TU Dresden, Fakultät Informatik
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Complexity Theory

Exercise 7: Circuit Complexity

Exercise 7.1. Define the function $\mathsf{maj}_n \colon \{0,1\}^n \to \{0,1\}^n$ by

$$\mathsf{maj}_n(x_1, \ldots, x_n) \coloneqq \begin{cases} 0 & \text{if } \sum x_i < n/2 \\ 1 & \text{if } \sum x_i \geq n/2. \end{cases}$$

Devise a circuit to compute maj_3 and test it on the example input 101 and 010.

Exercise 7.2. Denote with $\mathsf{add} \colon \{0,1\}^{2n} \to \{0,1\}^{n+1}$ the function that takes two binary n-bit numbers x and y and returns their $n+1$-bit sum. Show that add can be computed with size $\mathcal{O}(n)$ circuits.

Exercise 7.3. Show $\mathsf{NC}^1 \subseteq \mathsf{L}$.

Exercise 7.4. Show that every Boolean function with n variables can be computed with a circuit of size $\mathcal{O}(n \cdot 2^n)$.

Exercise 7.5. Show that every language $L \subseteq \{1^n \mid n \in \mathbb{N}\}$ is contained in $\mathsf{P/poly}$. Conclude that $\mathsf{P/poly}$ contains undecidable languages.

Exercise 7.6. Find a decidable language in $\mathsf{P/poly}$ that is not contained in P.

Hint:
Take a language over $\{0,1\}$ that is 2ExpTime-hard and consider its unary encoding.

Exercise 7.7. Show how to compute maj_n with circuits of size $\mathcal{O}(n \log n)$.

Exercise 7.8. Show that $\mathrm{NC} \neq \mathrm{PSPACE}$.