Exercise 4.1. Show that $P$ is closed under concatenation and star.

Exercise 4.2. Consider the problem CLIQUE:

Input: An undirected graph $G$ and some $k \in \mathbb{N}$
Question: Does there exist a clique in $G$ of size at least $k$?

Suppose CLIQUE can be solved in time $T(n)$ for some $T: \mathbb{N} \rightarrow \mathbb{N}$ with $T(n) \geq n$ for all $n \in \mathbb{N}$. Furthermore, show that then the optimisation problem

Input: An undirected graph $G$
Compute: A clique in $G$ of maximal size

can be computed in time $O(n \cdot T(n))$. You can assume that $T$ is monotone.

Exercise 4.3. Show that if a language $L$ is NP-complete, then $\overline{L}$ is coNP-complete.

Exercise 4.4. Show that if $P = NP$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given satisfiable propositional formula.

Exercise 4.5. Show that finding paths of a given length in undirected graphs, i.e.,

$\text{PATH} = \{ \langle G, s, t, k \rangle \mid G \text{ contains a simple path from } s \text{ to } t \text{ of length } k \}$

is NP-complete.