TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic Group

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Description Logics – Syntax and Semantics I

Lecture 4, 11th Nov 2024 // Foundations of Knowledge Representation, WS 2024/25

Motivation

Undecidable	FOL sat
Non-Elementary decidable	...
...	...
ExpTime	Datalog sat
PSpace	...
NP	PL sat
P	Horn PL sat

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Many KR applications do not require the full power of FOL

What can we leave out?

- Key reasoning problems should become decidable
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Description Logics are a family of FOL fragments that meet these requirements for many applications:

- Underlying formalisms of modern ontology languages
- Widely used in bio-medical information systems
- Core component of the Semantic Web

Motivation

Recall our arthritis example:

- A juvenile disease affects only children or teenagers
- Children and teenagers are not adults
- A person is either a child, a teenager, or an adult
- Juvenile arthritis is a kind of arthritis and a juvenile disease
- Every kind of arthritis damages some joint

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The important types of objects are given by unary FOL predicates:

juvenile disease, child, teenager, adult, ...

The types of relationships are given by binary FOL predicates:

affects, damages, ...

Motivation

The vocabulary of a Description Logic is composed of

- Unary FOL predicates

  Arthritis, Child, …

- Binary FOL predicates

  Affects, Damages, …

- FOL constants

  JohnSmith, *MaryJones*, *JRA*, …

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We are already restricting the expressive power of FOL

- No function symbols (of positive arity)
- No predicates of arity greater than 2

Motivation

Let us take a closer look at the FOL formulas for our example:

$$\forall x.(\mathit{JuvDis}(x) \to \forall y.(\mathit{Affects}(x,y) \to \mathit{Child}(y) \lor \mathit{Teen}(y)))$$
$$\forall x.(\mathit{Child}(x) \lor \mathit{Teen}(x) \to \neg \mathit{Adult}(x))$$
$$\forall x.(\mathit{Person}(x) \to \mathit{Child}(x) \lor \mathit{Teen}(x) \lor \mathit{Adult}(x))$$
$$\forall x.(\mathit{JuvArthritis}(x) \to \mathit{Arthritis}(x) \land \mathit{JuvDis}(x))$$
$$\forall x.(\mathit{Arthritis}(x) \to \exists y.(\mathit{Damages}(x,y) \land \mathit{Joint}(y)))$$

We can find several regularities in these formulas:

- There is an outermost universal quantifier on a single variable x
- The formulas can be split into two parts by the implication symbol
  - Each part is a formula with one free variable
- Atomic formulas involving a binary predicate occur only quantified in a syntactically restricted way.

Complexity

Undecidable	FOL-3 sat
Non-Elementary decidable	. . .
. . .	. . .
NExpTime	FOL-2 sat
ExpTime	Datalog sat
PSpace	. . .
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Let us look at all its sub-formulas at each side of the implication

$\mathit{Child}(x)$	Set of all children
$\mathit{Teen}(x)$	Set of all teenagers
$\mathit{Child}(x) \lor \mathit{Teen}(x)$	Set of all objects that are children or teenagers
$\mathit{Adult}(x)$	Set of all adults
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Important observations concerning formulas with one free variable:

- Some are atomic (e.g., $\mathit{Child}(x)$)

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Important observations concerning formulas with one free variable:

- Some are atomic (e.g., $\mathit{Child}(x)$)
  do not contain other formulas as subformulas
- Others are complex (e.g., $\mathit{Child}(x) \lor \mathit{Teen}(x)$)

Basic Definitions

Idea: Define operators for constructing complex formulas with one free variable out of simple building blocks

Atomic Concept: Represents an atomic formula with one free variable

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Complex concepts (part 1):

- Concept Union ($\sqcup$): applies to two concepts

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- Concept Intersection ($\sqcap$): applies to two concepts

$$\mathit{Arthritis} \sqcap \mathit{JuvDis} \quad \rightsquigarrow \quad \mathit{Arthritis}(x) \wedge \mathit{JuvDis}(x)$$

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- Concept Intersection ($\sqcap$): applies to two concepts

$$\mathit{Arthritis} \sqcap \mathit{JuvDis} \quad \leadsto \quad \mathit{Arthritis}(x) \land \mathit{JuvDis}(x)$$

- Concept Negation ($\neg$): applies to one concept

$$\neg \mathit{Adult} \quad \leadsto \quad \neg \mathit{Adult}(x)$$

Motivation

Consider examples with binary predicates:

$$\forall x.(\mathit{Arthritis}(x) \to \exists y.(\mathit{Damages}(x, y) \land \mathit{Joint}(y)))$$

$$\forall x.(\mathit{JuvDis}(x) \to \forall y.(\mathit{Affects}(x, y) \to (\mathit{Child}(y) \lor \mathit{Teen}(y))))$$

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- We have a concept and a binary predicate (called a role) mentioning the concept's free variable
- The role and the concept are connected via conjunction (existential quantification) or implication (universal quantification)
- Nested sub-concepts use a fresh (existentially/universally quantified) variable, and are connected to the surrounding concept by exactly one role atom (often called a guard)

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$$\exists \mathit{Damages}.\mathit{Joint} \quad \rightsquigarrow \quad \exists y.(\mathit{Damages}(x, y) \land \mathit{Joint}(y))$$

- Universal Restriction:

$$\forall \mathit{Affects}.(\mathit{Child} \sqcup \mathit{Teen}) \quad \rightsquigarrow \quad \forall y.(\mathit{Affects}(x, y) \to (\mathit{Child}(y) \lor \mathit{Teen}(y)))$$

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Concepts describe sets of objects with certain common features:

$\mathit{Woman} \sqcap \exists \mathit{hasChild}.(\exists \mathit{hasChild}.\mathit{Person})$	Women with a grandchild
$\mathit{Disease} \sqcap \forall \mathit{Affects}.\mathit{Child}$	Diseases affecting only children
$\mathit{Person} \sqcap \neg\exists \mathit{owns}.\mathit{DetHouse}$	People not owning a detached house
$\mathit{Man} \sqcap \exists \mathit{hasChild}.\top \sqcap \forall \mathit{hasChild}.\mathit{Man}$	Fathers having only sons

$\leadsto$ Very useful idea for Knowledge Representation

General Concept Inclusion Axioms

Recall our example formulas:

$$\forall x.(\mathit{JuvDis}(x) \rightarrow \forall y.(\mathit{Affects}(x,y) \rightarrow \mathit{Child}(y) \vee \mathit{Teen}(y)))$$

$$\forall x.(\mathit{Child}(x) \vee \mathit{Teen}(x) \rightarrow \neg \mathit{Adult}(x))$$

$$\forall x.(\mathit{Person}(x) \rightarrow \mathit{Child}(x) \vee \mathit{Teen}(x) \vee \mathit{Adult}(x))$$

$$\forall x.(\mathit{JuvArthritis}(x) \rightarrow \mathit{Arthritis}(x) \wedge \mathit{JuvDis}(x))$$

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They are of the following form, with $\alpha_C(x)$ and $\alpha_D(x)$ corresponding to $\mathcal{ALC}$ concepts C and D:

$$\forall x.(\alpha_C(x) \to \alpha_D(x))$$

Such sentences are $\mathcal{ALC}$ General Concept Inclusion Axioms (GCIs)

$$C \sqsubseteq D$$

where C and D are $\mathcal{ALC}$-concepts

General Concept Inclusion Axioms

$$\forall x.(\mathit{JuvDis}(x) \to$$
$$\forall y.(\mathit{Affects}(x,y) \to \mathit{Child}(y) \lor \mathit{Teen}(y))) \quad \rightsquigarrow$$
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Note that we often use $C \equiv D$ as an abbreviation for a symmetrical pair of GCIs $C \sqsubseteq D$ and $D \sqsubseteq C$, e.g.:

$$\left.\begin{array}{r} \mathit{Arth} \sqcap \mathit{JuvDis} \sqsubseteq \mathit{JuvArth} \\ \mathit{JuvArth} \sqsubseteq \mathit{Arth} \sqcap \mathit{JuvDis} \end{array}\right\} \quad \rightsquigarrow \quad \mathit{JuvArth} \equiv \mathit{Arth} \sqcap \mathit{JuvDis}$$

Terminological Statements

GCIs allow us to represent a surprising variety of terminological statements:

- Sub-type statements

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- Covering statements:

$$\forall x.(\mathit{Person}(x) \to \mathit{Adult}(x) \lor \mathit{Child}(x)) \quad \rightsquigarrow \quad \mathit{Person} \sqsubseteq \mathit{Adult} \sqcup \mathit{Child}$$

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- Disjointness statements:

$$\forall x.(\mathit{Child}(x) \rightarrow \neg \mathit{Adult}(x)) \quad \rightsquigarrow \quad \mathit{Child} \sqsubseteq \neg \mathit{Adult}$$

- Covering statements:

$$\forall x.(\mathit{Person}(x) \rightarrow \mathit{Adult}(x) \lor \mathit{Child}(x)) \quad \rightsquigarrow \quad \mathit{Person} \sqsubseteq \mathit{Adult} \sqcup \mathit{Child}$$

- Type (domain and range) restrictions:

$$\forall x.(\forall y.(\mathit{Affects}(x,y) \rightarrow \mathit{Arth}(x) \land \mathit{Person}(y))) \quad \rightsquigarrow \quad \begin{array}{l} \exists \mathit{Affects}.\top \sqsubseteq \mathit{Arth} \\ \top \sqsubseteq \forall \mathit{Affects}.\mathit{Person} \end{array}$$

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- E.g., axioms where the l.h.s. is atomic are sometimes called definitions:
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  - A primitive concept definition specifies only necessary conditions for instances, e.g.:

    $$Arth \sqsubseteq \exists Damages.Joint$$

Data Assertions

In description logics, we can also represent data:

$\mathit{Child}(\mathit{JohnSmith})$	John Smith is a child
$\mathit{JuvenileArthritis}(\mathit{JRA})$	JRA is a juvenile arthritis
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Examples of data assertions in $\mathcal{ALC}$:

$$\exists \mathit{hasChild}.\mathit{Teacher}(\mathit{John}) \quad \rightsquigarrow \quad \exists y.(\mathit{hasChild}(\mathit{John}, y) \land \mathit{Teacher}(y))$$

$$\mathit{HistorySt} \sqcup \mathit{ClassicsSt}(\mathit{John}) \quad \rightsquigarrow \quad \mathit{HistorySt}(\mathit{John}) \lor \mathit{ClassicsSt}(\mathit{John})$$

DL Knowledge Base: TBox + ABox

An $\mathcal{ALC}$ knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ is composed of:

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TBox:

$$\begin{aligned}
\mathit{JuvArthritis} &\sqsubseteq \mathit{Arthritis} \sqcap \mathit{JuvDisease} \\
\mathit{Arthritis} \sqcap \mathit{JuvDisease} &\sqsubseteq \mathit{JuvArthritis} \\
\mathit{Arthritis} &\sqsubseteq \exists \mathit{Damages}.\mathit{Joint} \\
\mathit{JuvDisease} &\sqsubseteq \forall \mathit{Affects}.(\mathit{Child} \sqcup \mathit{Teen}) \\
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ABox:

$$\mathit{Child}(\mathit{JohnSmith})$$
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Semantics via FOL Translation

$\mathcal{ALC}$ semantics can be defined via translation into FOL:

- Concepts translated as formulas with one free variable

$$\pi_x(A) = A(x) \qquad \pi_y(A) = A(y)$$
$$\pi_x(\neg C) = \neg \pi_x(C) \qquad \pi_y(\neg C) = \neg \pi_y(C)$$
$$\pi_x(C \sqcap D) = \pi_x(C) \land \pi_x(D) \qquad \pi_y(C \sqcap D) = \pi_y(C) \land \pi_y(D)$$
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- TBoxes, ABoxes and KBs are translated in the obvious way.

Semantics via FOL Translation

Note redundancy in concept-forming operators:

$$\begin{aligned} \bot &\rightsquigarrow \neg\top \\ C \sqcup D &\rightsquigarrow \neg(\neg C \sqcap \neg D) \\ \forall R.C &\rightsquigarrow \neg(\exists R.\neg C) \end{aligned}$$

These equivalences can be proved using FOL semantics:

$$\begin{aligned} \pi_x(\neg\exists R.\neg C) &= \neg\exists y.(R(x,y) \land \neg\pi_y(C)) \\ &\equiv \forall y.(\neg(R(x,y) \land \neg\pi_y(C))) \\ &\equiv \forall y.(\neg R(x,y) \lor \pi_y(C)) \\ &\equiv \forall y.(R(x,y) \to \pi_y(C)) \\ &= \pi_x(\forall R.C) \end{aligned}$$

We can define the syntax of $\mathcal{ALC}$ using (e.g.) only conjunction, negation, and existential restriction.

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Direct semantics: An alternative (and convenient) way of specifying semantics

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Consider the interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$

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\mathit{JuvDis}^{\mathcal{I}} &= \{u\} \\
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We can then interpret any concept as a subset of $\Delta^{\mathcal{I}}$:

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(\mathit{JuvDis} \sqcap \mathit{Child})^{\mathcal{I}} &= \\
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We can then interpret any concept as a subset of $\Delta^{\mathcal{I}}$:

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(\mathit{JuvDis} \sqcap \mathit{Child})^{\mathcal{I}} &= \emptyset \\
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We can now determine whether $\mathcal{I}$ is a model of …

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