Exercise 4.1. If $G$ is an undirected graph, a vertex cover of $G$ is a subset of the nodes where every edge of $G$ touches one of those nodes. The vertex cover problem asks whether a graph contains a vertex cover of a specified size.

\[
\text{VERTEX-COVER} = \{ \langle G, k \rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover.} \}
\]

Show that \text{VERTEX-COVER} is NP-complete.

Hint: Try to find a reduction from satisfiability of propositional formulas.

Exercise 4.2. Show that if $P = NP$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given satisfiable propositional formula.

Exercise 4.3. Show that finding paths of a given length in undirected graphs, i.e.,

\[
\text{PATH} = \{ \langle G, s, t, k \rangle \mid G \text{ contains a simple path from } s \text{ to } t \text{ of length } k \}
\]

is NP-complete.

Exercise 4.4. Show that if every NP-hard language is also PSPACE-hard, then $NP = PSPACE$.

Exercise 4.5. Let $A_{LBA}$ be the word problem of deterministic linear bounded automata. Show that $A_{LBA}$ is PSPACE-complete.

\[
A_{LBA} = \{ \langle M, w \rangle \mid w \in L(M) \text{ and } M \text{ is a deterministic linear bounded automata} \}
\]