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Normal-Form Games: Mixed Strategies

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Previously ...

- Games can model real-life situations, but model fidelity is important.
- Noncooperative (strategic) games in **normal form** comprise players, **strategies** for the players, and **gain functions** for all **strategy profiles**.
- Various concepts can help predict/analyse the outcome of a game:
 - **Dominant strategies**
 - **Pareto optimality**
 - (pure) **Nash equilibria**
- We have analysed a number of example games: **prisoner's dilemma**, **battle of the partners**, **chicken**, **penalties**, and **two thirds of the average**.
- Pure Nash equilibria need not always exist.

Chicken

Two people, **Eli** and **Fyn**, are racing towards each other in cars. Whoever swerves ("chickens out") loses face. If neither swerves, both get seriously injured.

(Eli, Fyn)	Swerve	RaceOn
Swerve	(2, 2)	(1, 3)
RaceOn	(3, 1)	(0, 0)

Overview

Motivation

Mixed Strategies and Equilibria

Computation of Nash Equilibria

Motivation

Motivation

- So far we considered (Nash equilibria in) **pure** strategies.
- For some games, such pure equilibria did not exist ..., e.g. penalties:

Penalties

Two football players face off at a (simplified) single penalty kick. The kicker can kick left or right; the goal keeper can jump left or right. The kicker scores a goal iff they choose a different side than the keeper.

(Kicker, Keeper)	JumpL	JumpR
KickL	$(-1, 1)$	$(1, -1)$
KickR	$(1, -1)$	$(-1, 1)$

What does that predict about how the game will be played?

Background

Recall

A (discrete) **probability distribution** on a countable set S is a function

$$\pi: S \rightarrow [0, 1] \quad \text{such that} \quad \sum_{s \in S} \pi(s) = 1$$

where $[0, 1] \subseteq \mathbb{R}$ is the real unit interval.

$\pi(s)$ represents the probability of a random variable taking the value $s \in S$.

Definition

A **lottery** (S, π) consists of a countable set S and a probability distribution π on S .

Assumption: Expected Utility Hypothesis

If (S, π) is a lottery and there is a function $u: S \rightarrow \mathbb{R}$ assigning a utility to each outcome $s \in S$, then the expected utility of the lottery is given by

$$U(S, \pi) := \sum_{s \in S} u(s)\pi(s)$$

Lotteries and Risk Neutrality

Example

Consider the following events: and lotteries:

- e_0 : "You get 0€."
- e_1 : "You get 100€."
- e_2 : "You get 200€."
- $L_1 = (\{e_1\}, \{e_1 \mapsto 1.0\})$
- $L_2 = (\{e_0, e_2\}, \{e_0 \mapsto 0.5, e_2 \mapsto 0.5\})$

Which of these lotteries would you prefer?

Terminology

- A **risk neutral** player is one who is indifferent between L_1 and L_2 ;
- a **risk averse** player is one who prefers L_1 over L_2 ;
- a **risk seeking** player is one who prefers L_2 over L_1 .

We assume throughout this course that players are risk neutral.

Mixed Strategies and Equilibria

Nash Equilibrium in Mixed Strategies

Definition

Let $(P, \mathbf{S}, \mathbf{u})$ be a game in normal form with S_i finite for all $i \in P$.

1. A **mixed strategy for player** $i \in P$ is a probability distribution π_i on S_i .
 - $\pi_i(s_j)$ is the probability of the event that player i chooses strategy $s_j \in S_i$.
 - Π_i denotes the set of all probability distributions on S_i , for each $1 \leq i \leq n$.
 - Denote $\Pi := \Pi_1 \times \cdots \times \Pi_n$ and $\Pi_{-i} := \Pi_1 \times \cdots \times \Pi_{i-1} \times \Pi_{i+1} \times \cdots \times \Pi_n$.
2. The **expected utility of a mixed-strategy profile** $\boldsymbol{\pi} = (\pi_1, \dots, \pi_n)$ for i is

$$U_i(\boldsymbol{\pi}) := \sum_{\mathbf{s}=(s_1, \dots, s_n) \in \mathcal{S}} u_i(\mathbf{s}) \cdot \prod_{j=1}^n \pi_j(s_j)$$

- Π is the set of all mixed-strategy profiles for all players.
- Π_{-i} is the set of all mixed-strategy profiles for all players **except** i .
- Likewise, for $\boldsymbol{\pi} \in \Pi$ and $i \in P$, we have $\boldsymbol{\pi}_{-i} \in \Pi_{-i}$.

Mixed Strategies: Examples

Notation

Let $(P, \mathbf{S}, \mathbf{u})$ be a game with $P = \{1, \dots, n\}$ and $S_i = \{s_1, \dots, s_{k_i}\}$ for $1 \leq i \leq n$. We denote a mixed strategy π_i for player i as a k_i -tuple $\pi_i = (\pi_i(s_1), \dots, \pi_i(s_{k_i}))$.

Examples

- In penalties, a mixed strategy for **Kicker** is

$$\pi_{\text{Kicker}} = (\pi_{\text{Kicker}}(\text{KickL}), \pi_{\text{Kicker}}(\text{KickR})) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

Another mixed strategy for **Kicker** is $\pi_{\text{Kicker}} = \left(\frac{2}{3}, \frac{1}{3}\right)$.

- In rock-paper-scissors, a mixed strategy for player **Ann** is

$$\pi_{\text{Ann}} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

Best Responses and Mixed Nash Equilibria

Definition

Let $(P, \mathbf{S}, \mathbf{u})$ be a game in normal form and assume that all S_i are finite.

1. A mixed strategy $\pi_i \in \Pi_i$ is a **best response** of player i to the mixed-strategy profile $\boldsymbol{\pi}_{-i} \in \boldsymbol{\Pi}_{-i}$ iff for all mixed strategies $\pi'_i \in \Pi_i$, we have

$$U_i(\pi_1, \dots, \pi_{i-1}, \pi_i, \pi_{i+1}, \dots, \pi_n) \geq U_i(\pi_1, \dots, \pi_{i-1}, \pi'_i, \pi_{i+1}, \dots, \pi_n)$$

2. A profile $\boldsymbol{\pi} = (\pi_1, \dots, \pi_n)$ is (in) a **Nash equilibrium in mixed strategies** iff mixed strategy π_i is a best response to $\boldsymbol{\pi}_{-i}$ for all players $1 \leq i \leq n$.

Examples

- In penalties, assume **Kicker** plays $\left(\frac{1}{2}, \frac{1}{2}\right)$. A best response of **Keeper** to this is $\left(\frac{1}{2}, \frac{1}{2}\right)$; other best responses are $(1, 0)$ and $(0, 1)$.
- In Rock-Paper-Scissors, a best response to $\left(\frac{2}{3}, \frac{1}{6}, \frac{1}{6}\right)$ is $(0, 1, 0)$.

Mixed Nash Equilibria Generalise Pure NE

Observation

A pure strategy $s_j \in S_i$ is a special case of a mixed strategy π_i with $\pi_i(s_j) = 1$ and $\pi_i(s_k) = 0$ for all $s_k \in S_i$ with $k \neq j$. We conveniently denote such π_i by s_j .

Theorem

Every pure Nash equilibrium is also a mixed Nash equilibrium.

Proof.

s is a pure NE

$\implies \forall i \in P: s_i$ is a pure best response to \mathbf{s}_{-i}

$\implies \forall i \in P: \forall s_j \in S_j: u_i(\mathbf{s}_{-i}, s_i) \geq u_i(\mathbf{s}_{-i}, s_j)$

$\implies \forall i \in P: \forall s_j \in S_j: U_i(\mathbf{s}_{-i}, s_i) \geq U_i(\mathbf{s}_{-i}, s_j)$

$\implies \forall i \in P: s_i$ is a mixed best response to \mathbf{s}_{-i}

$\implies \mathbf{s}$ is a mixed NE □

The converse is not the case: there are “proper” mixed Nash equilibria.

Mixed Nash Equilibria: Characterisation (1)

Definition

The **support** of a mixed strategy π_i for player i is the set $\{s_j \mid \pi_i(s_j) > 0\}$.

Theorem

Let $\boldsymbol{\pi} = (\pi_1, \dots, \pi_n)$ be a mixed-strategy profile in a game in normal form. A mixed strategy π_i is a best response to mixed-strategy profile $\boldsymbol{\pi}_{-i}$ if and only if all pure strategies in the support of π_i are best responses to $\boldsymbol{\pi}_{-i}$.

Proof (Sketch).

“if”: Denote by S_i^+ the support of π_i . For any two $s, t \in S_i^+$, we get $U_i(\boldsymbol{\pi}_{-i}, s) = U_i(\boldsymbol{\pi}_{-i}, t)$ and for any mix $\pi: S_i^+ \rightarrow [0, 1]$, we obtain $U_i(\boldsymbol{\pi}_{-i}, s) = U_i(\boldsymbol{\pi}_{-i}, t) = U_i(\boldsymbol{\pi}_{-i}, \pi) = U_i(\boldsymbol{\pi}_{-i}, \pi_i)$.

“only if”: If $s \in S_i^+$ is in the support of π_i but not a best response to $\boldsymbol{\pi}_{-i}$, we can remove s from π_i to obtain a better response, contradiction. \square

Mixed Nash Equilibria: Characterisation (2)

By definition, a mixed-strategy profile $\boldsymbol{\pi}$ is a Nash equilibrium iff for all $i \in P$,

$$U_i(\boldsymbol{\pi}) = \max_{\pi'_i \in \Pi_i} U_i(\boldsymbol{\pi}_{-i}, \pi'_i)$$

where $U_i(\boldsymbol{\pi}_{-i}, \pi'_i)$ denotes $U_i(\pi_1, \dots, \pi_{i-1}, \pi'_i, \pi_{i+1}, \dots, \pi_n)$.

By the previous theorem it is enough to focus on the pure strategies, thus

$$\max_{\pi'_i \in \Pi_i} U_i(\boldsymbol{\pi}_{-i}, \pi'_i) = \max_{s_j \in S_i} U_i(\boldsymbol{\pi}_{-i}, s_j)$$

Additionally, for all $i \in P$ and for any two $s_i, t_i \in \{s \in S_i \mid \pi_i(s) > 0\}$:

$$U_i(\boldsymbol{\pi}) = \max_{s_j \in S_i} U_i(\boldsymbol{\pi}_{-i}, s_j) = U_i(\boldsymbol{\pi}_{-i}, s_i) = U_i(\boldsymbol{\pi}_{-i}, t_i)$$

Thus in a Nash equilibrium, players are **indifferent** between their pure strategies.

Mixed Nash Equilibria: Examples (1)

Battle of the Partners

- By the previous theorem, both pure Nash equilibria are also mixed ones.
- A third, mixed Nash equilibrium $\boldsymbol{\pi}$ is obtained by reasoning as follows:

$$U_{\text{Cat}}(\text{Cinema}) = 10 \cdot \pi_{\text{Dee}}(\text{Cinema}) + 2 \cdot \pi_{\text{Dee}}(\text{Dancing})$$

$$U_{\text{Cat}}(\text{Dancing}) = 0 \cdot \pi_{\text{Dee}}(\text{Cinema}) + 7 \cdot \pi_{\text{Dee}}(\text{Dancing})$$

To make **Cat** indifferent between the two choices, **Dee** must choose the values for $\pi_{\text{Dee}}(\text{Cinema})$ and $\pi_{\text{Dee}}(\text{Dancing})$ such that $U_{\text{Cat}}(\text{Cinema}) = U_{\text{Cat}}(\text{Dancing})$, i.e.

$$10 \cdot \pi_{\text{Dee}}(\text{Cinema}) + 2 \cdot \pi_{\text{Dee}}(\text{Dancing}) = 7 \cdot \pi_{\text{Dee}}(\text{Dancing})$$

With $\pi_{\text{Dee}}(\text{Cinema}) + \pi_{\text{Dee}}(\text{Dancing}) = 1$, we obtain $\pi_{\text{Dee}} = \left(\frac{1}{3}, \frac{2}{3}\right)$.

By symmetry, we get $\boldsymbol{\pi} = (\pi_{\text{Cat}}, \pi_{\text{Dee}}) = \left(\left(\frac{2}{3}, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{2}{3}\right)\right)$ with $U_{\text{Cat}}(\boldsymbol{\pi}) = 4\frac{2}{3}$.

Mixed Nash Equilibria: Examples (2)

Penalties

- For the mixed strategies $\pi_{\text{Kicker}} = \left(\frac{1}{2}, \frac{1}{2}\right)$ and $\pi_{\text{Keeper}} = \left(\frac{1}{2}, \frac{1}{2}\right)$, the mixed-strategy profile $\boldsymbol{\pi} = (\pi_{\text{Kicker}}, \pi_{\text{Keeper}})$ is a (strict) Nash equilibrium with expected utilities $U_{\text{Kicker}}(\boldsymbol{\pi}) = 2 \cdot \frac{1}{4} \cdot (-1) + 2 \cdot \frac{1}{4} \cdot 1 = 0 = U_{\text{Keeper}}(\boldsymbol{\pi})$:
- If (e.g.) **Kicker** were to deviate by (e.g.) playing $\boldsymbol{\pi}' = \left(\frac{2}{3}, \frac{1}{3}\right)$, then **Keeper** would best-respond by playing (1, 0), that is, playing **JumpL**, leading to expected utilities $U_{\text{Kicker}}(\boldsymbol{\pi}') = \frac{2}{3} \cdot (-1) + \frac{1}{3} \cdot 1 = -\frac{1}{3} = -U_{\text{Keeper}}(\boldsymbol{\pi}')$.

Rock-Paper-Scissors

- Similarly, for $\pi_{\text{Ann}} = \pi_{\text{Bob}} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, the mixed-strategy profile $\boldsymbol{\pi} = (\pi_{\text{Ann}}, \pi_{\text{Bob}})$ is a (strict) Nash equilibrium in mixed strategies.
- Note that every pure strategy is a best response to $\boldsymbol{\pi}$.

Mixed Strategies: Discussion

Question

What does it mean to play a mixed strategy?

There are a (at least) four answers:

- Players may randomise to **confuse** their opponents.
- Players randomise because they are **uncertain** about actions of others.
- Mixed strategies describe what might happen in **repeated** play.
- Mixed strategies describe **population dynamics**: they describe the probability of choosing a specific pure strategy out of a population of pure strategies.

Nash's Theorem

Theorem (Nash, 1950)

Let $G = (P, \mathbf{S}, \mathbf{u})$ be a noncooperative game in normal form.

If $P = \{1, \dots, n\}$ is finite and for $\mathbf{S} = (S_1, \dots, S_n)$ each S_i is finite, then there exists a Nash equilibrium in mixed strategies.

Proof Sketch.

- View pure strategies $s_j \in S_j$ as unit vectors in $\mathbb{R}^{|S_j|}$; mixed strategies $\pi_j \in \Pi_j$ are then points of a simplex, a convex subset of $\mathbb{R}^{|S_j|}$; Π is a simplotope.
- Define functions $\varphi_{ij}(\boldsymbol{\pi}) = \max \{0, U_i(\boldsymbol{\pi}_{-i}, s_j) - U_i(\boldsymbol{\pi})\}$ for $i \in P, 1 \leq j \leq |S_i|$.
- Define (continuous) function $f: \Pi \rightarrow \Pi$ with $\boldsymbol{\pi} \mapsto \boldsymbol{\pi}' = (\pi'_1, \dots, \pi'_n)$, where

$$\pi'_i(s_j) := \frac{\pi_i(s_j) + \varphi_{ij}(\boldsymbol{\pi})}{\sum_{s_k \in S_i} (\pi_i(s_k) + \varphi_{ik}(\boldsymbol{\pi}))}$$

- Use Brouwer's fixpoint theorem to deduce that f has at least one fixpoint.
- Show that $f(\boldsymbol{\pi}) = \boldsymbol{\pi}$ if and only if $\boldsymbol{\pi}$ is a mixed Nash equilibrium for G . □

Computation of Nash Equilibria

Approaches to Find Equilibria for Two Players

Lemke-Howson algorithm (1964)

- Path-finding approach with geometrical interpretation
- Needs exponentially many steps in the worst case

Porter-Nudelman-Shoham (2004)

- Enumerates possible supports of mixed strategies, checks for equilibria
- Dominance checks and search bias for optimisation

Mixed Integer Programming (Sandholm, Gilpin, and Conitzer, 2005)

- Encode equilibrium property for a given game as a mixed integer program, i.e., as a mathematical (numerical) feasibility problem
- “Mixed” expresses that values for some variables may be real numbers

Mixed Integer Programming (in a Nutshell)

Definition

- A **mixed integer (linear) program** is of the form

$$\begin{aligned} & \text{maximise} && \mathbf{c}^T \mathbf{x} \\ & \text{subject to} && \mathbf{Ax} \leq \mathbf{b}, \\ & && \mathbf{x} \geq 0, \\ & && \text{and } \mathbf{x} \in \mathbb{Z}^k \times \mathbb{R}^\ell \end{aligned}$$

where \mathbf{x} is a vector of **decision variables**, and \mathbf{A} , \mathbf{b} , \mathbf{c} are a matrix and two vectors of real values; the expression $\mathbf{c}^T \mathbf{x}$ is the **objective function**.

- If there is no objective function the program is a **feasibility problem**.
- A **solution** is a variable-value assignment that satisfies all constraints.

E.g.: maximise $2x_1 - 3x_2$ subject to $x_1 + x_2 \leq 7$, $2x_1 - x_2 \leq 12$, $x_1, x_2 \geq 0$, and $(x_1, x_2) \in \mathbb{Z} \times \mathbb{R}$

Area of active research; used in industrial applications; solvers exist, e.g. SCIP.

Regret

Definition

Let $(P, \mathbf{S}, \mathbf{u})$ be a noncooperative game in normal form, $i \in P$, and $s_j \in S_j$. The **regret** of i playing s_j w.r.t. opponent profile $\boldsymbol{\pi}_{-i}$ is

$$r_{\boldsymbol{\pi}_{-i}, s_j} := \left(\max_{\pi_k \in \Pi_i} U_i(\boldsymbol{\pi}_{-i}, \pi_k) \right) - U_i(\boldsymbol{\pi}_{-i}, s_j)$$

Example: Prisoner's Dilemma

- The regret of playing **Silent** in response to **Confess** is 1.
- The regret of **Confess** in response to any opponent strategy (profile) is 0.

More generally: The regret of any best response is zero.

Observation

A mixed strategy profile $\boldsymbol{\pi}$ is a Nash equilibrium if and only if every pure strategy is either played with probability zero or has zero regret.

Computing Nash Equilibria via MIP (1)

For every player $i \in P$ and pure strategy $s_j \in S_i$, introduce decision variables

- b_{s_j} ... to express that s_j is *not* played by i , i.e.
 - $b_{s_j} = 1$ expresses that $\pi_i(s_j) = 0$, and
 - $b_{s_j} = 0$ expresses that $\pi_i(s_j) > 0$ and $r_{\pi_{-i}, s_j} = 0$;
- u_i ... to express the maximal utility of i given π_{-i} ;
- p_{s_j} ... to express the probability $\pi_i(s_j)$ with which s_j is played;
- u_{s_j} ... to express the expected utility $U(\pi_{-i}, s_j)$ from playing s_j ;
- r_{s_j} ... to express the regret r_{π_{-i}, s_j} from playing s_j .

The formulation also uses the constants v_i , denoting the maximally possible difference between two payoffs for player i :

$$v_i := \max_{\substack{s_u^{(i)}, s_\ell^{(i)} \in S_i, \\ s_u^{(3-i)}, s_\ell^{(3-i)} \in S_{3-i}}} \left\{ u_i(s_u^{(1)}, s_u^{(2)}) - u_i(s_\ell^{(1)}, s_\ell^{(2)}) \right\}$$

(Note that $3 - i$ for $i \in P = \{1, 2\}$ just refers to the player other than i .)

Computing Nash Equilibria via MIP (2)

Definition

Let $G = (P, \mathbf{S}, \mathbf{u})$ be a strategic game with $P = \{1, 2\}$. Its MIP formulation is

find $\mathbf{p}_{S_j}, \mathbf{u}_i, \mathbf{u}_{S_j}, \mathbf{r}_{S_j}, \mathbf{b}_{S_j}$ such that

$$\forall i: \sum_{S_j \in S_i} \mathbf{p}_{S_j} = 1 \quad (1)$$

$$\forall i: \forall S_j \in S_i: \mathbf{u}_{S_j} = \sum_{S_k \in S_{3-i}} u_i(s_k, S_j) \mathbf{p}_{S_k} \quad (2)$$

$$\forall i: \forall S_j \in S_i: \mathbf{r}_{S_j} = \mathbf{u}_i - \mathbf{u}_{S_j} \quad (3)$$

$$\forall i: \forall S_j \in S_i: \mathbf{p}_{S_j} \leq 1 - \mathbf{b}_{S_j} \quad (4)$$

$$\forall i: \forall S_j \in S_i: \mathbf{r}_{S_j} \leq \nu_i \mathbf{b}_{S_j} \quad (5)$$

where $\mathbf{p}_{S_j}, \mathbf{u}_i, \mathbf{u}_{S_j}, \mathbf{r}_{S_j} \geq 0$ and $\mathbf{b}_{S_j} \in \{0, 1\}$

Computing Nash Equilibria via MIP (3)

The intuition behind the constraints is as follows:

- (1) Guarantees that the values of the p_{s_j} constitute a valid probability distribution for each player.
- (2) Guarantees the correct utility value for playing s_j (where $u_i(s_k, s_j)$ denotes the constant $u_1(s_j, s_k)$ for $i = 1$ and the constant $u_2(s_k, s_j)$ otherwise).
- (3) Guarantees the correct regret value for playing s_j .
- (4) Guarantees that the probability of playing s_j is indeed zero whenever the strategy is chosen not to be played (otherwise the constraint is vacuous).
- (5) Guarantees that the regret for playing s_j is indeed zero whenever the strategy is chosen to be played (otherwise the constraint is vacuous).

Proposition

For any two-player strategic game, the solutions of the MIP formulation correspond one-to-one to the mixed Nash equilibria of the game.

Computing Nash Equilibria via MIP: Example

Consider Battle of the Partners with
 $P = \{\text{Cat}, \text{Dee}\}$ and the payoffs on the right.

(Cat, Dee)	Cinema	Dancing
Cinema	(10, 7)	(2, 2)
Dancing	(0, 0)	(7, 10)

We obtain the following feasibility problem:

$$\begin{array}{l}
 \text{find } p_s^i, u_i, u_s^i, r_s^i, b_s^i \text{ for } i \in P, s \in \{\text{Cinema}, \text{Dancing}\} \text{ such that} \\
 p_{\text{Cinema}}^{\text{Cat}} + p_{\text{Dancing}}^{\text{Cat}} = 1 \qquad \text{and } p_{\text{Cinema}}^{\text{Dee}} + p_{\text{Dancing}}^{\text{Dee}} = 1 \\
 u_{\text{Cinema}}^{\text{Cat}} = 10 \cdot p_{\text{Cinema}}^{\text{Dee}} + 2 \cdot p_{\text{Dancing}}^{\text{Dee}} \qquad \text{and } u_{\text{Cinema}}^{\text{Dee}} = 7 \cdot p_{\text{Cinema}}^{\text{Cat}} + 0 \cdot p_{\text{Dancing}}^{\text{Cat}} \\
 u_{\text{Dancing}}^{\text{Cat}} = 0 \cdot p_{\text{Cinema}}^{\text{Dee}} + 7 \cdot p_{\text{Dancing}}^{\text{Dee}} \qquad \text{and } u_{\text{Dancing}}^{\text{Dee}} = 2 \cdot p_{\text{Cinema}}^{\text{Cat}} + 10 \cdot p_{\text{Dancing}}^{\text{Cat}} \\
 \forall i \in P: r_{\text{Cinema}}^i = u_i - u_{\text{Cinema}}^i \qquad \text{and } r_{\text{Dancing}}^i = u_i - u_{\text{Dancing}}^i \\
 \forall i \in P: p_{\text{Cinema}}^i \leq 1 - b_{\text{Cinema}}^i \qquad \text{and } p_{\text{Dancing}}^i \leq 1 - b_{\text{Dancing}}^i \\
 \forall i \in P: r_{\text{Cinema}}^i \leq 10 \cdot b_{\text{Cinema}}^i \qquad \text{and } r_{\text{Dancing}}^i \leq 10 \cdot b_{\text{Dancing}}^i \\
 \text{where } p_s, u_i, u_s, r_s \geq 0 \text{ and } b_s^i \in \{0, 1\}
 \end{array}$$

Conclusion

Summary

- A **mixed strategy** is a probability distribution on pure strategies.
- In a **mixed Nash equilibrium**, all players play best responses.
- **Nash's Theorem:** Mixed Nash equilibria always exist (for finite games).
- Nash equilibria for concrete games can be obtained via a translation to a **mixed integer program**:
 - Binary variables model the choices of pure strategies to put in the support;
 - real-valued variables model probabilities, utilities, and regret.

Action Points

- Obtain all (mixed) Nash equilibria for chicken and interpret them.