Weak Completion Semantics 3

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► Revision
► Conditionals

"Logic is everywhere ..."
Conditionals

- **Conditionals** are statements of the form \( \text{if condition then consequence} \)
- **Indicative conditionals** are conditionals
  - whose condition may or may not be \textit{true}
  - whose consequence may or may not be \textit{true}
  - but the consequence is asserted to be \textit{true} if the condition is \textit{true}
- **Subjunctive conditionals** are conditionals
  - whose condition is \textit{false}
  - whose consequence may or may not be \textit{true}
  - but in the counterfactual circumstance of the condition being \textit{true}, the consequence is asserted to be \textit{true} as well
In the sequel, let $\text{cond}(C, D)$ be a conditional, where

- condition $C$ and consequence $D$ are finite and consistent sets of literals

Conditionals are evaluated wrt a given $\mathcal{P}$ and $\mathcal{IC}$

- We assume that $\mathcal{M}_\mathcal{P}$ satisfies $\mathcal{IC}$
Logic Programs

- Program clauses

\[ A \leftarrow B_1, \ldots, B_n \quad (n > 0) \quad A \leftarrow \top \quad A \leftarrow \bot \]

- Let \( P \) be a finite program
- Let \( S \) be a finite set of literals

\[ \text{def}(S, P) = \{ A \leftarrow \text{body} \in P \mid A \in S \lor \neg A \in S \} \]
Revision

Dietz, H. 2015: A New Computational Logic Approach to Reason with Conditionals
In: Calimeri et.al. (eds), Logic Programming and Nonmonotonic Reasoning, LPNMR,
LNAI 9345: 2015

Let $S$ be a finite and consistent set of literals

$$\text{rev}(P, S) = (P \setminus \text{def}(S, P)) \cup \{A \leftarrow \top \mid A \in S\} \cup \{A \leftarrow \bot \mid \neg A \in S\}$$

is called the revision of $P$ with respect to $S$

Proposition

- $\text{rev}$ is nonmonotonic,
  i.e., there exist $P$, $S$ and $F$ such that $P \models_{\text{wcs}} F$ and $\text{rev}(P, S) \not\models_{\text{wcs}} F$
- If $M_P(L) = U$ for all $L \in S$, then $\text{rev}$ is monotonic
- $M_{\text{rev}(P, S)}(S) = \top$
Conditionals – The Firing Squad Example

▶ Pearl: Causality: Models, Reasoning, and Inference
   Cambridge University Press, New York, USA: 2000

▶ If the court orders an execution, then the captain will give the signal
   upon which rifleman A and B will shoot the prisoner; consequently, the prisoner will be dead

▶ We assume that
  ▶ the court’s decision is unknown
  ▶ both riflemen are accurate, alert and law-abiding
  ▶ the prisoner is unlikely to die from any other causes

▶ Evaluate the following conditionals (true, false, unknown)
  ▶ If the prisoner is not dead, then the captain did not signal
  ▶ If rifleman A shot, then rifleman B shot as well
  ▶ If rifleman A did not shoot, then the prisoner is not dead
  ▶ If the captain gave no signal and rifleman A decides to shoot,
    then the court did not order an execution
Evaluating Conditionals – Our Approach

Given $\mathcal{P}$, $\mathcal{IC}$, and $\text{cond}(\mathcal{C}, \mathcal{D})$

- If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ then $\text{cond}(\mathcal{C}, \mathcal{D}) = \mathcal{M}_\mathcal{P}(\mathcal{D})$
- If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \bot$ then evaluate $\text{cond}(\mathcal{C}, \mathcal{D})$ wrt $\mathcal{M}_{\text{rev} (\mathcal{P}, S)}$, where

  $S = \{L \in \mathcal{C} \mid \mathcal{M}_\mathcal{P}(L) = \bot\}$

- If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \bot$ then evaluate $\text{cond}(\mathcal{C}, \mathcal{D})$ wrt $\mathcal{M}_\mathcal{P}'$, where

  $\mathcal{P}' = \text{rev}(\mathcal{P}, S) \cup \mathcal{E}$,

  $S$ is a smallest subset of $\mathcal{C}$ and

  $\mathcal{E} \subseteq \mathcal{A}_{\text{rev}(\mathcal{P}, S)}$ is an explanation for $\mathcal{C} \setminus S$ such that $\mathcal{P}' \models_{\text{wcs}} \mathcal{C}$ and $\mathcal{M}_\mathcal{P}'$ satisfies $\mathcal{IC}$

Minimal revision followed by abduction
Modeling the Firing Squad Example

\[\mathcal{P}\]

\[
\begin{align*}
  s &\leftarrow e \land \neg ab_1 & ab_1 &\leftarrow \bot \\
  ra &\leftarrow s \land \neg ab_2 & ab_2 &\leftarrow \bot \\
  rb &\leftarrow s \land \neg ab_3 & ab_3 &\leftarrow \bot \\
  d &\leftarrow ra \land \neg ab_4 & ab_4 &\leftarrow \bot \\
  d &\leftarrow rb \land \neg ab_5 & ab_5 &\leftarrow \bot \\
  a &\leftarrow \neg d \land \neg ab_6 & ab_6 &\leftarrow \bot \\
\end{align*}
\]

\[\mathcal{M}_\mathcal{P}\]

\[
\langle \emptyset, \{ab_1, ab_2, ab_3, ab_4, ab_5, ab_6\} \rangle
\]

\[\mathcal{A}_\mathcal{P}\]

\[
\{e \leftarrow \top, e \leftarrow \bot\}
\]

\[\mathcal{E}_\top = \{e \leftarrow \top\} \text{ explains } \{s, ra, rb, d, \neg a\}\]

\[\mathcal{E}_\bot = \{e \leftarrow \bot\} \text{ explains } \{\neg s, \neg ra, \neg rb, \neg d, a\}\]

\[\{\neg s, ra\} \text{ cannot be explained}\]
The Firing Squad Conditionals

Observations

- $\mathcal{E}_\top = \{ e \leftarrow \top \}$ explains $\{ s, ra, rb, d, \neg a \}$
- $\mathcal{E}_\bot = \{ e \leftarrow \bot \}$ explains $\{ \neg s, \neg ra, \neg rb, \neg d, a \}$
- $\{ \neg s, ra \}$ cannot be explained

If the prisoner is alive, then the captain did not signal

$$\text{cond}(a, \neg s) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\bot \Rightarrow \text{true}$$

If rifleman A shot, then rifleman B shot as well

$$\text{cond}(ra, rb) : \mathcal{P} \Rightarrow \mathcal{P} \cup \mathcal{E}_\top \Rightarrow \text{true}$$

If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution

$$\text{cond}(\{ \neg s, ra \}, \neg e) : \mathcal{P} \Rightarrow \text{rev}(\mathcal{P}, ra) \cup \mathcal{E}_\bot \Rightarrow \text{true}$$
The Last Firing Squad Example Revisited

- If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution

\[ \mathcal{P} \Rightarrow \text{rev}(\mathcal{P}, ra) \cup \mathcal{E}_{\bot} \]
Subjunctive Conditionals – The Forest Fire Example

Byrne: The Rational Imagination: How People Create Alternatives to Reality
MIT Press 2007

Lightning causes a forest fire if nothing abnormal is taking place
Lightning happened
The absence of dry leaves is an abnormality
Dry leaves are present

\[ P = \{ f \leftarrow \ell \land \neg ab_1, \ell \leftarrow \top, ab_1 \leftarrow \neg d, d \leftarrow \top \} \]

If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred

\[
\begin{array}{c|c|c}
\uparrow 0 & \Phi_P & \Phi_{\text{rev}(P, \neg d)} \\
\uparrow 1 & \langle \{d, \ell\}, \emptyset \rangle & \langle \{\ell\}, \{d\} \rangle \\
\uparrow 2 & \langle \{d, \ell\}, \{ab_1\} \rangle & \langle \{\ell, ab_1\}, \{d\} \rangle \\
\uparrow 3 & \langle \{d, \ell, f\}, \{ab_1\} \rangle & \langle \{\ell, ab_1\}, \{d, f\} \rangle
\end{array}
\]

Subjunctive conditional \( \text{cond}(-d, -f) \)

\[ \text{rev}(P, \neg d) = \{ f \leftarrow \ell \land \neg ab_1, \ell \leftarrow \top, ab_1 \leftarrow \neg d, d \leftarrow \bot \} \]
The Extended Forest Fire Example


- Arson causes a forest fire if nothing abnormal is taking place

- If there had not been so many dry leaves on the forest floor, then the forest fire would not have occurred

\[
\mathcal{P} = \{ f \leftarrow \ell \land \neg ab_1, f \leftarrow a \land \neg ab_2, \\
\ell \leftarrow \top, ab_1 \leftarrow \neg d, d \leftarrow \top, ab_2 \leftarrow \bot \}
\]

\[
\mathcal{M}_\mathcal{P} = \langle \{d, \ell, f\}, \{ab_1, ab_2\} \rangle
\]

\[
rev(\mathcal{P}, \neg d) = \{ f \leftarrow \ell \land \neg ab_1, f \leftarrow a \land \neg ab_2, \\
\ell \leftarrow \top, ab_1 \leftarrow \neg d, d \leftarrow \bot, ab_2 \leftarrow \bot \}
\]

\[
\mathcal{M}_{rev(\mathcal{P}, \neg d)} = \langle \{\ell, ab_1\}, \{d, ab_2\} \rangle
\]

- Dietz, H., Pereira: On Conditionals
  Global Conference on Artificial Intelligence, Epic Series in Computing: 2015
Some Open Questions

► Do humans reason with multi-valued logics and, if they do, which multi-valued logic are they using?

► Can an answer ’I don’t know’ be qualified as a truth value assignment or is it a meta-remark?

► What do we have to tell humans such that they fully understand the background information?

► Do humans apply abduction and/or revision if the condition of a conditional is unknown and, if they apply both, do they prefer one over the other?

► Do they prefer skeptical over credulous abduction?

► Do they prefer minimal revision?

► How important is the order in which multiple conditions of a conditional are considered?

► Do humans consider abduction and/or revision steps which turn an indicative conditional into a subjunctive one?