Syllogistic Reasoning in Seven Spaces

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Motivation

- **Research Question:**
  How does logical thinking work?
- **Syllogistic argumentation** can be traced back to Aristotle.
- A classical syllogism consists of two quantified assertions about categories, e.g. A, B, and C, like
  - 'Some A are B' and
  - 'No B are C'

  connecting exactly two categories each time.
- What are the logical consequences from these assertions?
  - 'Some A are not C'

Aristotle Altemps Inv8575.jpg
Wikipedia

Example

Example 1 – with concrete categories [3]

Some artists are bakers.
All bakers are chemists.
∴ Some artists are chemists.

- Assertions can be expressed by the logic of monadic assertions, i.e. subset of first-order logic with predicates having exactly one argument:

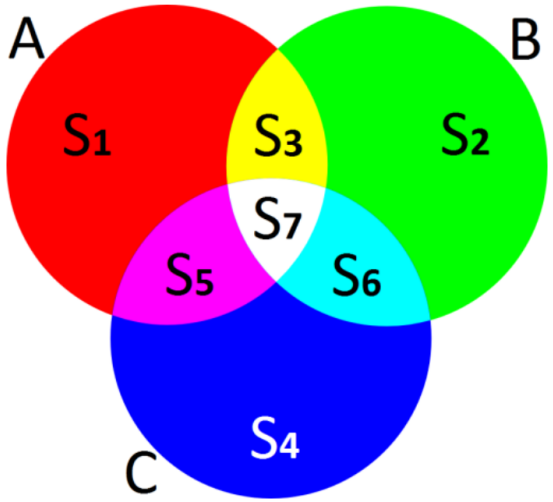
$$\text{‘All } A \text{ are } B\text{’} \quad \approx \quad \forall x \big(A(x) \to B(x)\big)$$

- Machines may employ classical first-order logic, but what do human reasoners do?
- They maybe consider several cases – in seven spaces.
- Human rationality is bounded. Reasoning may interfere with concrete knowledge (cf. Wason selection task [7]).

Overview

Categories and Spaces

- Although people may not think formally, we define notions precisely, laying a clear foundation for modeling and implementation.
- A **category** C stands for a set of individuals or objects.
- A classical syllogism combines in total three categories: A, B, and C.
- A **space** S is one of the seven possible subsets of $A \cup B \cup C$ in the set diagram for the three involved categories.
- The set of all such spaces is:



Venn diagram for three categories [5]

$$M = \{A \cap \overline{B} \cap \overline{C}, \overline{A} \cap B \cap \overline{C}, A \cap B \cap \overline{C}, \overline{A} \cap \overline{B} \cap C, A \cap \overline{B} \cap C, \overline{A} \cap B \cap C, A \cap B \cap C\}$$

Assertions and Moods

- An **assertion** for two categories A and B is a statement of the form 'A certain quantity of A is (not) B'.
- They are combination of **quantifier** 'All' or 'Some' and optional **negation** of the second category.
- This leads to four so-called classical **moods**.

Moods – interpreted as conditional probabilities $P(B|A)$

A:	'All A are B'	$P(B\|A) = 1 \rightsquigarrow P(A \cap \overline{B}) = 0$
E:	'All A are not B'	$P(\overline{B}\|A) = 1 \rightsquigarrow P(A \cap B) = 0$
I:	'Some A are B'	$P(B\|A) > 0 \rightsquigarrow P(A \cap B) > 0$
O:	'Some A are not B'	$P(\overline{B}\|A) > 0 \rightsquigarrow P(A \cap \overline{B}) > 0$

- Mood **U** 'Some but not all' (**I** $\wedge$ **O**) more specific than **I**:
  1. For non-empty categories, **A** entails **I**, but not **U**.
  2. Order of categories does not matter for **I**, but for **U**.

Syllogism and Figures

- Three assertions φ_1, φ_2 (**premises**), and φ_3 (**conclusion**) form a **syllogism**:

$$\varphi_1 \wedge \varphi_2 \rightarrow \varphi_3$$

- In each assertion, categories may occur in two orders (leading to different **figures**).

Figures – orders (only) of the premises

Assertion	Figure 1	Figure 2	Figure 3	Figure 4
φ_1	A–B	B–A	A–B	B–A
φ_2	B–C	C–B	C–B	B–C

- For n moods, there are $N = (n \cdot 2)^3$ syllogisms:

n	N	
4	512	(classical moods)
5	1000	(with mood **U**)

Related Works

- **Common result:** Conclusions humans draw differ from those of classical first-order logical reasoning $\rightsquigarrow$ many theories.
- **Heuristic atmosphere theory** [9]: People may be predisposed to accept a conclusion that fits the mood of the premises.
- **Set-theoretic approaches** including fuzzy set theory [4]: Fuzzy existential quantifiers cover the range from ‘Some’ to ‘All’ in several steps.
- **Mental model theory** [2] postulates: Human reasoners can represent a set iconically and build a mental model of its members.
- **Remark:** No single theory may provide an adequate account, because different persons apply very distinct reasoning strategies.

Assumptions

Human reasoners adopt assumptions, possibly not consciously.

1. If a space S is non-empty, it has a **non-zero** probability, i.e. $P(S) > 0$ for $S \neq \emptyset$. In the following, we always adopt this assumption.
2. Quantification with 'Some' (moods **I** and **O**) induces the existence of individuals or objects of the respective category. Furthermore it can be understood **non-inclusive**, i.e. as 'Some but not all' (mood **U**).
3. All categories are **non-empty**, i.e. $A, B, C \neq \emptyset$ and thus $P(A) > 0$, $P(B) > 0$, and $P(C) > 0$ (because of Assumption 1).
4. All categories are **non-equivalent**, i.e. $A \neq B \neq C \neq A$. Each of these three inequalities can be interpreted probabilistically, e.g. $A \neq B$ by $P(A \backslash B \cup B \backslash A) > 0$, because $A \neq B$ is equivalent to $A \not\subseteq B$ or $B \not\subseteq A$ and hence to $A \backslash B \ \cup \ B \backslash A \neq \emptyset$.

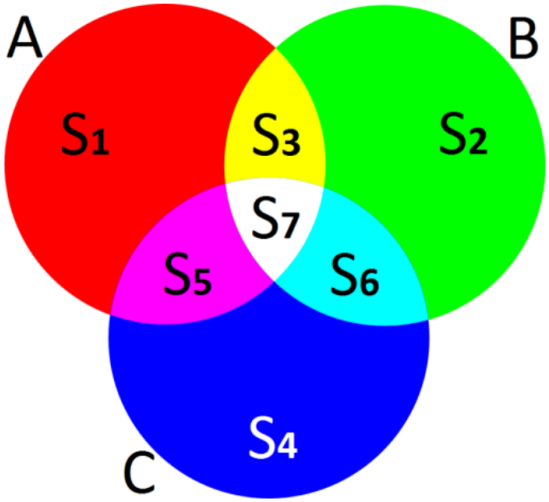
Implementation

- Assumptions (except last one) can be traced back to Aristotle's original works on analytics (*Analytica priora* and *Analytica posteriora*, cf. [1]).
- They allow to model several human reasoning strategies. Hence theory with seven spaces is parametrizable.
- Syllogistic reasoning in seven spaces can be implemented by **constraint logic programming** in SWI Prolog [8].
- We employ **finite domain constrains** with binary space variables $S_1, \ldots, S_7$: empty vs. not empty.
- Every assumption and every mood can be expressed by one or more conditions of the form
  - $P(E) = 0$ or
  - $P(E) > 0$

  for some event (set of spaces) E.

Details

- **Assumption 1**: space is empty or non-empty:
  `[S1,S2,S3,S4,S5,S6,S7] ins 0..1`
- **Mood A** (similarly **E**):
  'All A are B' $\rightsquigarrow P(A \cap \overline{B}) = 0$
  $\rightsquigarrow A \cap \overline{B} = S_1 \cup S_5$
  $\rightsquigarrow$ `S1 + S5 #= 0`
- **Mood I** (similarly **O**):
  'Some A are B' $\rightsquigarrow P(A \cap B) > 0$
  $\rightsquigarrow A \cap B = S_3 \cup S_7$
  $\rightsquigarrow$ `S3 + S7 #> 0`



- **Assumption 3**: category (here C) is non-empty
  $C = S_4 \cup S_5 \cup S_6 \cup S_7 \rightsquigarrow$ `S4 + S5 + S6 + S7 #> 0`
- **Assumption 4**: $A \neq B$ (others analogously)
  $A \backslash B \cup B \backslash A \neq \emptyset \rightsquigarrow A \backslash B = S_1 \cup S_5 \wedge B \backslash A = S_2 \cup S_6$
  $\rightsquigarrow$ `S1 + S2 + S5 + S6 #> 0`

Example

Example 2 – with abstract categories

All A are B.
All B are C.
∴ All C are A.

- The premises are in mood **AA1**, conclusion is also in mood **A** but categories in reverse order.
- The premises induce that S_1, S_2, S_3, and S_5 must be empty, remaining three spaces S_4, S_6, and S_7 open.
- Conclusion enforces that S_4 and S_6 must be empty, too. Thus all spaces are empty except possibly S_7.
- The fraction (ratio) of valid cases is only $2/8 = 25\%$.
- If all categories are non-empty (i.e. with assumption 3, then space S_7 must be non-empty.
- Hence the syllogism holds in $1/4 = 25\%$ of the cases.

Table with Ratios of Cases (with assumption 3)

Syllogism	Premises	Aac			Aca			Iac			Ica			Oac			Oca			Eac			Eca		
AA1	Aab,Abc	4	4	100%	1	4	25%	4	4	100%	4	4	100%	0	4	0%	3	4	75%	0	4	0%	0	4	0%
AA2	Aba,Acb	1	4	25%	4	4	100%	4	4	100%	4	4	100%	3	4	75%	0	4	0%	0	4	0%	0	4	0%
AA3	Aab,Acb	4	10	40%	4	10	40%	8	10	80%	8	10	80%	6	10	60%	6	10	60%	2	10	20%	2	10	20%
AA4	Aba,Abc	4	8	50%	4	8	50%	8	8	100%	8	8	100%	4	8	50%	4	8	50%	0	8	0%	0	8	0%
AI1	Aab,Ibc	8	20	40%	4	20	20%	16	20	80%	16	20	80%	12	20	60%	16	20	80%	4	20	20%	4	20	20%
AI2	Aba,Icb	4	16	25%	8	16	50%	16	16	100%	16	16	100%	12	16	75%	8	16	50%	0	16	0%	0	16	0%
AI3	Aab,Icb	8	20	40%	4	20	20%	16	20	80%	16	20	80%	12	20	60%	16	20	80%	4	20	20%	4	20	20%
AI4	Aba,Ibc	4	16	25%	8	16	50%	16	16	100%	16	16	100%	12	16	75%	8	16	50%	0	16	0%	0	16	0%
AO1	Aab,Obc	4	18	22.22%	3	18	16.67%	12	18	66.67%	12	18	66.67%	14	18	77.78%	15	18	83.33%	6	18	33.33%	6	18	33.33%
AO2	Aba,Ocb	3	18	16.67%	6	18	33.33%	16	18	88.89%	16	18	88.89%	15	18	83.33%	12	18	66.67%	2	18	11.11%	2	18	11.11%
AO3	Aab,Ocb	4	12	33.33%	0	12	0%	8	12	66.67%	8	12	66.67%	8	12	66.67%	12	12	100%	4	12	33.33%	4	12	33.33%
AO4	Aba,Obc	0	14	0%	6	14	42.86%	12	14	85.71%	12	14	85.71%	14	14	100%	8	14	57.14%	2	14	14.29%	2	14	14.29%
AE1	Aab,Ebc	0	2	0%	0	2	0%	0	2	0%	0	2	0%	2	2	100%	2	2	100%	2	2	100%	2	2	100%
AE2	Aba,Ecb	0	6	0%	2	6	33.33%	4	6	66.67%	4	6	66.67%	6	6	100%	4	6	66.67%	2	6	33.33%	2	6	33.33%
AE3	Aab,Ecb	0	2	0%	0	2	0%	0	2	0%	0	2	0%	2	2	100%	2	2	100%	2	2	100%	2	2	100%
AE4	Aba,Ebc	0	6	0%	2	6	33.33%	4	6	66.67%	4	6	66.67%	6	6	100%	4	6	66.67%	2	6	33.33%	2	6	33.33%
IA1	Iab,Abc	8	16	50%	4	16	25%	16	16	100%	16	16	100%	8	16	50%	12	16	75%	0	16	0%	0	16	0%
IA2	Iba,Acb	4	20	20%	8	20	40%	16	20	80%	16	20	80%	16	20	80%	12	20	60%	4	20	20%	4	20	20%
IA3	Iab,Acb	4	20	20%	8	20	40%	16	20	80%	16	20	80%	16	20	80%	12	20	60%	4	20	20%	4	20	20%
IA4	Iba,Abc	8	16	50%	4	16	25%	16	16	100%	16	16	100%	8	16	50%	12	16	75%	0	16	0%	0	16	0%
II1	Iab,Ibc	16	80	20%	16	80	20%	72	80	90%	72	80	90%	64	80	80%	64	80	80%	8	80	10%	8	80	10%
II2	Iba,Icb	16	80	20%	16	80	20%	72	80	90%	72	80	90%	64	80	80%	64	80	80%	8	80	10%	8	80	10%
II3	Iab,Icb	16	80	20%	16	80	20%	72	80	90%	72	80	90%	64	80	80%	64	80	80%	8	80	10%	8	80	10%
II4	Iba,Ibc	16	80	20%	16	80	20%	72	80	90%	72	80	90%	64	80	80%	64	80	80%	8	80	10%	8	80	10%
IO1	Iab,Obc	8	76	10.53%	16	76	21.05%	64	76	84.21%	64	76	84.21%	68	76	89.47%	60	76	78.95%	12	76	15.79%	12	76	15.79%
IO2	Iba,Ocb	12	72	16.67%	12	72	16.67%	64	72	88.89%	64	72	88.89%	60	72	83.33%	60	72	83.33%	8	72	11.11%	8	72	11.11%
IO3	Iab,Ocb	12	72	16.67%	12	72	16.67%	64	72	88.89%	64	72	88.89%	60	72	83.33%	60	72	83.33%	8	72	11.11%	8	72	11.11%
IO4	Iba,Obc	8	76	10.53%	16	76	21.05%	64	76	84.21%	64	76	84.21%	68	76	89.47%	60	76	78.95%	12	76	15.79%	12	76	15.79%
IE1	Iab,Ebc	0	12	0%	4	12	33.33%	8	12	66.67%	8	12	66.67%	12	12	100%	8	12	66.67%	4	12	33.33%	4	12	33.33%
IE2	Iba,Ecb	0	12	0%	4	12	33.33%	8	12	66.67%	8	12	66.67%	12	12	100%	8	12	66.67%	4	12	33.33%	4	12	33.33%
IE3	Iab,Ecb	0	12	0%	4	12	33.33%	8	12	66.67%	8	12	66.67%	12	12	100%	8	12	66.67%	4	12	33.33%	4	12	33.33%
IE4	Iba,Ebc	0	12	0%	4	12	33.33%	8	12	66.67%	8	12	66.67%	12	12	100%	8	12	66.67%	4	12	33.33%	4	12	33.33%

Hypotheses

Hypothesis 1

People draw a specific conclusion the higher the percentage of its computed ratio is according to our model.

- Best (moderate) correlation to empirical results [3, Table 6] without any of the assumptions ($r = 0.482$).
- Model allows more than one conclusion.
- Case of no valid conclusions is ignored yet.

Hypothesis 2

People draw a specific conclusion only if the percentage of its computed ratio is 100% and no valid conclusions otherwise.

Correlation and Coincidence

Results

assumptions 2	3	4	hypothesis 1 correlation	hypothesis 2 coincidence	no valid conclusions
no	no	no	0.482	89.6%	65.6%
no	no	yes	0.377	89.2%	65.6%
no	yes	no	0.370	87.3%	73.4%
no	yes	yes	0.314	87.0%	73.4%
yes	no	no	0.450	88.9%	65.6%
yes	no	yes	0.367	88.5%	65.6%
yes	yes	no	0.371	87.5%	71.9%
yes	yes	yes	0.317	86.8%	71.9%

- Root mean square error (RMSE) = 0.161 (with hypothesis 1).
- Place 3 (of 4) in **Syllogistic Challenge** (Dortmund, 26/9/2017).
- Model has to be refined. More settings should be tested.

Summary

- Set-based approach to syllogistic reasoning allows a fine-grained analysis of syllogistic reasoning.
- It is cognitively simple, because it makes use of only seven spaces.
- Constraint logic programming implementation is very fast.
- Model may be parametrized by assumptions.
- Theory differs from empirical findings and has therefore to be improved.
- Possibly no single, monolithic theory can explain the whole picture. In consequence, the individual behavior of a test person may be derived from sample conclusions taken by that person.
- Techniques from machine learning like case-based reasoning, clustering, decision tree learning, or neural networks may be employed.

Future Work

Vision: Detect argument structures in dialogues from more than one sentence.

Example 3

Aba *. . . If people have a job they are happy . . .*
Ecb *. . . AI often replaces jobs by machines . . .*
Eca *. . . AI makes people unhappy . . .*

1. Analyze with a part-of-speech tagger like KNEWS (or MultiNet), extract topics of discourse, e.g. *happy*, *jobs*, *AI*.
2. Find essential categories of the discourse and hence arguments (e.g. by latent semantic analysis).
3. We assume that always two categories are connected in one assertion; determine quantifier (e.g. ‘often’) and possibly negation (e.g. ‘unhappy’).
4. Form syllogism; reconstruct argument structure.

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