# Towards an Error-Tolerant Construction of $\mathcal{EL}^{\perp}$-Ontologies from Data using Formal Concept Analysis

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## Goal

Learn Knowledge from Data.

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- From which kind of data? *Interpretations*
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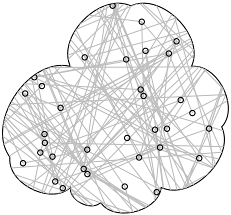
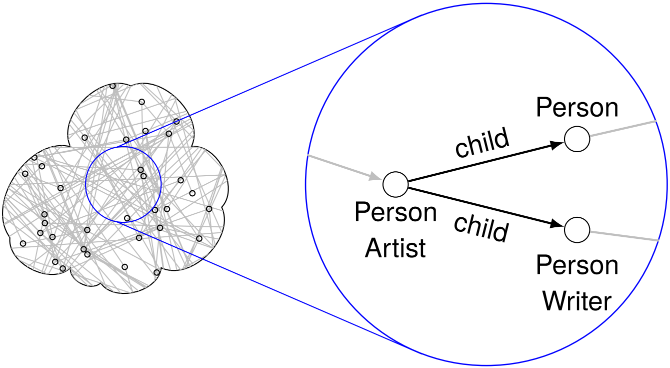
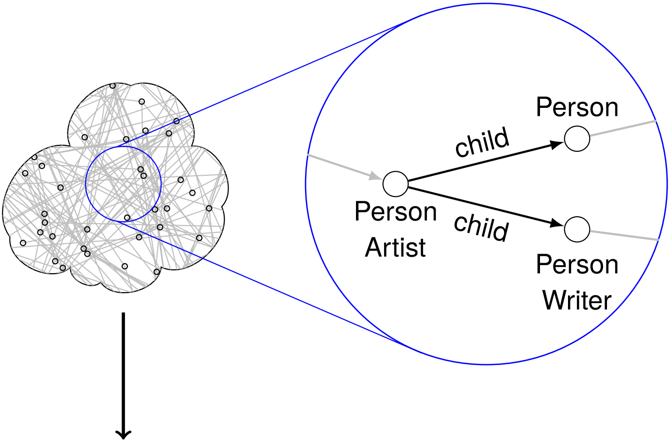
- From which kind of data? *Interpretations*
- Which knowledge? $\mathcal{EL}^{\bot}$*-General Concept Inclusions*
- How to learn? *Approach by Baader and Distel*




Person
child
Person
Artist
child
Person
Writer

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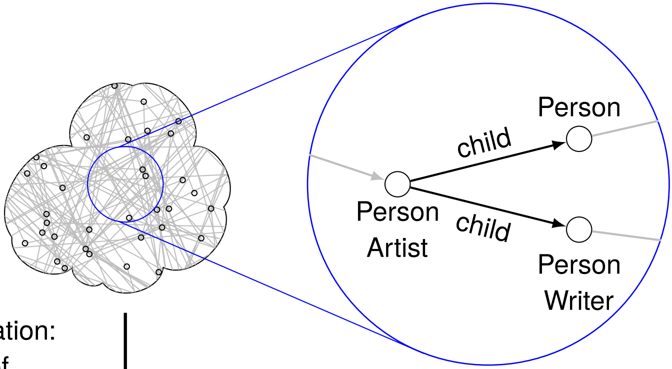


Axiomatization:
Base of
*valid* GCIs

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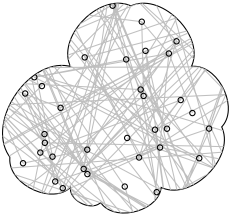

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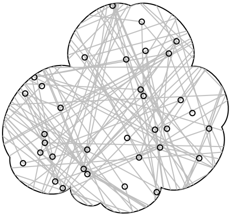

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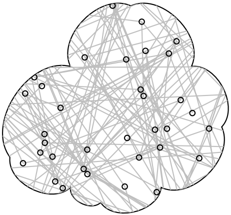
What about *errors* in the data?



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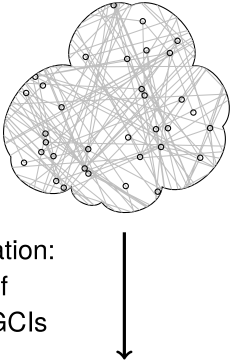

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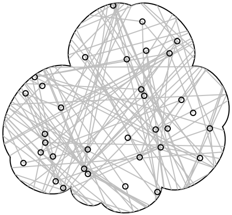

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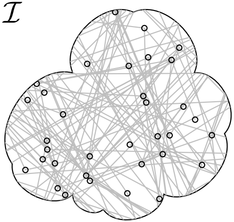
$\mathcal{I}$



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| --- | --- |
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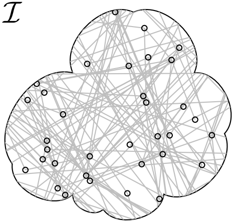

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Axiomatization
(Base of *valid* Implications)

$\{ U \to U'' \mid \ldots \}$

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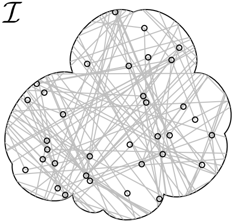

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Axiomatization
Base of valid GCIs

$\downarrow$

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Multiple labels per vertex/edge allowed

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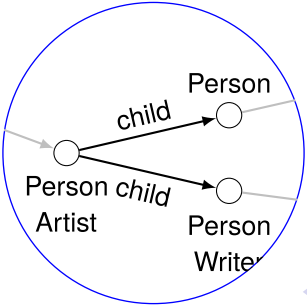

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Consider expressions of the form

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## Definition

For $A \in N_C$, $C, D$ two $\mathcal{EL}^\bot$-concept descriptions, $r \in N_R$:

- $\bot^\mathcal{I} = \emptyset$, $\top^\mathcal{I} = \Delta^\mathcal{I}$,
- $A^\mathcal{I}$ set of all vertices labeled with $A$,
- $(C \sqcap D)^\mathcal{I} := C^\mathcal{I} \cap D^\mathcal{I}$
- $(\exists r.C)^\mathcal{I} := \{\, x \in \Delta^\mathcal{I} \mid \exists y \in \Delta^\mathcal{I} \colon (x, y) \in r^\mathcal{I}, y \in C^\mathcal{I} \,\}$,

# General Concept Inclusions

*General Concept Inclusions* (GCIs) are of the form

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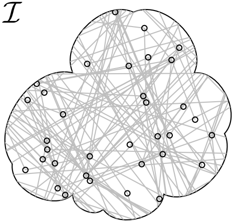
## Learning Goal

Find *finite bases* of *valid* GCIs of $\mathcal{I}$.

$\mathcal{I}$



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Axiomatization (Base of *valid* GCIs) $\downarrow$

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## Theorem (Baader, Distel 2008)

*If $\mathcal{L}$ is a base of $\mathbb{K}_{\mathcal{I}}$ then the set*

$$\{\, \bigsqcap U \sqsubseteq ((\bigsqcap U)^{\mathcal{I}})^{\mathcal{I}} \mid (U \to U'') \in \mathcal{L} \,\}$$

*is a finite base of $\mathcal{I}$.*

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## Observation

$$\text{conf}_{\mathcal{I}_{\text{DBpedia}}}(\exists \text{child}.\top \sqsubseteq \text{Person}) = \frac{2547}{2551}$$

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The *confidence* of $C \sqsubseteq D$ in $\mathcal{I}$ is defined as

$$\mathrm{conf}_{\mathcal{I}}(C \sqsubseteq D) := \begin{cases} 1 & \text{if } C^{\mathcal{I}} = \emptyset, \\ \frac{|(C \sqcap D)^{\mathcal{I}}|}{|C^{\mathcal{I}}|} & \text{otherwise.} \end{cases}$$

Let $c \in [0, 1]$. Define $\mathrm{Th}_c(\mathcal{I})$ as the set of all GCIs having confidence of at least $c$ in $\mathcal{I}$.

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### New Goal

Axiomatize $\mathrm{Th}_c(\mathcal{I})$, i. e. find a *finite base* of $\mathrm{Th}_c(\mathcal{I})$.

## Question

How to find bases for $\mathrm{Th}_c(\mathcal{I})$?

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Let $\mathcal{B}$ base of $\mathbb{K}$,

$$\mathcal{C} = \{ X'' \to Y'' \mid 1 > \operatorname{conf}_{\mathbb{K}}(X'' \to Y'') \geqslant c \}$$

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### Definition

$$\mathsf{Conf}(\mathcal{I}, c) := \{\, X^{\mathcal{I}} \sqsubseteq Y^{\mathcal{I}} \mid Y \subseteq X \subseteq \Delta_{\mathcal{I}}, 1 > \mathsf{conf}_{\mathcal{I}}(X^{\mathcal{I}} \sqsubseteq Y^{\mathcal{I}}) \geqslant c \,\}.$$

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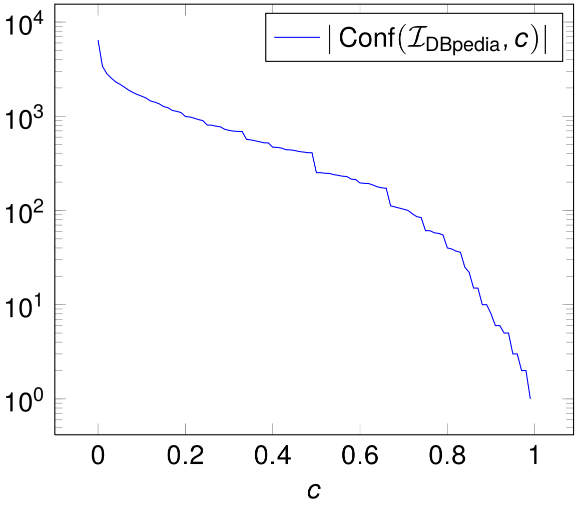
**Theorem**

*Let $\mathcal{B}$ be a finite base of $\mathcal{I}$, and $c \in [0, 1]$. Then $\mathcal{B} \cup \mathrm{Conf}(\mathcal{I}, c)$ is a finite base of $\mathrm{Th}_c(\mathcal{I})$.*


$|\mathrm{Conf}(\mathcal{I}_{\mathrm{DBpedia}}, c)|$
$10^4$
$10^3$
$10^2$
$10^1$
$10^0$
0
0.2
0.4
0.6
0.8
1
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- Exploration of $\mathsf{Th}_c(\mathcal{I})$?

Thank You