Exercise Sheet 11

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January 27, 2026

Exercise 5

Review the slides from Lecture 24, and especially slides 13–18 on calculating the odds in a quantum-based game strategy.

1. Can Bob use the experiment to learn about Alice's choice, possibly by repeating the experiment with many pairs of entangled particles? Compute the chances of Bob measuring 0 in each case considered on the slides.
2. Suppose Alice measures her bit without any rotation happening before. Specify the possible states of Bob's remaining 1-qubit system after this.
3. Consider the case $x = 0$ and $y = 1$, and the following order of events: Alice measures, Bob rotates, Bob measures. What is the probability of the game being won in this case.

Exercise 5

Two players, Alice and Bob, are in physically separated locations:

- The game master throws two coins: $x, y \in \{0, 1\}$
- The value of x is communicated to Alice, the value of y to Bob
- Alice must answer with a bit $a \in \{0, 1\}$, and Bob with a bit $b \in \{0, 1\}$
- The players win if $a \oplus b = x \wedge y$.

A strategy that uses Quantum Mechanics:

- Alice and Bob create an entangled quantum state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
- Alice takes the first bit (particle); Bob the second.
- Alice's strategy: if $x = 1$, rotate the first bit by $\pi/8$ (22.5 degrees), otherwise do nothing; then measure the first bit and answer with its value
- Bob's strategy: if $y = 1$, rotate the second bit by $-\pi/8$ (-22.5 degrees), otherwise do nothing; then measure the second bit and answer with its value

Exercise 5

Can Bob use the experiment to learn about Alice's choice, possibly by repeating the experiment with many pairs of entangled particles? Compute the chances of Bob measuring 0 in each case considered on the slides.

Case 1: $x = 0$ and $y = 0$.
None of them rotate.

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{matrix} \text{corresponds to } |00\rangle \\ \text{corresponds to } |01\rangle \\ \text{corresponds to } |10\rangle \\ \text{corresponds to } |11\rangle \end{matrix}$$

The players win if $a \oplus b = x \wedge y$. That is, they win if $a = b$.
The probability of this happening is $(\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = 1$.

The probability of Bob's bit (i.e., the second bit) being 0 is $(\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$.

Exercise 5

Can Bob use the experiment to learn about Alice's choice, possibly by repeating the experiment with many pairs of entangled particles? Compute the chances of Bob measuring 0 in each case considered on the slides.

Case 2: $x = 0$ and $y = 1$.
Bob rotates; Alice does not do anything.

$$\begin{pmatrix} \cos\frac{\pi}{8} & \sin\frac{\pi}{8} & 0 & 0 \\ -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} & 0 & 0 \\ 0 & 0 & \cos\frac{\pi}{8} & \sin\frac{\pi}{8} \\ 0 & 0 & -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{8} \\ \frac{1}{\sqrt{2}}\sin\frac{\pi}{8} \\ \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} \end{pmatrix} \begin{matrix} \text{corresponds to } |00\rangle \\ \text{corresponds to } |01\rangle \\ \text{corresponds to } |10\rangle \\ \text{corresponds to } |11\rangle \end{matrix}$$

The players win if $a \oplus b = x \wedge y$. That is, they win if $a = b$.
The probability of this happening is $2(\frac{1}{\sqrt{2}}\cos\frac{\pi}{8})^2 = (\cos\frac{\pi}{8})^2 > 0.853$

The probability of Bob's bit (i.e., the second bit) being 0 is $(\frac{1}{\sqrt{2}}\cos\frac{\pi}{8})^2 + (\frac{1}{\sqrt{2}}\sin\frac{\pi}{8})^2 = \frac{1}{2}$.

Exercise 5

Can Bob use the experiment to learn about Alice's choice, possibly by repeating the experiment with many pairs of entangled particles? Compute the chances of Bob measuring 0 in each case considered on the slides.

Case 3: $x = 1$ and $y = 0$.
Alice rotates; Bob does not do anything.

$$\begin{pmatrix} \cos\frac{\pi}{8} & 0 & -\sin\frac{\pi}{8} & 0 \\ 0 & \cos\frac{\pi}{8} & 0 & -\sin\frac{\pi}{8} \\ \sin\frac{\pi}{8} & 0 & \cos\frac{\pi}{8} & 0 \\ 0 & \sin\frac{\pi}{8} & 0 & \cos\frac{\pi}{8} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{8} \\ \frac{1}{\sqrt{2}}\sin\frac{\pi}{8} \\ \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} \end{pmatrix} \quad \begin{matrix} \text{corresponds to } |00\rangle \\ \text{corresponds to } |01\rangle \\ \text{corresponds to } |10\rangle \\ \text{corresponds to } |11\rangle \end{matrix}$$

The players win if $a \oplus b = x \wedge y$. That is, they win if $a = b$.
The probability of this happening is $2(\frac{1}{\sqrt{2}}\cos\frac{\pi}{8})^2 = (\cos\frac{\pi}{8})^2 > 0.853$

The probability of Bob's bit (i.e., the second bit) being 0 is $(\frac{1}{\sqrt{2}}\cos\frac{\pi}{8})^2 + (\frac{1}{\sqrt{2}}\sin\frac{\pi}{8})^2 = \frac{1}{2}$.

Exercise 5

Case 4: $x = 1$ and $y = 1$.

Bob rotates.

$$\begin{pmatrix} \cos\frac{\pi}{8} & \sin\frac{\pi}{8} & 0 & 0 \\ -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} & 0 & 0 \\ 0 & 0 & \cos\frac{\pi}{8} & \sin\frac{\pi}{8} \\ 0 & 0 & -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{8} \\ \frac{1}{\sqrt{2}}\sin\frac{\pi}{8} \\ \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} \end{pmatrix} \begin{matrix} \text{corresponds to } |00\rangle \\ \text{corresponds to } |01\rangle \\ \text{corresponds to } |10\rangle \\ \text{corresponds to } |11\rangle \end{matrix}$$

Then Alice rotates.

$$\begin{pmatrix} \cos\frac{\pi}{8} & 0 & -\sin\frac{\pi}{8} & 0 \\ 0 & \cos\frac{\pi}{8} & 0 & -\sin\frac{\pi}{8} \\ \sin\frac{\pi}{8} & 0 & \cos\frac{\pi}{8} & 0 \\ 0 & \sin\frac{\pi}{8} & 0 & \cos\frac{\pi}{8} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{8} \\ \frac{1}{\sqrt{2}}\sin\frac{\pi}{8} \\ \frac{1}{\sqrt{2}}\cos\frac{\pi}{8} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} (\cos\frac{\pi}{8})^2 - (\sin\frac{\pi}{8})^2 \\ -2\cos\frac{\pi}{8}\sin\frac{\pi}{8} \\ 2\cos\frac{\pi}{8}\sin\frac{\pi}{8} \\ (\cos\frac{\pi}{8})^2 - (\sin\frac{\pi}{8})^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

The players win if $a \oplus b = x \wedge y$. That is, they win if $a \neq b$.
The probability of this happening is $(-\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

The probability of Bob's bit (i.e., the second bit) being 0 is $(\frac{1}{2})^2 + (\frac{1}{2})^2 = \frac{1}{2}$.

Exercise 5

Suppose Alice measures her bit without any rotation happening before. Specify the possible states of Bob's remaining 1-qubit system after this.

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{matrix} \text{corresponds to } |00\rangle \\ \text{corresponds to } |01\rangle \\ \text{corresponds to } |10\rangle \\ \text{corresponds to } |11\rangle \end{matrix}$$

Exercise 5

Consider the case $x = 0$ and $y = 1$, and the following order of events: Alice measures, Bob rotates, Bob measures. What is the probability of the game being won in this case.

Case 1: Alice reads a 0.

$$\begin{pmatrix} \cos\frac{\pi}{8} & \sin\frac{\pi}{8} & 0 & 0 \\ -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} & 0 & 0 \\ 0 & 0 & \cos\frac{\pi}{8} & \sin\frac{\pi}{8} \\ 0 & 0 & -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{8} \\ -\sin\frac{\pi}{8} \\ 0 \\ 0 \end{pmatrix} \begin{matrix} \text{corresponds to } |00\rangle \\ \text{corresponds to } |01\rangle \\ \text{corresponds to } |10\rangle \\ \text{corresponds to } |11\rangle \end{matrix}$$

Alice and Bob win the game if $a = b$.
The probability of this happening is $(\cos\frac{\pi}{8})^2 > 0.853$.

Exercise 5

Consider the case $x = 0$ and $y = 1$, and the following order of events: Alice measures, Bob rotates, Bob measures. What is the probability of the game being won in this case.

Case 2: Alice reads a 1.

$$\begin{pmatrix} \cos\frac{\pi}{8} & \sin\frac{\pi}{8} & 0 & 0 \\ -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} & 0 & 0 \\ 0 & 0 & \cos\frac{\pi}{8} & \sin\frac{\pi}{8} \\ 0 & 0 & -\sin\frac{\pi}{8} & \cos\frac{\pi}{8} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sin\frac{\pi}{8} \\ \cos\frac{\pi}{8} \end{pmatrix} \begin{matrix} \text{corresponds to } |00\rangle \\ \text{corresponds to } |01\rangle \\ \text{corresponds to } |10\rangle \\ \text{corresponds to } |11\rangle \end{matrix}$$

Alice and Bob win the game if $a = b$.
The probability of this happening is $(\cos\frac{\pi}{8})^2 > 0.853$.