EXERCISE 3

Science of Computational Logic

Steffen Hölldobler, Marcos Cramer

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Problem 3.1

We have three tiles, called a, b and c, placed in a squared ordered field that can contain exactly four tiles, as illustrated in the figure below. Tile X can be moved via the action $move(X)$ either horizontally or vertically to occupy the adjacent free portion of the field. It cannot be moved diagonally and it cannot overlap another tile.

state (A) state (B)

1. Formalize the actions $move(X)$ in fluent calculus.
2. Formalize the state (A).
3. Give a plan that transforms state (A) to state (B).

Solution

1. We use the following fluents to represent states:

   $\text{on}(X, R, C)$ – tile X is on row R, column C
   $\text{empty}(R, C)$ – no tile on row R, column C

   with (row,column):

$(1,1)$	$(1,2)$
$(2,1)$	$(2,2)$

   action move(X) :

   horizontal move: $\text{action}\ (\ \text{on}(X,R,C) \circ \text{empty}(R,C'),\ \text{move}(X),\ \text{on}(X,R,C') \circ \text{empty}(R,C)\ )$

vertical move: action ($\text{on}(X, R, C) \circ \text{empty}(R', C)$,
$\text{move}(X)$,
$\text{on}(X, R', C) \circ \text{empty}(R, C)$)

2. $\text{on}(a, 2, 1) \circ \text{on}(b, 1, 2) \circ \text{on}(c, 2, 2) \circ \text{empty}(1, 1)$

3. $[\text{move}(a)]$

Problem 3.2

Consider the decision variant of the knapsack problem:

Given a set of items $\{i_1, \ldots, i_n\}$, each with a mass $m_i \in \mathbb{N}$ and a value $v_i \in \mathbb{N}$, can we include them in in a collection so that the total weight is less than m and the total value is more than v?

Specify a planning problem that has a solution if and only if the above problem can be answered with 'yes'.

Solution

We have fluents *mass* and fluents *value*. The intution is that the fluent *mass* is present n times if we have collected objects with a mass of n. Likewise for *value*.

So for every $i \in \{i_1, \ldots, i_n\}$, we include an action $\dot{\{}i_1, mass, \ldots, mass\dot{\}} \rightarrow \dot{\{}value, \ldots value\dot{\}}$ with m_i occurrences of $mass$ and v_i occurrences of $value$.

The initial state is $\dot{\{}i_1, \ldots, i_2\dot{\}} \dot{\cup} \dot{\{}\underbrace{mass, \ldots, mass}_{m-1}\dot{\}}$. The goal state is $\dot{\{}\underbrace{value, \ldots, value}_{v+1}\dot{\}}$.

Problem 3.3

Let $\mathcal{I}$ be an initial state containing, and $\mathcal{G}$ be a goal state and let $\mathcal{A}$ be the set of actions of Blocks World as specified in the lecture.

Specify a propositional formula F_n such that F_n is satisfiabile if and only if the planning problem has a solution of length n.

Solution

We assume that the initial and goal state contains at most one occurence of each fluent, i.e. they are sets.

Since the actions of Blocks World do not introduce new fluents, we model a state as a set. Then, we use propositional variables p_i that denotes that the fluent p is present at time point $0 \leq i \leq n$. We will only consder ground actions.

Initial State

$$\mathcal{F}_0 = \{p_0 \mid p \in \mathcal{I}\} \cup \{\neg p_0 \mid p \notin \mathcal{I}\}$$

Goal state

$$\mathcal{F}_n = \{p_n \mid p \in \mathcal{G}\}$$

Now we need to solve the frame problem, ramification problem, qualification problem.

We use the propositional variable a_i denoting that we execute action a in time point $1 \leq i \leq n$.

At most one action can be executed in each step

$$Aux = \{a_i \wedge a'_i \rightarrow \bot \mid a \neq a' \text{ and } 1 \leq i \leq n\}$$

At least one action must be executed in each step

$$Aux = \{a_i^1 \vee \cdots \vee a_i^m \mid 1 \leq i \leq n\}, \text{ where } \mathcal{A} = \{a^1, \ldots, a^m\}$$

If an action is executed at time i, its precondition must be satisfied at time point $i-1$

$$P = \{a_i \rightarrow (p_{i-1}^1 \wedge p_{i-1}^2 \wedge \ldots \wedge p^m{}_{i-1}) \mid \text{action } a \text{ has preconditions } p^1 \ldots p^m\}$$

If an action is executed at time i, the effects must be satisfied at time point i

$$E = \{a_i \rightarrow (e_i^1 \wedge e_i^2 \wedge \ldots \wedge e^m{}_i) \mid \text{action } a \text{ has effects } e^1 \ldots e^m\}$$

If an action is executed, some fluents are not present any more and must be false

$$PE = \{a_i \rightarrow (\neg p_i^1 \wedge \neg p_i^2 \wedge \ldots \wedge \neg p^m{}_i) \mid \text{action } a \text{ has preconditions } p^1 \ldots p^m \text{ that are not in the effects of } a\}$$

Fluents that are not affected

$$FA = \{a_i \wedge p_{i-1} \rightarrow p_i \mid p \text{ does not occur in preconditions and effects of } a\}$$

$$FA' = \{a_i \wedge \neg p_{i-1} \rightarrow \neg p_i \mid p \text{ does not occur in preconditions and effects of } a\}$$

Now we define F_n to be the conjunction of all the formulas in Aux, Aux', P, E, PE, FA and FA':

$$F_n = \bigwedge (Aux \cup Aux' \cup P \cup E \cup PE \cup FA \cup FA')$$