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Complexity Theory

Exercise 4: Time Complexity

Exercise 4.1. Show that P is closed under concatenation and star.

Exercise 4.2. Consider the problem **CLIQUE**:

Input: An undirected graph G and some $k \in \mathbb{N}$
Question: Does there exists a clique in G of size at least k?

Suppose **CLIQUE** can be solved in time $T(n)$ for some $T\colon \mathbb{N} \to \mathbb{N}$ with $T(n) \geq n$ for all $n \in \mathbb{N}$. Furthermore, show that then the optimisation problem

Input: An undirected graph G
Compute: A clique in G of maximal size

can be computed in time $\mathcal{O}(n \cdot T(n))$. You can assume that T is monotone.

Exercise 4.3. Show that if a language **L** is NP-complete, then $\overline{\mathbf{L}}$ is coNP-complete.

Exercise 4.4. Show that if $\mathrm{P} = \mathrm{NP}$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given satisfiable propositional formula.

Exercise 4.5. Show that finding paths of a given length in undirected graphs, i.e.,

$$\textsc{Path} = \{\, \langle G, s, t, k\rangle \mid G \text{ contains a simple path from } s \text{ to } t \text{ of length } k \,\}$$

is NP-complete.