Weak Completion Semantics 3

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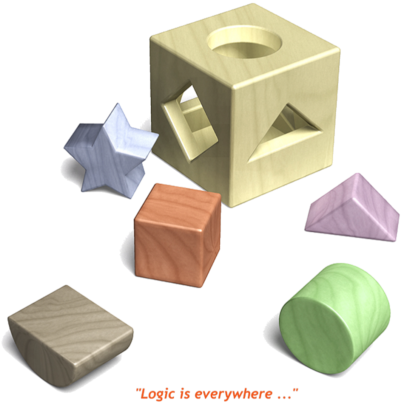


- **Revision**
- **Conditionals**

Conditionals

- **Conditionals** are statements of the form ***if condition then consequence***
- **Indicative conditionals** are conditionals
  - whose condition may or may not be *true*
  - whose consequence may or may not be *true*
  - but the consequence is asserted to be *true* if the condition is *true*
- **Subjunctive conditionals** are conditionals
  - whose condition is *false*
  - whose consequence may or may not be *true*
  - but in the counterfactual circumstance of the condition being *true*, the consequence is asserted to be *true* as well

More on Conditionals

- In the sequel, let *cond*$(\mathcal{C}, \mathcal{D})$ be a conditional, where
  - condition $\mathcal{C}$ and consequence $\mathcal{D}$ are finite and consistent sets of literals
- Conditionals are evaluated wrt a given $\mathcal{P}$ and $\mathcal{IC}$
  - We assume that $\mathcal{M}_{\mathcal{P}}$ satisfies $\mathcal{IC}$

Logic Programs

- **Program clauses**

$$A \leftarrow B_1, \ldots, B_n \ (n > 0) \qquad A \leftarrow \top \qquad A \leftarrow \bot$$

- **Let $\mathcal{P}$ be a finite program**
- **Let $\mathcal{S}$ be a finite set of literals**

$$def(\mathcal{S}, \mathcal{P}) = \{A \leftarrow body \in \mathcal{P} \mid A \in \mathcal{S} \vee \neg A \in \mathcal{S}\}$$

Revision

- Dietz, H. 2015: A New Computational Logic Approach to Reason with Conditionals In: Calimeri et.al. (eds), Logic Programming and Nonmonotonic Reasoning, LPNMR, LNAI 9345: 2015

- **Let $\mathcal{S}$ be a finite and consistent set of literals**

$$rev(\mathcal{P}, \mathcal{S}) = (\mathcal{P} \setminus def(\mathcal{S}, \mathcal{P})) \cup \{A \leftarrow \top \mid A \in \mathcal{S}\} \cup \{A \leftarrow \bot \mid \neg A \in \mathcal{S}\}$$

  is called the revision of $\mathcal{P}$ with respect to $\mathcal{S}$

- **Proposition**
  - ***rev* is nonmonotonic,**
    i.e., there exist $\mathcal{P}$, $\mathcal{S}$ and F such that $\mathcal{P} \models_{wcs} F$ and $rev(\mathcal{P}, \mathcal{S}) \not\models_{wcs} F$
  - **If $\mathcal{M}_{\mathcal{P}}(L) = U$ for all $L \in \mathcal{S}$, then *rev* is monotonic**
  - $\mathcal{M}_{rev(\mathcal{P},\mathcal{S})}(\mathcal{S}) = \top$

Conditionals – The Firing Squad Example

- Pearl: Causality: Models, Reasoning, and Inference
  Cambridge University Press, New York, USA: 2000
- **If the court orders an execution, then the captain will give the signal upon which rifleman *A* and *B* will shoot the prisoner; consequently, the prisoner will be dead**
- **We assume that**
  - **the court's decision is *unknown***
  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**
- **Evaluate the following conditionals (*true*, *false*, *unknown*)**
  - **If the prisoner is not dead, then the captain did not signal**
  - **If rifleman A shot, then rifleman B shot as well**
  - **If rifleman A did not shoot, then the prisoner is not dead**
  - **If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution**

Evaluating Conditionals – Our Approach

- Given $\mathcal{P}$, $\mathcal{IC}$, and $\mathit{cond}(\mathcal{C}, \mathcal{D})$
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ then $\mathit{cond}(\mathcal{C}, \mathcal{D}) = \mathcal{M}_{\mathcal{P}}(\mathcal{D})$
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \bot$ then evaluate $\mathit{cond}(\mathcal{C}, \mathcal{D})$ wrt $\mathcal{M}_{\mathit{rev}(\mathcal{P},\mathcal{S})}$, where
    - $\mathcal{S} = \{L \in \mathcal{C} \mid \mathcal{M}_{\mathcal{P}}(L) = \bot\}$
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \mathrm{U}$ then evaluate $\mathit{cond}(\mathcal{C}, \mathcal{D})$ wrt $\mathcal{M}_{\mathcal{P}'}$, where
    - $\mathcal{P}' = \mathit{rev}(\mathcal{P}, \mathcal{S}) \cup \mathcal{E}$,
    - $\mathcal{S}$ is a smallest subset of $\mathcal{C}$ and
      $\mathcal{E} \subseteq \mathcal{A}_{\mathit{rev}(\mathcal{P},\mathcal{S})}$ is an explanation for $\mathcal{C} \setminus \mathcal{S}$ such that
      $\mathcal{P}' \models_{\mathit{wcs}} \mathcal{C}$ and $\mathcal{M}_{\mathcal{P}'}$ satisfies $\mathcal{IC}$

Minimal revision followed by abduction

Modeling the Firing Squad Example

- $\mathcal{P}$

$$
\begin{array}{lcl@{\qquad\qquad}lcl}
s & \leftarrow & e \wedge \neg ab_1 & ab_1 & \leftarrow & \bot \\
ra & \leftarrow & s \wedge \neg ab_2 & ab_2 & \leftarrow & \bot \\
rb & \leftarrow & s \wedge \neg ab_3 & ab_3 & \leftarrow & \bot \\
d & \leftarrow & ra \wedge \neg ab_4 & ab_4 & \leftarrow & \bot \\
d & \leftarrow & rb \wedge \neg ab_5 & ab_5 & \leftarrow & \bot \\
a & \leftarrow & \neg d \wedge \neg ab_6 & ab_6 & \leftarrow & \bot
\end{array}
$$

- $\mathcal{M}_\mathcal{P}$

$$\langle \emptyset, \{ab_1, ab_2, ab_3, ab_4, ab_5, ab_6\}\rangle$$

- $\mathcal{A}_\mathcal{P}$

$$\{e \leftarrow \top, e \leftarrow \bot\}$$

- **Observations**
  - $\mathcal{E}_\top = \{e \leftarrow \top\}$ explains $\{s, ra, rb, d, \neg a\}$
  - $\mathcal{E}_\bot = \{e \leftarrow \bot\}$ explains $\{\neg s, \neg ra, \neg rb, \neg d, a\}$
  - $\{\neg s, ra\}$ cannot be explained

The Firing Squad Conditionals

- Observations
  - $\mathcal{E}_\top = \{e \leftarrow \top\}$ explains $\{s, ra, rb, d, \neg a\}$
  - $\mathcal{E}_\bot = \{e \leftarrow \bot\}$ explains $\{\neg s, \neg ra, \neg rb, \neg d, a\}$
  - $\{\neg s, ra\}$ cannot be explained
- **If the prisoner is alive, then the captain did not signal**

$$cond(a, \neg s) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\bot \Rightarrow true$$

- **If rifleman A shot, then rifleman B shot as well**

$$cond(ra, rb) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\top \Rightarrow true$$

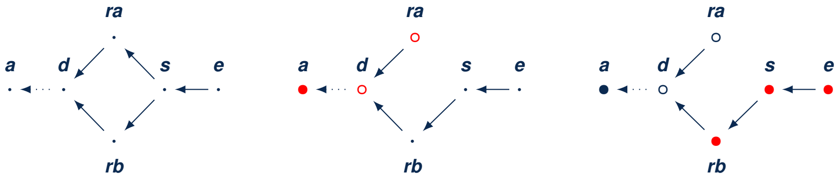
- **If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution**

$$cond(\{\neg s, ra\}, \neg e) : \mathcal{P} \Rightarrow rev(\mathcal{P}, ra) \cup \mathcal{E}_\bot \Rightarrow true$$

The Last Firing Squad Example Revisited

- If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution

$$\mathcal{P} \Rightarrow \mathit{rev}(\mathcal{P}, \mathit{ra}) \cup \mathcal{E}_\perp$$



Subjunctive Conditionals – The Forest Fire Example

- Byrne: The Rational Imagination: How People Create Alternatives to Reality
  MIT Press 2007

- **Lightning causes a forest fire if nothing abnormal is taking place**
  Lightning happened
  The absence of dry leaves is an abnormality
  Dry leaves are present

$$\mathcal{P} = \{f \leftarrow \ell \wedge \neg ab_1,\ \ell \leftarrow \top,\ ab_1 \leftarrow \neg d,\ d \leftarrow \top\}$$

- **If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred**

	$\Phi_{\mathcal{P}}$	$\Phi_{rev(\mathcal{P},\neg d)}$
$\uparrow 0$	$\langle \emptyset, \emptyset \rangle$	$\langle \emptyset, \emptyset \rangle$
$\uparrow 1$	$\langle \{d, \ell\}, \emptyset \rangle$	$\langle \{\ell\}, \{d\} \rangle$
$\uparrow 2$	$\langle \{d, \ell\}, \{ab_1\} \rangle$	$\langle \{\ell, ab_1\}, \{d\} \rangle$
$\uparrow 3$	$\langle \{d, \ell, f\}, \{ab_1\} \rangle$	$\langle \{\ell, ab_1\}, \{d, f\} \rangle$

- **Subjunctive conditional** $cond(\neg d, \neg f)$

$$rev(\mathcal{P}, \neg d) = \{f \leftarrow \ell \wedge \neg ab_1,\ \ell \leftarrow \top,\ ab_1 \leftarrow \neg d,\ d \leftarrow \bot\}$$

The Extended Forest Fire Example

- Pereira, Dietz, H.: Contextual Abductive Reasoning with Side-Effects
  Theory and Practice of Logic Programming 14, 633-648: 2014
- **Arson causes a forest fire if nothing abnormal is taking place**
- **If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred**

$$
\begin{array}{rcl}
\mathcal{P} & = & \{f \leftarrow \ell \wedge \neg ab_1,\ f \leftarrow a \wedge \neg ab_2, \\
 & & \ \ell \leftarrow \top,\ ab_1 \leftarrow \neg d,\ d \leftarrow \top,\ ab_2 \leftarrow \bot\} \\
\mathcal{M}_{\mathcal{P}} & = & \langle \{d, \ell, f\}, \{ab_1, ab_2\}\rangle \\
rev(\mathcal{P}, \neg d) & = & \{f \leftarrow \ell \wedge \neg ab_1,\ f \leftarrow a \wedge \neg ab_2, \\
 & & \ \ell \leftarrow \top,\ ab_1 \leftarrow \neg d,\ d \leftarrow \bot,\ ab_2 \leftarrow \bot\} \\
\mathcal{M}_{rev(\mathcal{P}, \neg d)} & = & \langle \{\ell, ab_1\}, \{d, ab_2\}\rangle
\end{array}
$$

- Dietz, H., Pereira: On Conditionals
  Global Conference on Artificial Intelligence, Epic Series in Computing: 2015

Some Open Questions

- **Do humans reason with multi-valued logics and, if they do, which multi-valued logic are they using?**
- **Can an answer 'I don't know' be qualified as a truth value assignment or is it a meta-remark?**
- **What do we have to tell humans such that they fully understand the background information?**
- **Do humans apply abduction and/or revision if the condition of a conditional is *unknown* and, if they apply both, do they prefer one over the other?**
- **Do they prefer skeptical over creduluous abduction?**
- **Do they prefer minimal revision?**
- **How important is the order in which multiple conditions of a conditional are considered?**
- **Do humans consider abduction and/or revision steps which turn an indicative conditional into a subjunctive one?**