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Complexity Theory

Exercise 4: Time Complexity

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Exercise 4.1. If G is an undirected graph, a *vertex cover* of G is a subset of the nodes where every edge of G touches one of those nodes. The vertex cover problem asks whether a graph contains a vertex cover of a specified size.

$$\textsc{Vertex-Cover} = \{\langle G, k\rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover.}\}$$

Show that **Vertex-Cover** is NP-complete.

Hint:
Try to find a reduction from satisfiability of propositional formulas.

Exercise 4.2. Show that if $\mathrm{P} = \mathrm{NP}$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given satisfiable propositional formula.

Exercise 4.3. Show that finding paths of a given length in undirected graphs, i.e.,

$$\textsc{Path} = \{\, \langle G, s, t, k\rangle \mid G \text{ contains a simple path from } s \text{ to } t \text{ of length } k \,\}$$

is NP-complete.

Exercise 4.4. Show that if every NP-hard language is also PSPACE-hard, then $\mathrm{NP} = \mathrm{PSPACE}$.

Exercise 4.5. Let $\mathbf{A}_{\text{LBA}}$ be the word problem of deterministic linear bounded automata. Show that $\mathbf{A}_{\text{LBA}}$ is PSPACE-complete.

$$\mathbf{A}_{\text{LBA}} = \{\langle \mathcal{M}, w\rangle \mid w \in \mathbf{L}(\mathcal{M}) \text{ and } \mathcal{M} \text{ is a deterministic linear bounded automata}\}$$