Exercise 2.1. Transform the following concepts into negation normal form:

(a) \( \neg(A \sqcap \forall r.B) \)

(b) \( \neg\forall r.\exists s. (\neg B \sqcup \exists r.A) \)

(c) \( \neg((\neg A \sqcap \exists r.\top) \sqcup \exists s. (A \sqcup \neg B)) \)

Exercise 2.2. Apply the tableau algorithm in order to check if the axiom \( A \sqsubseteq B \) is a logical consequence of the TBox \( \{ \neg C \sqsubseteq B, A \sqcap C \sqsubseteq \bot \} \).

Exercise 2.3. Apply the tableau algorithm in order to check satisfiability of the concept \( A \sqcap \forall r.B \) w.r.t. the TBox \( \{ A \sqsubseteq \exists r.A, B \sqsubseteq \exists r.\neg C, C \sqsubseteq \forall r.\forall r.B \} \).

Exercise 2.4. Lukas wants to apply the tableau algorithm for checking the satisfiability of the concept \( B \sqcap \exists r.\neg A \) w.r.t. the TBox \( \{ A \sqsubseteq \exists r.\neg A \sqcap \exists r.B, \top \sqsubseteq 1 r \} \). He arrives at the situation depicted below and concludes that no further rules are applicable, since \( v_2 \) is blocked by \( v_1 \). What is Lukas’ error? Continue the algorithm until its termination.

\[
\begin{array}{c}
\text{Exercise 2.5. }
\text{Extend the } \leq 1 \text{ rule in a way that also qualified functionality axioms of the form } C \subseteq 1 r.A \text{ can be treated correctly, where } A \text{ is an atomic concept. Can you also treat arbitrary axioms of the form } C \subseteq 1 r.D?}
\end{array}
\]