Complexity Theory

Exercise 4: Space Complexity

Exercise 4.1. Let $\mathsf{A}_{\mathsf{LBA}}$ be the word problem of deterministic linear bounded automata. Show the $\mathsf{A}_{\mathsf{LBA}}$ is PSpace-complete.

$$\mathbf{A}_{\mathsf{LBA}} = \{\langle \mathcal{M}, w\rangle \mid \mathcal{M} \text{ is a (deterministic) LBA and } w \in \mathbf{L}(\mathcal{M})\}$$

Exercise 4.2. Consider the Japanese game *go-moku* that is played by two players X and O on a 19×19 board. Players alternately place their markers on the board. The first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalized version of go-moku on an n×n board. Say that a *position* of go-moku is a placement of markers on such a board as it could occur during the game, together with a marker which player moves next. We define

$$\mathbf{GM} = \{\langle B\rangle \mid B \text{ is a position of go-moku where X has a winning strategy}\}.$$

Show that **GM** is in PSpace.

Exercise 4.3. Show that the universality problem of nondeterministic finite automata

$$\mathbf{ALL}_{\mathsf{NFA}} = \{\langle \mathcal{A}\rangle \mid \mathcal{A} \text{ is an NFA accepting every valid input}\}$$

is in PSpace.

Exercise 4.4. Show that the composition of logspace reductions again yields a logspace reduction.

Exercise 4.5. Show that the word problem $\mathbf{A}_{\mathsf{NFA}}$ of nondeterministic finite automata is NL-complete.

Exercise 4.6. Show that

$$\mathsf{BIPARTITE} = \{\langle G\rangle \mid G \text{ is a finite bipartite graph}\}$$

is in NL. For this, show that $\overline{\mathsf{BIPARTITE}} \in \mathrm{NL}$ and use $\mathrm{NL} = \mathrm{coNL}$.