Chase Termination Beyond Polynomial Time

43rd Symposium on Principles of Database Systems (PODS'24)

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Introduction

Tuple-generating dependencies (tgds): rules with existential quantifiers

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Example

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$$\mathit{elem}(e) \wedge \mathit{set}(S) \to \exists V.\, \mathit{add}(e, S, V) \tag{1}$$

$$\mathit{add}(e, S, T) \to \mathit{add}(e, T, T) \tag{2}$$

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Chase: 'apply rules until nothing new follows'
$\to$ but might run forever

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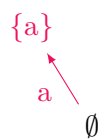

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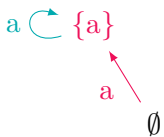

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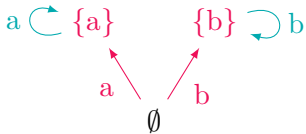

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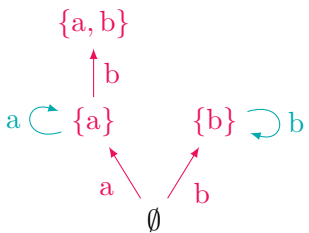

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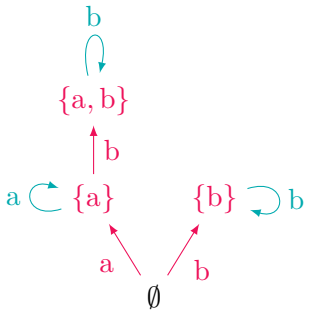

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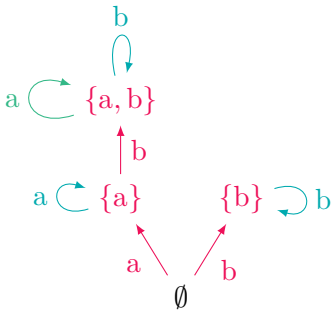

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Chase: relational structure of nulls,
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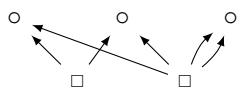

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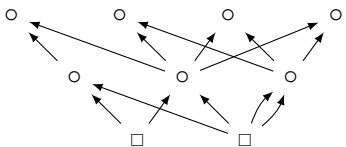

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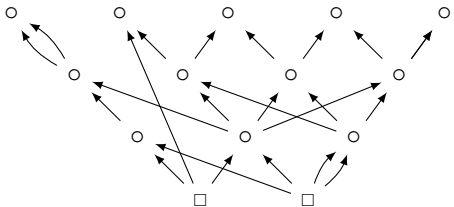

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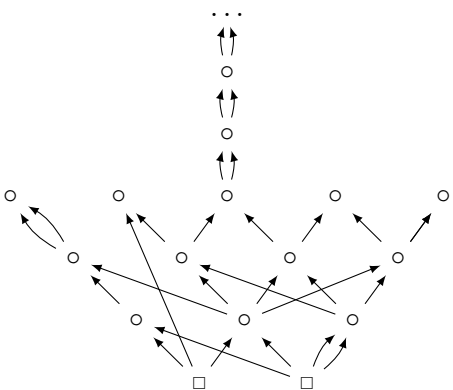

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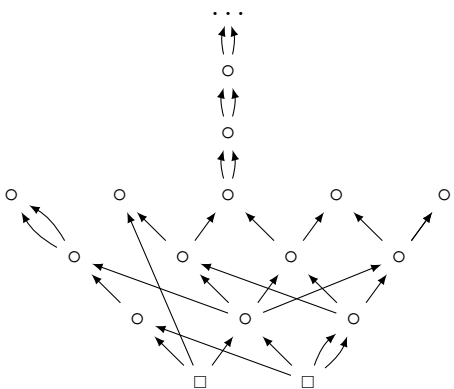

Abstraction of the chase structure
$\rightarrow$ dependency graph

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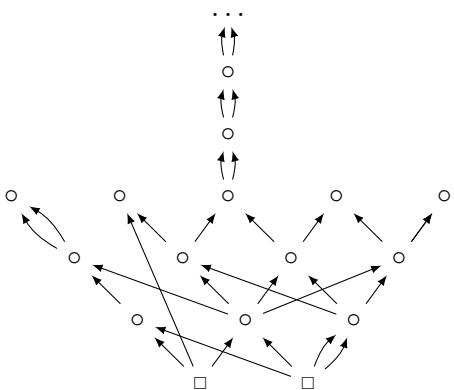

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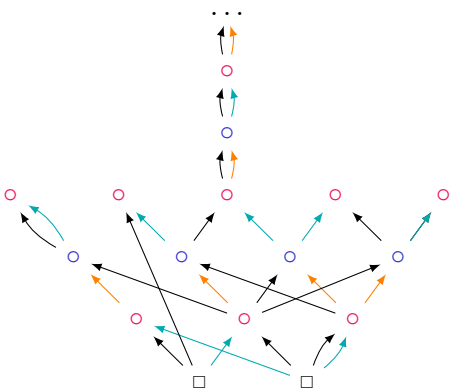

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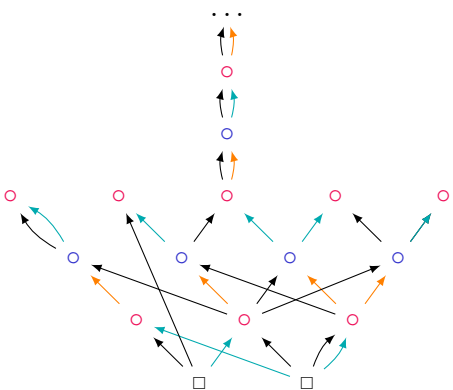

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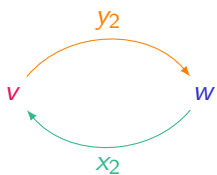

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Acyclic dependency graph $\rightarrow$ chase terminates
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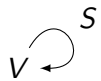

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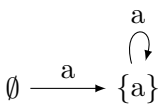

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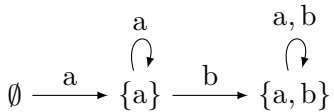

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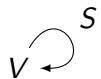

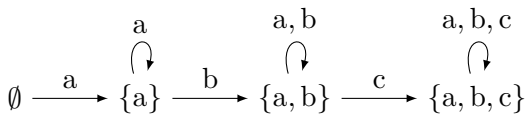

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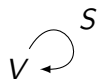

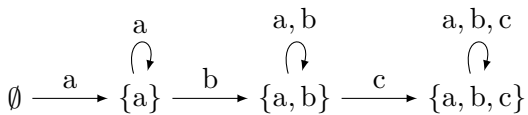

Tgd (2) 'blocks' application along a path for latest element

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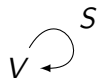

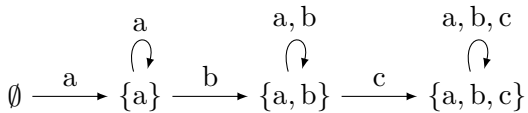

Tgd (2) 'blocks' application along a path for latest element
Tgd (3) 'blocks' application along a path for previously added elements

Generalising this idea

$$elem(e) \wedge set(S) \rightarrow \exists V.\, add(e, S, V)$$

We identify three kinds of variables for the tgds we 'block':

- the existential variable that creates a new null
- the variable that may be matched with a previous null: the 'predecessor'
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Saturation conditions:

1. A null satisfies the context that was used to create it.
2. A null satisfies all contexts satisfied by its predecessor null.

Generalising this idea

$$elem(e) \wedge set(S) \rightarrow \exists V.\, add(e, S, V)$$

We identify three kinds of variables for the tgds we 'block':

- the existential variable that creates a new null
- the variable that may be matched with a previous null: the 'predecessor'
- the remaining variables: the 'context'

Goal: nulls should satisfy the tgd for all contexts that led to its creation

Saturation conditions:

1. A null satisfies the context that was used to create it.
2. A null satisfies all contexts satisfied by its predecessor null.

→ Datalog-entailment checks, based on tgds and dependency graph

Breaking the cycles

Not all tgds need to be blocked:

- select ‘blocking’ tgds to break cycles
- identify predecessor and context variables
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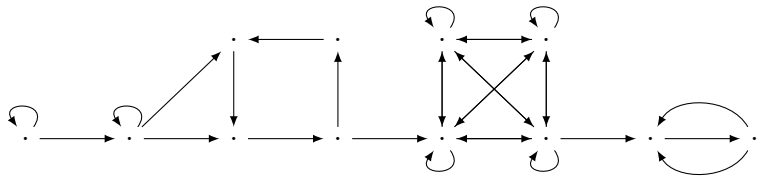

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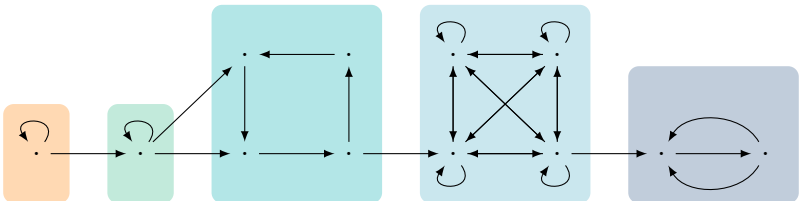

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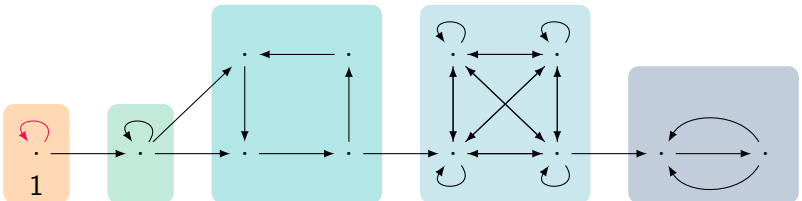

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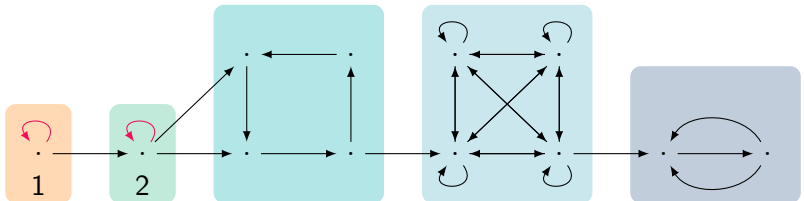

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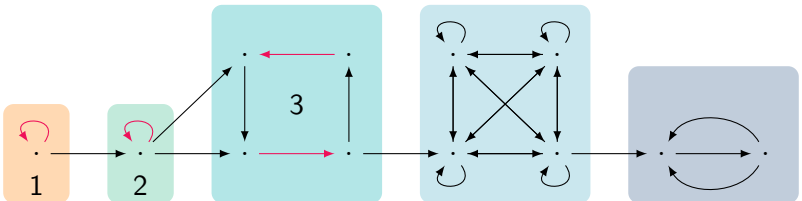

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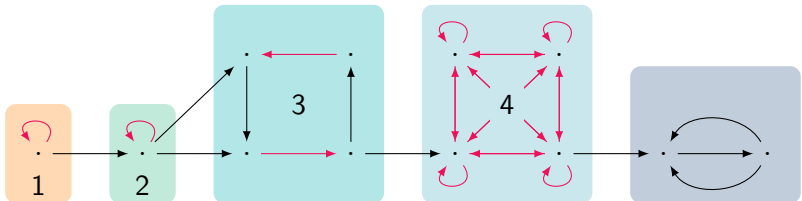

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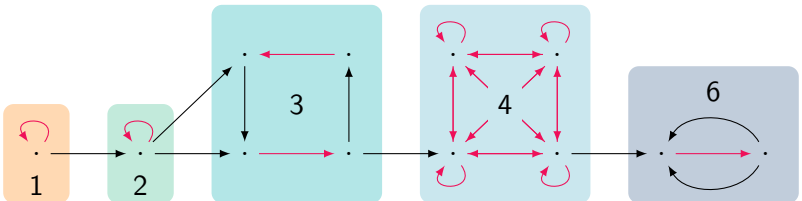

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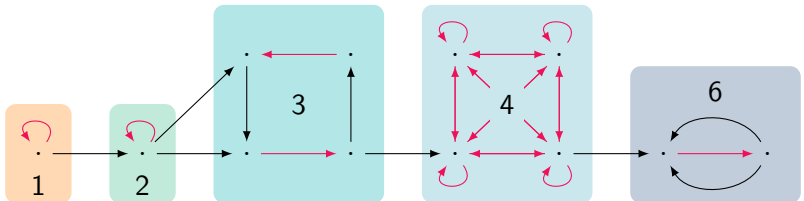
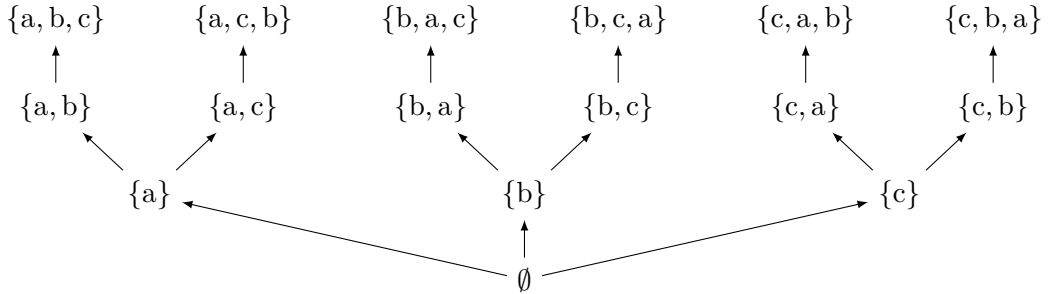


The saturation criteria define decidable classes of tgds with k-ExpTime-complete data complexity for every k.

Teaser



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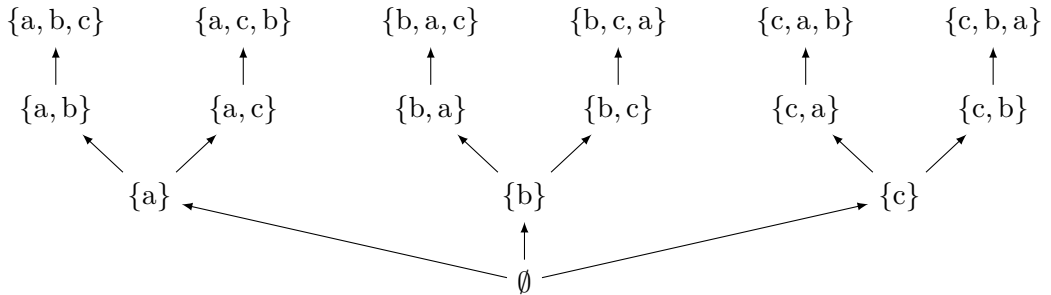

There are exponentially many nulls, but they form a tree:
$\rightarrow$ query answering might be possible in PSPACE.

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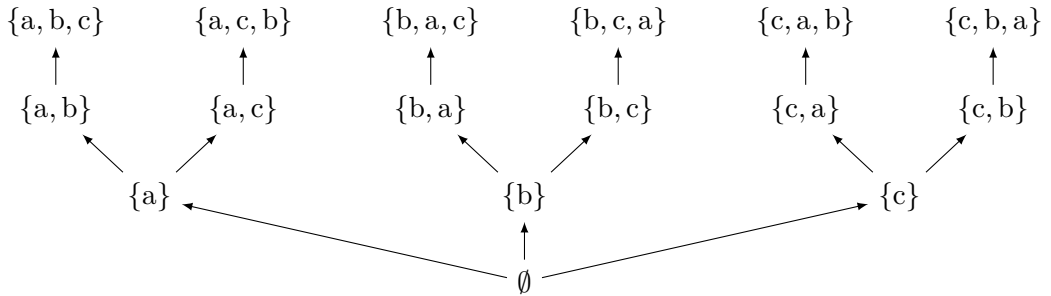

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We define a syntactic condition and a chase that realises this complexity.

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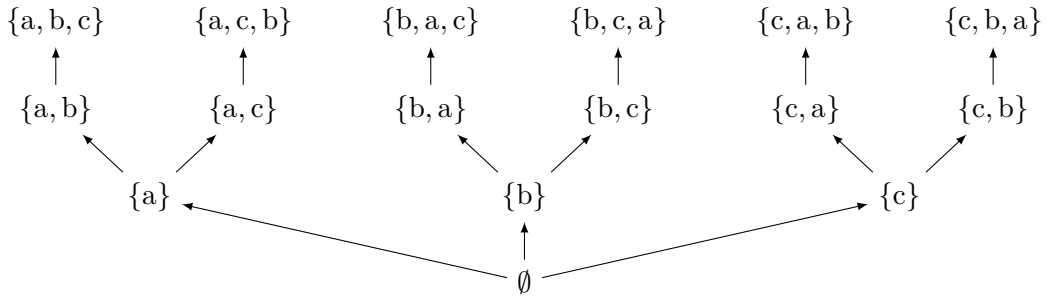

There are exponentially many nulls, but they form a tree:
$\rightarrow$ query answering might be possible in PSPACE.

We define a syntactic condition and a chase that realises this complexity
and obtain decidable classes of tgds with PSPACE- and k-EXPSPACE-complete data complexity for all k.

Summary

Main results:

- decidable classes of tgds with PSPACE, k-EXPSPACE, k-EXPTIME data complexity for all k
- new methods for analysing the structure of the standard chase
- new chase procedures that are optimal for space-bound complexity classes

Open questions:

- refinement of criteria for further complexity classes
- capturing of queries in the covered complexity classes