Problem 6.1

1. Consider the following definite programs $P_1$ and $P_2$:

   $$P_1 = \{ \text{even}(0) \leftarrow \top, \text{odd}(s(0)) \leftarrow \top, \text{even}(s(X)) \leftarrow \text{odd}(X), \text{odd}(s(X)) \leftarrow \text{even}(X) \}$$

   $$P_2 = \{ \text{even}(0) \leftarrow \top, \text{even}(s(s(0))) \leftarrow \text{even}(X), \text{even}(X) \leftarrow \text{even}(s(s(X))) \}$$

Which interpretation is a two-valued model of $P_1$ and $P_2$?

2. Consider the following definition for definite programs:

   **Acceptable Definite Program** Let $P$ be a definite program, $\ell$ a level mapping for $P$ and $M$ a two-valued model of $P$. $P$ is called **acceptable with respect to $\ell$ and $M$** if for every clause $A \leftarrow B_1, \ldots, B_m \in gP$ the following implication holds for all $i$ with $1 \leq i \leq m$:

   $$M(B_1 \land \cdots \land B_{i-1}) = \top \implies \ell(A) > \ell(B_i).$$

   $P$ is called **acceptable** if it is acceptable with respect to some level mapping and some model of $P$.

3. Are $P_1$ and $P_2$ acceptable programs? Motivate your answer.

Problem 6.2

Show that the following proposition holds:

**Proposition 19** Let $P$ be a program, $\ell$ a (total) level mapping for $P$, $I$ the set of (three-valued) interpretations for $P$, and $I, J \in I$. The function $d_\ell : I \times I \rightarrow \mathbb{R}$ defined as

$$d_\ell(I, J) = \begin{cases} 
\left(\frac{1}{2}\right)^n & I \neq J \text{ and } I(A) = J(A) \neq U \text{ for all } A \text{ with } \ell(A) < n \\
0 & I(A) \neq J(A) \text{ or } I(A) = J(A) = U \text{ for some } A \text{ with } \ell(A) = n 
\end{cases}$$

is a metric.

Problem 6.3

Show that the deduction theorem is not satisfied under Łukasiewicz and Kleene logic.

Problem 6.4

Show that the following lemma holds:

**Lemma 22** Let $I$ be the least fixed point of $\Phi_P$ and $J$ be a model of $\text{wc} \ P$. Then for every ground atom $A$, the following holds:

(1) If $I(A) = \top$ then $J(A) = \top$ and (2) If $I(A) = \bot$ then $J(A) = \bot$. 