)
- **connected** (defined by the – possibly distant – connections between objects)

Moreover, knowledge graphs are typically:

- **explicit** (created purposefully with an intended meaning)
- **declarative** (meaningful in itself, independent of a particular implementation or algorithm)
- **annotated** (enriched with contextual information to record additional details and meta-data)
- **non-hierarchical** (more than just a tree-structure)
- **large** (millions rather than hundreds of elements)

(Counter-)Examples

Typical knowledge graphs:

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Typical knowledge graphs:

- Wikidata, Yago, Freebase, DBpedia (though hardly annotated)
- OpenStreetMap
- Google Knowledge Graph, Microsoft Bing Satori (presumably; we can't really know)

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- Document stores (Lucene, MongoDB, etc.): structured, but not normalised; connections sub-ordinary

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Primarily not knowledge graphs:

- Wikipedia: mostly unstructured text; not normalised; connections (links) important but sub-ordinary (similar: [The Web](#))
- Relational database of some company: structured and possibly normalised, but no focus on connections (traditional RDBMS support connectivity queries only poorly)

Graphs in Computer Science and Mathematics

What is a graph?

Definition 1.1: A **simple undirected graph** G consists of a set V of **vertices** and a set E of **edges**, where each edge is a set of two vertices. Two vertices $v_1, v_2 \in V$ are **adjacent** (in G) if there is an edge $\{v_1, v_2\} \in E$.

Vertices are sometimes also called **nodes**; undirected edges are sometimes also called **arcs**.

Unless otherwise noted, we assume all graphs to be finite.

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Discrete mathematics considers a variety of other kinds of “graphs”:

- Directed or undirected
- Simple graph or multi-graph
- Possibly labelled edges or vertices
- Possibly with self-loops
- Possibly with higher arity edges (hypergraphs)

Directed and other graphs

Definition 1.2: A simple directed graph (a.k.a. simple digraph) G consists of a set V of vertices and a set $E \subseteq V \times V$ of (directed) edges from a source vertex to a target vertex.

Other terms are similar to undirected graphs; directed edges are also known as arrows and are often denoted as such, e.g., $v_1 \xrightarrow{e_1} v_2$.

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Other terms are similar to undirected graphs; directed edges are also known as **arrows** and are often denoted as such, e.g., $v_1 \xrightarrow{e_1} v_2$.

Definition 1.3: The following generalisations apply to directed and to undirected graphs.

- A **graph with self-loops** is a graph extended with the option of having edges that relate a vertex to itself.
- A **multi-graph** is a graph that may have multiple edges with the same vertices (in the same direction).
- An **edge-labelled graph** is a graph that has an additional labelling function $\lambda : E \rightarrow L$ that maps each edge in E to an element from a set of labels L (similarly for vertex-labelled graphs).

Other basic notions

Definition 1.4: An edge is said to be **incidental** to the vertices it connects. The **degree** of a vertex is the number of edges that are incidental to it. In a digraph, the **in-degree** of a vertex is the number of edges pointing towards it; analogously for **out-degree**.

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Definition 1.5: A **directed path** in a digraph is a sequence of consecutive edges $v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} \dots \xrightarrow{e_n} v_n$. An **undirected path** is a sequence of edges that may point either way (or that are simply undirected).

A **simple path** (directed or undirected) is a path without repeated vertices other than possibly the first and last node.

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Definition 1.6: Two vertices are **connected** if there is an undirected path from one to the other. A graph is connected if any pair of two distinct vertices is connected. A digraph is **strongly connected** if there is a directed path from any vertex to any other vertex (hence: one directed path in either direction).

Representing graphs (1)

There are several obvious ways of representing graphs in computer science.

Definition 1.7: The **adjacency matrix** of a graph $G = \langle V, E \rangle$ is the boolean $|V| \times |V|$ matrix that contains, at any coordinate $\langle v_1, v_2 \rangle$, the value **1** if there is an edge connecting v_1 and v_2 .

Notes:

- Adjacency matrices for undirected graphs are symmetric.
- Loops (if allowed) show up as **1** in the diagonal.
- The matrix could be adapted to multi-graphs by storing the numbers of edges.
- The matrix could be adapted to labelled simple graphs by storing the labels.

Representing graphs (2)

There are several obvious ways of representing graphs in computer science.

Definition 1.8: The **adjacency list** of a graph $G = \langle V, E \rangle$ is the list of all of its edges.

Notes:

- We can write edges as pairs (order is irrelevant for undirected graphs).
- Loops (if allowed) show up as edges with repeated vertices.
- The list could be adapted to multi-graphs by adding the number of edges to each line, or by allowing repeated lines.
- The list could be adapted to labelled graphs by adding labels to each line (for multi-graph: repeat lines rather than also storing number).
- The list does not encode V : vertices without edges are missing (might be listed separately if relevant to application)

Representing graphs (3)

There are also other options.

Example 1.9: We could also encode the adjacency matrix by giving, for each row, a list of all vertices whose column is set to **1**. This is equivalent to ordering edges by first vertex and combining them into a single line.

Which graph representation to pick?

Each representation has its pros and cons:

- **Matrix:**
 - + space efficient for dense graphs (1 bit per edge);
 - + can be processed with matrix operations (highly parallel);
 - space inefficient for sparse graphs;
 - not natural for labelled multi-graphs
- **List:**
 - + space efficient for sparse graphs;
 - + easy to use for labelled multi-graphs;
 - harder to process (esp. if edge order can be random);
 - not space efficient for dense graphs

Note: Knowledge graphs are typically sparse and labelled, but parallel processing still makes matrices attractive in some applications.

Summary

The course will be 50% lectures and 50% tutorial classes

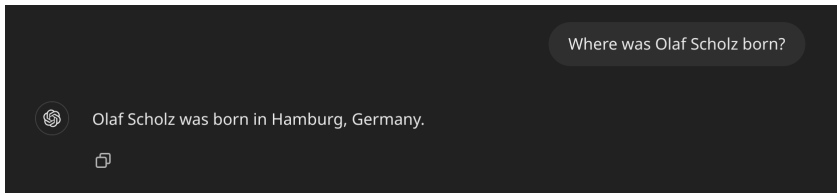
Active participation is important to succeed

Knowledge Graphs are a data management concept of practical importance

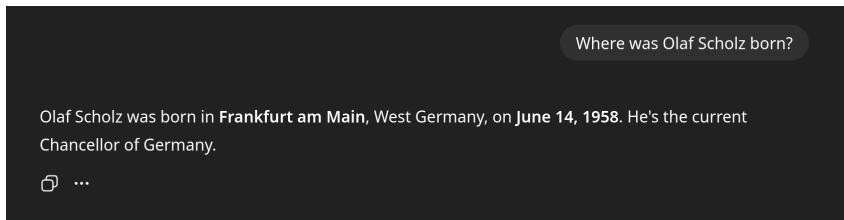
Mathematics studies various types of graphs, which can be represented by several common data structures

What's next?

- Encoding graphs in technical systems: the Resource Description Format RDF
- Formats for exchanging RDF
- Modelling knowledge in RDF graphs



(ChatGPT mini4o, 15 Oct 2024)



(ChatGPT (mini?), 14 Oct 2025)