Theories of Reasoning (Lecture 2)

Marco Ragni

Albert-Ludwigs-Universität Freiburg
Foundations of AI
ragni@cs.uni-freiburg.de

14. September 2015

What types of reasoning exist?

- Logic is the science of correct reasoning
- Human reasoning is an epistemic, mental activity: inferences are drawn from given knowledge
  - It requires to understand the question, have all facts at disposal, analyze the problem, note similarities and differences, with previous problems

What types of reasoning problems exist?

- Deductive Reasoning:
  - from general statements to specific instances
- Inductive Reasoning:
  - from a series of specific cases to a general statement; only possible conclusions
- Abductive Reasoning:
  - from an observation to a theory accounting for the observation; ideally simplest and most likely explanation

- Deductive reasoning

  Prem. 1: If it rains the street is wet
  Prem. 2: It rains
  Concl.: The street is wet

What types of reasoning problems exist?

- Deductive Reasoning:
  - from general statements to specific instances
- Inductive Reasoning:
  - from a series of specific cases to a general statement; only possible conclusions
- Abductive Reasoning:
  - from an observation to a theory accounting for the observation; ideally simplest and most likely explanation

- Deductive reasoning

  Prem. 1: If it rains the street is wet
  Prem. 2: It rains
  Concl.: The street is wet

- Inductive reasoning

  Observ. 1 It rains and
  Observ. 2 the street often gets wet

What types of reasoning problems exist?

- Deductive Reasoning:
  - from general statements to specific instances
- Inductive Reasoning:
  - from a series of specific cases to a general statement; only possible conclusions
- Abductive Reasoning:
  - from an observation to a theory accounting for the observation; ideally simplest and most likely explanation

- Deductive reasoning

  Prem. 1: If it rains the street is wet
  Prem. 2: It rains
  Concl.: The street is wet

- Inductive reasoning

  Observ. 1 It rains and
  Observ. 2 the street often gets wet
  Concl.: If it rains the street is wet

- Abductive reasoning

  Observ. 1 If it rains the street is wet.
  Observ. 2 the street is wet

What types of reasoning problems exist?

- Deductive Reasoning:
  - from general statements to specific instances
- Inductive Reasoning:
  - from a series of specific cases to a general statement; only possible conclusions
- Abductive Reasoning:
  - from an observation to a theory accounting for the observation; ideally simplest and most likely explanation

- Deductive reasoning

  Prem. 1: If it rains the street is wet
  Prem. 2: It rains
  Concl.: The street is wet

- Inductive reasoning

  Observ. 1 It rains and
  Observ. 2 the street often gets wet
  Concl.: If it rains the street is wet

- Abductive reasoning

  Observ. 1 If it rains the street is wet.
  Observ. 2 the street is wet
  Concl.: It rains

What types of reasoning problems exist?

Three main areas: Relations, Conditionals, and Quantifiers

- Relational reasoning:
  - A is left of B and B is left of C
- Conditional reasoning:
  - If – then
- Syllogistic reasoning:
  - all, some, some – not, none, many, few

Finding I: The influence of language in reasoning (Grice, 1975)

- Difference in the meaning of quantifiers in logic and everyday language
- Principles of cooperation (1975)
  - Quantity (not too much and not too less information)
  - Quality (says the "truth")
  - Relevance
  - Clarity
- Errors occur particularly with "Some"
- Logic: Some = at least one and maybe all
- Everyday language: Some = at least one, but not all

Finding II: Two Types of Reasoning

Cognitive psychologists distinguish between

- Type 1: rapid automatic inferences based on heuristics
- Type 2: slower conscious deliberations based on systematic and perhaps normative principles

  (see, e.g., Evans, 2003; Sloman, 1996; Stanovich, 1999; Verschueren, Schaeken, & d'Ydewalle, 2005)

An Example

All frenchmen drink wine

Some wine drinkers are gourmets

Therefore, . . . ?

Finding III: Belief Bias

$$\frac{\begin{array}{l}\text{All frenchmen drink wine}\\ \text{Some wine drinkers are gourmets}\end{array}}{\text{Some frenchmen are gourmets}}$$

Finding III: Belief Bias

All frenchmen drink wine
Some wine drinkers are gourmets

Some frenchmen are gourmets

Although the argument is widely accepted, it is not valid!

All frenchmen drink wine
Some wine drinkers are italians

Some frenchmen are italians

- Belief Bias Effect!

Types of Syllogism

- 4 types of moods

  All a are b

  Some a are b

  No a are b

  Some a are not b

- 2 assertions, 3 diff. terms
- 4 figures:

  Figure 1 (a-b b-c)

  Figure 2 (b-a c-b)

  Figure 3 (a-b c-b)

  Figure 4 (b-a b-c)

  All b are a

  Some b are not c

  What follows?
  ... Some a are not c.

- 64 problems =

  4 (moods for 1st premise) x

  4 (moods for 2nd premise) x

  4 figures

- 9 possible responses =

  8 responses (Aac, Aca, ...)
  + "no valid conclusion"

Heuristic Theories

1. Atmosphere heuristic
   (Woodworth & Sells, 1936; Begg & Denny, 1969; Revlis, 1975; Revlin et al., 1980)
2. Matching heuristic
   (Wetherick & Gilhooly, 1990)
3. Illicit conversion
   (Chapman & Chapman, 1959; Revlis, 1975)
4. Probability heuristics
   (Chater & Oaksford, 1999)

... formal rules of logic ...

5. First-order predicate calculus
   (Rips, 1994; Braine & Rumain, 1983; Braine, 1998)
6. Verbal substitutions
   (Storring, 1908; Ford, 1995)
7. Monotonicity
   (Geurts, 2003; Politzer, 2007)

... of diagrams

8. Venn diagrams
   (Newell, 1981)
9. Euler circles
   (Erickson, 1974; Guyote & Sternberg, 1981; Ford, 1995)
10. Source founding
    (Stenning & Yule, 1997)
11. Verbal models
    (Polk & Newell, 1995)
12. Mental models
    (Johnson-Laird & Steedman, 1978; Bucciarelli & Johnson-Laird, 1999)

Theories of Reasoning: Mental Logic (Rips, 1994)

In a nutshell: Human reasoning follows the syntactic rules of formal logic (cp. Inhelder & Piaget, 1958):

- Application of (mental) inference rules
- Analogy to syntactic approaches
- Sequences of rules are a mental proof for conclusion
  - Premises are interpreted and kept in working memory
  - This activates schemata to find conclusions
  - Due to the involvement of many processes, errors are made

Theories of Reasoning: Mental Logic

Syllogistic sentence (Rips, 1994, Ch7) in words (first lines), their translations into PSYCOP (second), and implicatures (third)

Mood		
A	All A are B	
	IF A(x) THEN B(x)	[if x is A then x is B]
	A(a) AND B(a)	[there are things, a, which are A and B]
I	Some A are B	
	A(b) AND B(b)	[there are things, b, which are A and B]
	A(a) AND NOT B(a)	[there are things, a, which are A and not B]
E	No A are B	
	Not (A(x) AND B(x))	[it is not the case that x is A and x is B]
	A(a) AND NOT B(a)	[there are things, a, which are A and not B]
O	Some A are not B	
	A(b) AND NOT B(b)	[there are things, b, which are A and not B]
	A(a) AND B(a)	[there are things, a, which are A and B]

x is a variable, a and b are "temporary names"

Theories of Reasoning: Diagrammatic

- Theory of Mental Models (Johnson-Laird, 1983)
  - Mental representation of situations
  - Construction upon perception, comprehension or imagination
  - Structure correspondence to reality

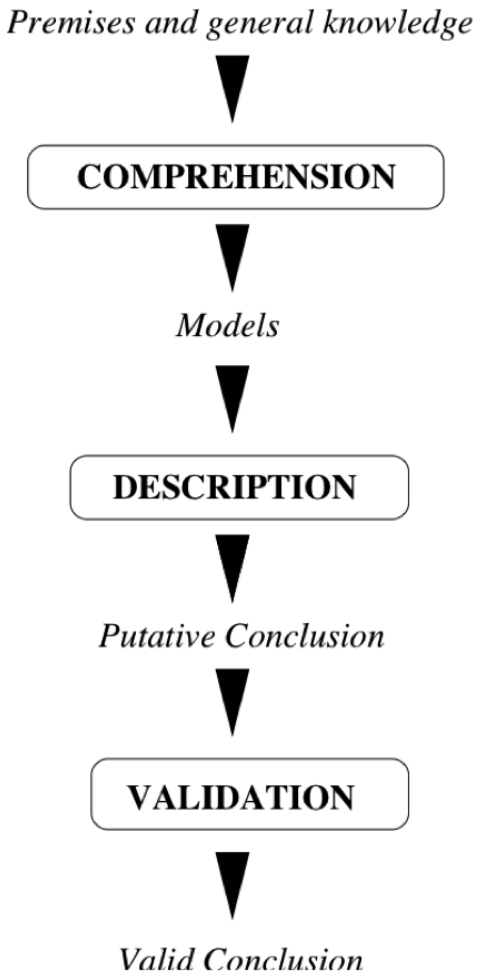
Theories of Reasoning: Mental Models

	Initial Model	Explicit Model ("fleshed out")
A: All A are B	[a] b [a] b ...	[a] [b] [a] [b] [¬a] [b] [¬a] [¬b]
I: Some A are B	a b a b ...	a b a b ¬a b a ¬b ¬a ¬b
E: No A are B	[a] [a] [b] [b] ...	[a] [¬b] [a] [¬b] [¬a] [b] [¬a] [b] ¬a ¬b

Notation:

- [] defines set of elements; none of elements appear elsewhere
- ¬ defines negation
- . . . defines implicit information

Theories of Reasoning: Mental Models



- Comprehension: Construction of a model using existing knowledge about the meaning of given premises
- Description: Combination of premises for premature conclusion
- Validation: Search models for falsification of premature conclusion holding given premises as true.

Theories of Reasoning: Mental Models

- Mental Models represent by definition what is true but not what is false ("principle of truth")
- Mental Models represent a single possibility which could be true ("initial model") **(= System 1)**
- The more models a task elicit, the more difficult the task is ("flesh out process") **(= System 2)**
- A putative conclusion formed after the description phase is validated after a search for counter-examples

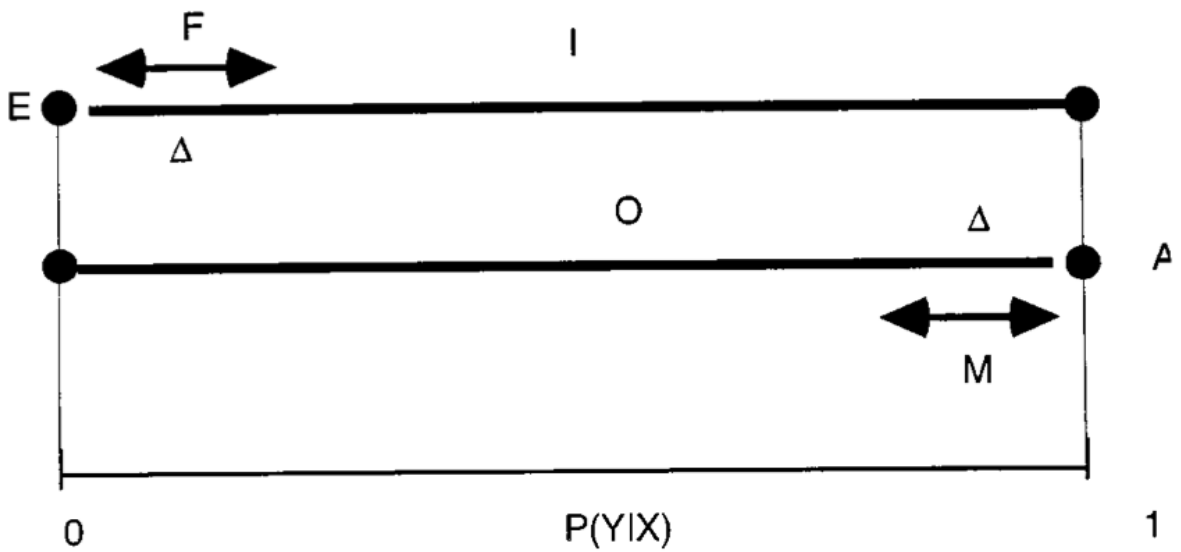
Theories of Reasoning: Probabilistic Approach

- Probabilistic Reasoning (Oaksford & Chater, 1994)
  - If a conditional rule (If . . . then) needs to be interpreted, humans try to find out if the given states rather have a probabilistic relationship or a null model
  - Assumes that humans interpret implications based on probability, i.e., "If X then Y" means:
    - If X then most probably Y
    - Bayesian approach: $P(Y \mid X)$

Theories of Reasoning: Probabilistic approach (Chater & Oaksford, 1999)

- Probabilistic semantics for quantifiers
- Estimation of probability adheres to the following scheme:

All X are Y:	$P(Y \mid X) = 1$
Some X are Y:	$P(Y \mid X) > 0$
Some X are not Y:	$P(Y \mid X) < 1$



Theories of Reasoning: Probabilistic approach (Chater & Oaksford, 1999)

Probability heuristics model (PHM) is based on an informational ordering of the quantifiers: $A > I > E > O$

1. Min-heuristic: quantifier of the conclusion is the one of the least informative premise
2. Max-heuristic: Subject of the conclusion is end-term of the min-premise else it is the end-term of the max-premise
3. p-entailment: the next most preferred conclusion will be the p-entailment of the conclusion predicted by the min-heuristic

Generalized Syllogisms

Some frenchmen are wine drinkers

None of the wine drinkers are beer drinkers

Therefore, . . . ?

Generalized Syllogisms

Some frenchmen are wine drinkers

None of the wine drinkers are beer drinkers

Therefore, . . . ?

- Premises in classical syllogisms use one of four quantifiers: all (**A**), some (**I**), some not (**O**), or none (**E**)

Generalized Syllogisms

Some frenchmen are wine drinkers

None of the wine drinkers are beer drinkers

Therefore, . . . ?

- Premises in classical syllogisms use one of four quantifiers: all (**A**), some (**I**), some not (**O**), or none (**E**)
- *Therefore, some of the frenchmen are not beer drinkers.*

Generalized Syllogisms

Some frenchmen are wine drinkers

None of the wine drinkers are beer drinkers

Therefore, . . . ?

- Premises in classical syllogisms use one of four quantifiers: all (**A**), some (**I**), some not (**O**), or none (**E**)
- *Therefore, some of the frenchmen are not beer drinkers.*
- Everyday human reasoning is "based [. . .] on beliefs, in which there are varying degrees of confidence" (Evans, 2002, p.980)

Generalized Syllogisms

Some frenchmen are wine drinkers

None of the wine drinkers are beer drinkers

Therefore, . . . ?

- Premises in classical syllogisms use one of four quantifiers: all (**A**), some (**I**), some not (**O**), or none (**E**)
- *Therefore, some of the frenchmen are not beer drinkers.*
- Everyday human reasoning is "based [. . .] on beliefs, in which there are varying degrees of confidence" (Evans, 2002, p.980)
- We consider generalized quantifiers most (**M**) and few (**F**)

A Syllogism

Some frenchmen are wine drinkers

Few wine drinkers are beer drinkers

Therefore, . . . ?

A Syllogism

Some frenchmen are wine drinkers

Few wine drinkers are beer drinkers

Therefore, few frenchmen are beer drinkers.

Therefore, some frenchmen are beer drinkers.

Theory Predictions and Extensions

- Probability Heuristics Model (PHM) (Chater & Oaksford, 1999) use 3 heuristics and predict that conclusions can be ordered

$$A > M > F > I > E >> O$$

Theory Predictions and Extensions

- Probability Heuristics Model (PHM) (Chater & Oaksford, 1999) use 3 heuristics and predict that conclusions can be ordered

$$A > M > F > I > E >> O$$

- Extension 1: Matching Hypothesis Ordered from most to least conservative quantifier:

$$E > O = I >> A$$

Theory Predictions and Extensions

- Probability Heuristics Model (PHM) (Chater & Oaksford, 1999) use 3 heuristics and predict that conclusions can be ordered

$$A > M > F > I > E >> O$$

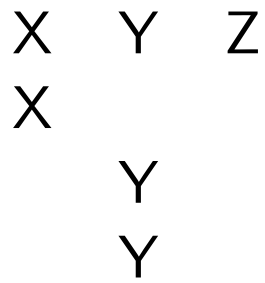
- Extension 1: Matching Hypothesis Ordered from most to least conservative quantifier:

$$E > O = I >> A$$

$$E > O = I > M = F >> A$$

Theory Predictions and Extensions

- Extension 2: Mental Models + Heuristics Construction of Mental Model and $E > I \geq F > O > M > A$



- Extension 3: Preferred Mental Models Formalization of Mental Models as (minimal) spatial models $\varphi_1 : \Omega_1 \to \mathbb{N}^2$ satisfying premise P_1 and P_2

Example Item

Some brokers are waiters.
Few waiters are agents.

What follows?

[ ] of the brokers are agents.

Quantifiers: All, Some, Some Not, Most, Few, None.
[*"Nothing follows" was not a provided option.*]

Experiment

- Online study (Amazon MT) with 25 participants.
- 40 items per participant
  - All items of Figure 1 (P1: *X* - *Y*, P2: *Y* - *Z*)
  - Conclusion: 20 trials *X* - *Z*, 20 trials *Z* - *X*
  - for each set of 20 items:
    - 6 syllogisms with *most* in P1
    - 6 syllogisms with *few* in P1
    - 4 syllogisms with *most* in P2
    - 4 syllogisms with *few* in P2
  - Different professions and hobbies constituted the content of the terms.

Predictions and Results

Observed responses and predictions of the four theories for selected syllogisms (X-Z conclusion).

Syll.	Data	PHM	Matching	PMM	Min. Models
MM	M(84%)	M, (I, O)	M	M	M
FF	F(84%)	F, (I, O)	F	F	F
IF	F(56%), I(32%)	I, (O)	I	F	F, I
FI	F(64%)	I, (O)	I	F	F, I
FO	F(44%), I(32%)	O, (I)	O	I	F
OF	F(48%), I(24%)	O, (I)	O	I	F
MO	I(56%)	O, (I)	O	I	O
OM	I(48%), F(36%)	O, (I)	O	I	O

Note. Predictions in parentheses indicate predictions by the non-preferred process, i.e., p-entailments for PHM.

Multinomial Processing Tree (MPT) models

- MPT models prominent class of measurement models for categorical data (Riefer & Batchelder, 1988).

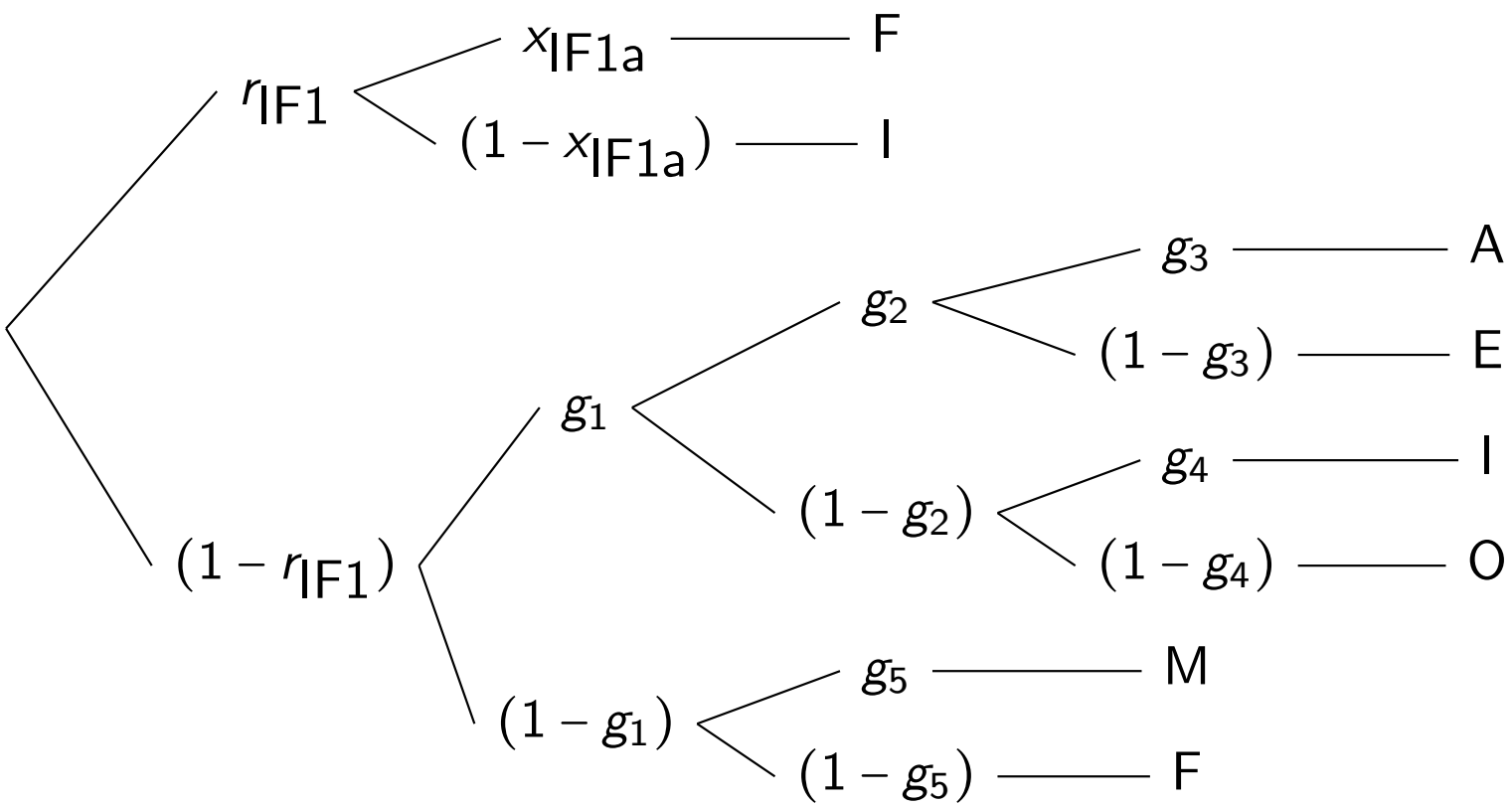
Multinomial Processing Tree (MPT) models

- MPT models prominent class of measurement models for categorical data (Riefer & Batchelder, 1988).
- Describe observed response frequencies as resulting from set of mutually exclusive latent cognitive states:
  - Reasoning state: response predicted by theories.
  - Uncertainty state: Any response can be guessed.

Multinomial Processing Tree (MPT) models

- MPT models prominent class of measurement models for categorical data (Riefer & Batchelder, 1988).
- Describe observed response frequencies as resulting from set of mutually exclusive latent cognitive states:
  - Reasoning state: response predicted by theories.
  - Uncertainty state: Any response can be guessed.
- Model parameters represent probability with which states are reached.

Multinomial Process Tree models for IF1 (MMT)



MPT model comparison

- Model for each theory consisted of 40 different trees
  - For each theory only one guessing tree (constant across all items)
  - Full dataset $40 \times 5 = 200$ available degrees of freedom

MPT model comparison

- Model for each theory consisted of 40 different trees
  - For each theory only one guessing tree (constant across all items)
  - Full dataset $40 \times 5 = 200$ available degrees of freedom
- Each model fitted to data summed across individuals
  - Full dataset had $40 \times 25 = 1000$ observations

MPT model comparison

- Model for each theory consisted of 40 different trees
  - For each theory only one guessing tree (constant across all items)
  - Full dataset $40 \times 5 = 200$ available degrees of freedom
- Each model fitted to data summed across individuals
  - Full dataset had $40 \times 25 = 1000$ observations
- Model selection: Weighing model fit and model flexibility
  - AIC and BIC: Employ number of parameters as proxy for complexity
  - FIA: Estimates the functional complexity (third term below)

$$\text{FIA} = \frac{1}{2} G^2 + \frac{k}{2} ln \frac{N}{2\pi} + ln \int \sqrt{\det I(\Theta)} \, d\Theta$$

Model Comparison

Model Comparison

Theory	k	G^2	AIC	BIC	FIA
PMM	45	235.8	325.8	**546.6**	197.8
Min. M.	49	223.5	**321.5**	562.0	195.7
Matching	49	261.7	359.7	600.1	214.2
PHM	101	**187.2**	389.2	884.9	**182.4**

Model Comparison

Model Comparison

Theory	k	G^2	AIC	BIC	FIA
PMM	45	235.8	325.8	**546.6**	197.8
Min. M.	49	223.5	**321.5**	562.0	195.7
Matching	49	261.7	359.7	600.1	214.2
PHM	101	**187.2**	389.2	884.9	**182.4**

- Matching Hypothesis outperformed which contrasts with meta-analysis on classical syllogisms (Khemlani & Johnson-Laird, 2012)

Comparison of Reasoning Parameters

Comparison of Reasoning (r_i) Parameters

Theory	Mean	SD	Median	Min	Max
PMM	.44	.21	.46	.00	.82
Minimal Models	.46	.21	.48	.00	.82
Matching	.38	.26	.39	.00	.83
PHM	.51	.25	.53	.08	.93

Note. Although .00 is the smallest value for three theories, it does not occur at the same syllogism for all of them.

Comparison of Reasoning Parameters

Comparison of Reasoning (r_i) Parameters

Theory	Mean	SD	Median	Min	Max
PMM	.44	.21	.46	.00	.82
Minimal Models	.46	.21	.48	.00	.82
Matching	.38	.26	.39	.00	.83
PHM	.51	.25	.53	.08	.93

Note. Although .00 is the smallest value for three theories, it does not occur at the same syllogism for all of them.

- r_i parameters overall larger for **M** than for **F**.

Result

- Language, Beliefs and Systems affect human reasoning
- Everyday reasoning is based on degrees of belief rather than absolute certainty (Evans, 2002)
  $\Rightarrow$ generalized quantifiers "Most" and "Few"
- Only one theory so far
  - Probability Heuristics Model (Chater & Oaksford, 1999)
  - Extended Matching Hypothesis and two MM approaches
- Formalized as MPT models and empirically evaluated
- PHM and MM approaches outperform Matching Hypothesis
  - (which shows a good fit to the data on classical syllogistic reasoning; Khemlani & Johnson-Laird, 2012)
- MPT can be (even) used to build better theories!

Thank your for your attention!

Supported by the DFG:

- Heisenberg-Programm
- "Nonmonotonic logic" im SPP "New Frameworks of Rationality"

Some fun with syllogisms

- Nothing is better than eternal happiness.
- A ham sandwich is better than nothing.
- Conclusion: A ham sandwich is better than eternal happiness.

Some fun with syllogisms

- Nothing is better than eternal happiness.
- A ham sandwich is better than nothing.
- Conclusion: A ham sandwich is better than eternal happiness.

Cave:

- "nothing is better" means the thing being named has the highest value possible;
- "better than nothing" means the thing being described has only marginal value.
- Therefore, "nothing" acts as two different words.