Exercise 9.1. Describe a polynomial-time ATM solving **EXACT INDEPENDENT SET**:

**Input:** Given a graph $G$ and some number $k$.
**Question:** Does there exists a maximal independent set in $G$ of size exactly $k$?

Find a level of the polynomial hierarchy where this problem is contained in.

Exercise 9.2. Consider the Japanese game *go-moku* that is played by two players $X$ and $O$ on a 19x19 board. Players alternately place markers on the board, and the first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalised version of go-moku on an $n \times n$ board. Say that a *position* of go-moku is a placement of markers on such a board as it could occur during the game. Define

$$ GM = \{ \langle B \rangle \mid B \text{ is a position of go-moku where } X \text{ has a winning strategy} \}.$$

Describe a polynomial-time ATM solving $GM$.

Exercise 9.3. Show that $A\text{ExpTime} = \text{ExpSpace}$.

Exercise 9.4. Show the following result: If there is any $k$ such that $\Sigma^p_k = \Sigma^p_{k+1}$ then $\Sigma^p_j = \Pi^p_j = \Sigma^p_k$ for all $j > k$, and therefore $\text{PH} = \Sigma^p_k$.

Exercise 9.5. Show that $\text{PH} \subseteq \text{PSPACE}$.