Finite and algorithmic model theory – Exercises no. 2

Exercise 1

Read again the definition of extension axioms $\sigma_{s,t}$. Do you have any idea why they are called like this?

Exercise 2

Show that extensions axioms are true with the limit probability 1.

Exercise 3

Read Section 5.3 from Graedel's notes and show that FO has 0–1 law for arbitrary purely relational symbols.

Exercise 4

Show that the presence of constants in the signature spoils the 0–1 law of FO.

Exercise 5

Does the 0–1 law of FO help to show that the following properties are not FO-definable [for suitable signatures]? (i) that a graph is two-colorable (ii) that a graph is a tree (c) that for a a unary symbol U and all structures $\mathfrak{A}$ we have that $|U^{\mathfrak{A}}| \leq |A \setminus U^{\mathfrak{A}}|$.

Below you can find two nice research ideas, related to zero-one laws. I would be happy to work on it with an ambitious student (it should be good enough for a master thesis or research project).

- Show that FO extended with percentage quantifiers $\exists^{=k\%}\varphi$ (stating that exactly $k\%$ of domain element satisfy φ) does not have the zero-one law, but it has a convergence law (i.e. that μ_∞ always exists).
- We say that a regular language of words $\mathcal{L}$ has the zero-one law, if $\lim_{n\to\infty} \frac{|\{w \in \Sigma^* \,:\, |w|=n\} \cap \mathcal{L}|}{|\{w \in \Sigma^* \,:\, |w|=n\}|}$ is either equal to 0 or 1. Not so long ago it was shown that $\mathcal{L}$ has the zero-one law iff its syntactic monoid has zero, or equivalently, that either $\mathcal{L}$ or its complement contains the language of the form $\Sigma^* w \Sigma^*$ for some word w. It would be nice to show a similar result for regular tree languages.