Problem 1.1
Given the following program $\mathcal{P}$:
\[
\begin{align*}
q(X, X, c) \\
q(f(X), Y, f(Z)) & \leftarrow q(X, Y, Z) \\
p(X, X, f(c)) \\
p(X, Y, f(Z)) & \leftarrow p(X', Y, Z) \land q(X, X', Y)
\end{align*}
\]
1. Compute $T^n_\mathcal{P}(\emptyset)$ for $n = 1, \ldots, 4$.
2. Find the least Herbrand model of the program $\mathcal{P}$.

Problem 1.2
Given the following program $\mathcal{P}$:
\[
\begin{align*}
q(X) & \leftarrow \\
p(a) & \leftarrow p(a) \\
p(b) & \leftarrow q(X) \\
p(s(X)) & \leftarrow p(X)
\end{align*}
\]
1. Let $I_0 = \{p(a)\}$ and $I_{n+1} = T_\mathcal{P}(I_n)$ for all $n \in \mathbb{N}$. Compute $I_n$ for all $n \in \mathbb{N}$.
2. Show that $\bigcup_{n \in \mathbb{N}} I_n$ is not the least Herbrand model of $\mathcal{P}$.

Problem 1.3
Given the following program $\mathcal{P}$:
\[
\begin{align*}
p(a, X, X) \\
p(s(X), Y, s(Z)) & \leftarrow p(X, Y, Z)
\end{align*}
\]
Compute $T_\mathcal{P} \uparrow n$ for every $n \in \mathbb{N}$ and compute lfp($T_\mathcal{P}$).

Problem 1.4
Given a definite program $\mathcal{P}$. Show that $T_\mathcal{P}$ is monotonic.