

Human Reasoning and Computational Logic

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Problem 1.1

Given the following program \mathcal{P} :

$$\begin{aligned} & q(X, X, c) \\ q(f(X), Y, f(Z)) & \leftarrow q(X, Y, Z) \\ & p(X, X, f(c)) \\ p(X, Y, f(Z)) & \leftarrow p(X', Y, Z) \wedge q(X, X', Y) \end{aligned}$$

1. Compute $T_{\mathcal{P}}^n(\emptyset)$ for $n = 1, \dots, 4$.
2. Find the least Herbrand model of the program \mathcal{P} .

Problem 1.2

Given the following program \mathcal{P} :

$$\begin{aligned} q(X) & \leftarrow \\ p(a) & \leftarrow p(a) \\ p(b) & \leftarrow q(X) \\ p(s(X)) & \leftarrow p(X) \end{aligned}$$

1. Let $I_0 = \{p(a)\}$ and $I_{n+1} = T_{\mathcal{P}}(I_n)$ for all $n \in \mathbb{N}$. Compute I_n for all $n \in \mathbb{N}$.
2. Show that $\bigcup_{n \in \mathbb{N}} I_n$ is not the least Herbrand model of \mathcal{P} .

Problem 1.3

Given the following program \mathcal{P} :

$$\begin{aligned} & p(a, X, X) \\ p(s(X), Y, s(Z)) & \leftarrow p(X, Y, Z) \end{aligned}$$

Compute $T_{\mathcal{P}} \uparrow n$ for every $n \in \mathbb{N}$ and compute $\text{lfp}(T_{\mathcal{P}})$.

Problem 1.4

Given a definite program \mathcal{P} . Show that $T_{\mathcal{P}}$ is monotonic.