Syllogistic Reasoning in Seven Spaces

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Hochschule Harz
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Research Question:
How does logical thinking work?

Syllogistic argumentation can be traced back to Aristotle.

A classical syllogism consists of two quantified assertions about categories, e.g. A, B, and C, like

- ‘Some A are B’ and
- ‘No B are C’

connecting exactly two categories each time.

What are the logical consequences from these assertions?

- ‘Some A are not C’
Example 1 – with concrete categories [3]

Some artists are bakers.
All bakers are chemists.
∴ Some artists are chemists.

- Assertions can be expressed by the logic of monadic assertions, i.e. subset of first-order logic with predicates having exactly one argument:

  ‘All A are B’  \( \approx \forall x (A(x) \rightarrow B(x)) \)

- Machines may employ classical first-order logic, but what do human reasoners do?
- They maybe consider several cases – in seven spaces.
- Human rationality is bounded. Reasoning may interfere with concrete knowledge (cf. Wason selection task [7]).
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Although people may not think formally, we define notions precisely, laying a clear foundation for modeling and implementation.

- A **category** $C$ stands for a set of individuals or objects.
- A classical syllogism combines in total three categories: $A$, $B$, and $C$.
- A **space** $S$ is one of the seven possible subsets of $A \cup B \cup C$ in the set diagram for the three involved categories.
- The set of all such spaces is:

$$M = \{A \cap \overline{B} \cap \overline{C}, \overline{A} \cap B \cap \overline{C}, A \cap B \cap \overline{C}, \overline{A} \cap \overline{B} \cap C, A \cap \overline{B} \cap C, \overline{A} \cap B \cap C, A \cap B \cap C\}$$
Assertions and Moods

- An **assertion** for two categories $A$ and $B$ is a statement of the form ‘A certain quantity of $A$ is (not) $B$’.
- They are combination of **quantifier** ‘All’ or ‘Some’ and optional **negation** of the second category.
- This leads to four so-called classical **moods**.

| Mood | Assertion | Conditional Probability $P(B|A)$ |
|------|-----------|----------------------------------|
| A    | ‘All $A$ are $B$’ | $P(B|A) = 1 \implies P(A \cap \overline{B}) = 0$ |
| E    | ‘All $A$ are not $B$’ | $P(\overline{B}|A) = 1 \implies P(A \cap B) = 0$ |
| I    | ‘Some $A$ are $B$’ | $P(B|A) > 0 \implies P(A \cap B) > 0$ |
| O    | ‘Some $A$ are not $B$’ | $P(\overline{B}|A) > 0 \implies P(A \cap \overline{B}) > 0$ |

- **Mood U** ‘Some but not all’ ($I \land O$) more specific than $I$:
  1. For non-empty categories, $A$ entails $I$, but not $U$.
  2. Order of categories does not matter for $I$, but for $U$.
Syllogism and Figures

- Three assertions $\varphi_1$, $\varphi_2$ (premises), and $\varphi_3$ (conclusion) form a syllogism:

$$\varphi_1 \land \varphi_2 \rightarrow \varphi_3$$

- In each assertion, categories may occur in two orders (leading to different figures).

<table>
<thead>
<tr>
<th>Figures – orders (only) of the premises</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assertion</strong></td>
</tr>
<tr>
<td>$\varphi_1$</td>
</tr>
<tr>
<td>$\varphi_2$</td>
</tr>
</tbody>
</table>

- For $n$ moods, there are $N = (n \cdot 2)^3$ syllogisms:

<table>
<thead>
<tr>
<th>$n$</th>
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<tr>
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</tr>
<tr>
<td>5</td>
<td>1000</td>
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</table>

(classical moods) (with mood $\mathbf{U}$)
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Related Works

- **Common result**: Conclusions humans draw differ from those of classical first-order logical reasoning $\leadsto$ many theories.

- **Heuristic atmosphere theory** [9]: People may be predisposed to accept a conclusion that fits the mood of the premises.

- **Set-theoretic approaches** including fuzzy set theory [4]: Fuzzy existential quantifiers cover the range from ‘Some’ to ‘All’ in several steps.

- **Mental model theory** [2] postulates: Human reasoners can represent a set iconically and build a mental model of its members.

- **Remark**: No single theory may provide an adequate account, because different persons apply very distinct reasoning strategies.
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Assumptions

Human reasoners adopt assumptions, possibly not consciously.

1. If a space $S$ is non-empty, it has a **non-zero** probability, i.e. $P(S) > 0$ for $S \neq \emptyset$. In the following, we always adopt this assumption.

2. Quantification with ‘Some’ (moods I and O) induces the existence of individuals or objects of the respective category. Furthermore it can be understood **non-inclusive**, i.e. as ‘Some but not all’ (mood U).

3. All categories are **non-empty**, i.e. $A, B, C \neq \emptyset$ and thus $P(A) > 0$, $P(B) > 0$, and $P(C) > 0$ (because of Assumption 1).

4. All categories are **non-equivalent**, i.e. $A \neq B \neq C \neq A$. Each of these three inequalities can be interpreted probabilistically, e.g. $A \neq B$ by $P(A \setminus B \cup B \setminus A) > 0$, because $A \neq B$ is equivalent to $A \nsubseteq B$ or $B \nsubseteq A$ and hence to $A \setminus B \cup B \setminus A \neq \emptyset$. 


Implementation

- Assumptions (except last one) can be traced back to Aristotle’s original works on analytics (*Analytica priora* and *Analytica posteriora*, cf. [1]).
- They allow to model several human reasoning strategies. Hence theory with seven spaces is parametrizable.
- Syllogistic reasoning in seven spaces can be implemented by constraint logic programming in SWI Prolog [8].
- We employ finite domain constraints with binary space variables $S_1, \ldots, S_7$: empty vs. not empty.
- Every assumption and every mood can be expressed by one or more conditions of the form
  - $P(E) = 0$ or
  - $P(E) > 0$
for some event (set of spaces) $E$. 
Details

- **Assumption 1**: space is empty or non-empty:
  \[ [S1, S2, S3, S4, S5, S6, S7] \text{ ins } 0..1 \]

- **Mood A** (similarly E):
  ‘All A are B’ \( \leadsto P(A \cap \overline{B}) = 0 \)
  \( \leadsto A \cap \overline{B} = S_1 \cup S_5 \)
  \( \leadsto S_1 + S_5 \neq 0 \)

- **Mood I** (similarly O):
  ‘Some A are B’ \( \leadsto P(A \cap B) > 0 \)
  \( \leadsto A \cap B = S_3 \cup S_7 \)
  \( \leadsto S_3 + S_7 \gg 0 \)

- **Assumption 3**: category (here C) is non-empty
  \[ C = S_4 \cup S_5 \cup S_6 \cup S_7 \leadsto S_4 + S_5 + S_6 + S_7 \gg 0 \]

- **Assumption 4**: \( A \neq B \) (others analogously)
  \( A \setminus B \cup B \setminus A \neq \emptyset \leadsto A \setminus B = S_1 \cup S_5 \land B \setminus A = S_2 \cup S_6 \)
  \( \leadsto S_1 + S_2 + S_5 + S_6 \gg 0 \)
Example

Example 2 – with abstract categories

All A are B.
All B are C.
∴ All C are A.

- The premises are in mood AA1, conclusion is also in mood A but categories in reverse order.
- The premises induce that $S_1$, $S_2$, $S_3$, and $S_5$ must be empty, remaining three spaces $S_4$, $S_6$, and $S_7$ open.
- Conclusion enforces that $S_4$ and $S_6$ must be empty, too. Thus all spaces are empty except possibly $S_7$.
- The fraction (ratio) of valid cases is only $2/8 = 25\%$.
- If all categories are non-empty (i.e. with assumption 3, then space $S_7$ must be non-empty.
- Hence the syllogism holds in $1/4 = 25\%$ of the cases.
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</table>

*Note: The table above represents the ratios of cases for different syllogistic forms with assumption 3. The numbers in the table indicate the percentage of cases that fall into each category.*
Hypotheses

Hypothesis 1

People draw a specific conclusion the higher the percentage of its computed ratio is according to our model.

- Best (moderate) correlation to empirical results [3, Table 6] without any of the assumptions ($r = 0.482$).
- Model allows more than one conclusion.
- Case of no valid conclusions is ignored yet.

Hypothesis 2

People draw a specific conclusion only if the percentage of its computed ratio is 100% and no valid conclusions otherwise.
## Root mean square error (RMSE) = 0.161 (with hypothesis 1).

- Place 3 (of 4) in **Syllogistic Challenge** (Dortmund, 26/9/2017).
- Model has to be refined. More settings should be tested.

### Results

<table>
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<th>assumptions 2 3 4</th>
<th>hypothesis 1 correlation</th>
<th>hypothesis 2 coincidence</th>
<th>no valid conclusions</th>
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<td>86.8%</td>
<td>71.9%</td>
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</table>
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Summary

- Set-based approach to syllogistic reasoning allows a fine-grained analysis of syllogistic reasoning.
- It is cognitively simple, because it makes use of only seven spaces.
- Constraint logic programming implementation is very fast.
- Model may be parametrized by assumptions.
- Theory differs from empirical findings and has therefore to be improved.
- Possibly no single, monolithic theory can explain the whole picture. In consequence, the individual behavior of a test person may be derived from sample conclusions taken by that person.
- Techniques from machine learning like case-based reasoning, clustering, decision tree learning, or neural networks may be employed.
**Future Work**

**Vision:** Detect argument structures in dialogues from more than one sentence.

**Example 3**

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Aba</strong></td>
<td>If people have a job they are happy ...</td>
</tr>
<tr>
<td><strong>Ecb</strong></td>
<td>AI often replaces jobs by machines ...</td>
</tr>
<tr>
<td><strong>Eca</strong></td>
<td>AI makes people unhappy ...</td>
</tr>
</tbody>
</table>

1. Analyze with a part-of-speech tagger like KNEWS (or MultiNet), extract topics of discourse, e.g. happy, jobs, AI.
2. Find essential categories of the discourse and hence arguments (e.g. by latent semantic analysis).
3. We assume that always two categories are connected in one assertion; determine quantifier (e.g. ‘often’) and possibly negation (e.g. ‘unhappy’).
4. Form syllogism; reconstruct argument structure.
References


