Technische Universität Dresden

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Formal Concept Analysis
Exercise Sheet 5, Winter Semester 2014/15

Exercise 1 (closure system)

Definition (closure system and closure operator).

(a) *A set* $\mathfrak{A} \subseteq \mathfrak{P}(G)$ *is a* closure system *on the set* G, *iff* $G \in \mathfrak{A}$ *and* $\mathfrak{X} \subseteq \mathfrak{A} \implies \bigcap \mathfrak{X} \in \mathfrak{A}$.

(b) *A* closure operator φ *on* G *is a map* φ *which maps each subset* $X \subseteq G$ *onto the corresponding* closure $\varphi X \subseteq G$ *such that*

   1) $X \subseteq Y \implies \varphi X \subseteq \varphi Y$ $\hfill$ *(monotone)*

   2) $X \subseteq \varphi X$ $\hfill$ *(extensive)*

   3) $\varphi\varphi X = \varphi X$ $\hfill$ *(idempotent)*

   holds.

Regard the "family context" $\mathbb{K} :=(\{\text{father, mother, daughter, son}\}, \{\text{old, young, male, female}\}, \{(\text{father, old}), (\text{father, male}), (\text{mother, old}), (\text{mother, female}), (\text{daughter, young}), (\text{daughter, female}), (\text{son, young}), (\text{son, male})\})$.

   a) Explicitly list the elements of the map $\varphi\colon \mathfrak{P}(M) \to \mathfrak{P}(M)$ with $\varphi\colon B \mapsto B''$ and verify that φ is a closure operator.

   b) Verify that the set of all concept intents of the family context is a closure system.

   c) Draw a line diagram of the powerset of $\{\text{father}, \text{mother}, \text{daughter}, \text{son}\}$ and highlight the sets that have the same closure. Compare the diagram with the diagram of the concept lattice of the family context.

Exercise 2 (Next-Closure)

	old (1)	young (2)	male (3)	female (4)
father	×		×	
mother	×			×
son		×	×	
daughter		×		×

Compute all concept intents of the above "family context" using the Next-Closure algorithm. Compare your result with the concept intents from Exercise 1.

A	i	$(A\cap\{1,2,...,i-1\})\cup\{i\}$ $A+i$	$(A+i)''$ $A\oplus i$	$A <_i A\oplus i$?	new intent