

Exercise Sheet 1: Relational Algebra
 David Carral, Markus Krötzsch
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Exercise 1.1. Consider a cinema database with tables of the following form (adapted from a similar example in the textbook of Abiteboul, Hull and Vianu):

| Title | Director | Actor |
|------------------------|---------------|-------------|
| The Imitation Game | Tyldum | Cumberbatch |
| The Imitation Game | Tyldum | Knightley |
| ... | ... | ... |
| The Internet's Own Boy | Knappenberger | Swartz |
| The Internet's Own Boy | Knappenberger | Lessig |
| The Internet's Own Boy | Knappenberger | Berners-Lee |
| ... | ... | ... |
| Dogma | Smith | Damon |
| Dogma | Smith | Affleck |
| Dogma | Smith | Morissette |
| Dogma | Smith | Smith |

| Cinema | Address | Phone |
|-----------|--------------------------|---------|
| UFA | St. Petersburger Str. 24 | 4825825 |
| Schauburg | Königsbräcker Str. 55 | 8032185 |
| CinemaxX | Hüblerstr. 8 | 3158910 |
| ... | ... | ... |

| Cinema | Title | Time |
|-----------|--------------------|-------|
| Schauburg | The Imitation Game | 19:30 |
| Schauburg | Dogma | 20:45 |
| UFA | The Imitation Game | 22:45 |
| CinemaxX | The Imitation Game | 19:30 |

Express the following queries in relational algebra:

1. Who is the director of “The Imitation Game”?
2. Which cinemas feature “The Imitation Game”?
3. What are the address and phone number of “Schauburg”?
4. Is a film directed by “Smith” playing in some cinema?
5. List the pairs of persons such that the first directed the second in a film and vice versa.
6. List the names of directors who have acted in a film they directed.
7. Always return $\{ Title \mapsto "Apocalypse Now", Director \mapsto "Coppola" \}$ as the answer.
8. Find the actors cast in at least one film by “Smith.”
9. Find the actors cast in every film by “Smith.”
10. Find the actors cast only in films by “Smith.”
11. Find all pairs of actors who act together in at least one film.
12. Find all pairs of actors cast in exactly the same films.
13. Find the directors such that every actor is cast in one of his or her films.

Exercise 1.2. We use ε to denote the *empty function*, i.e., the function with the empty domain, which is defined for no value. We use \emptyset to denote the empty table with no rows and no columns.

Now for a table R , what are the results of the following expressions?

$$R \bowtie R$$

$$R \bowtie \emptyset$$

$$R \bowtie \{\varepsilon\}$$

Exercise 1.3. Express the following operations using other operations presented in the lecture:

- Intersection $R \cap S$.
- Cross product (Cartesian product) $R \times S$.
- Selection $\sigma_{n=a}(R)$ with a a constant.
- Arbitrary constant tables in queries (the constants in the lecture only had one single column and one single row; generalise this to any number of constants and rows)

Exercise 1.4. Consider the following identities and decide for each whether it is true or false. If true, prove your answer using the definitions from the lecture; if false, give a counterexample.

1. $R \bowtie S = S \bowtie R$
2. $R \bowtie (S \bowtie T) = (R \bowtie S) \bowtie T$.
3. $\pi_X(R \circ S) = \pi_X(R) \circ \pi_X(S)$ for all $\circ \in \{\cup, \cap, -, \bowtie\}$.
4. $\sigma_{n=m}(R \circ S) = \sigma_{n=m}(R) \circ \sigma_{n=m}(S)$ for all $\circ \in \{\cup, \cap, -\}$.
5. $\sigma_{n=m}(R \bowtie S) = \sigma_{n=m}(R) \bowtie S$, for n and m attributes of R only.

Why are these identities of interest?

Exercise 1.5. Let R^I and S^I be tables of schema $R[U]$ and $S[V]$, respectively. The *division* of R^I by S^I , written as $(R^I \div S^I)$, is defined to be the maximal table over the attributes $U \setminus V$ that satisfies $(R^I \div S^I) \bowtie S^I \subseteq R^I$. Note that the joined tables here do not have any attributes in common, so the natural join works as a cross product.

Consider the following table and use the division operator to express a query for the cities that have been visited by all people.

| Visited | |
|---------|----------|
| Person | City |
| Tomas | Berlin |
| Markus | Santiago |
| Markus | Berlin |
| Fred | New York |
| Fred | Berlin |

Express division using the relational algebra operations introduced in the lecture.

Exercise 1.6. Suggest how to write the relational algebra operations for using the unnamed perspective. What changes?

Exercise 1.7. We have seen above that \cap can be expressed in terms of the other standard operators of relational algebra. Indeed, the set of operations $\{\sigma, \pi, \cup, -, \bowtie, \delta\}$ can express all queries of relational algebra: it is complete. Try to show that it is not possible to reduce this set any further:

For each operator, first try to find an example query that cannot be expressed when using only the other operators. Then try to find a general argument that shows that the operator is really needed.