Complexity Theory

Exercise 4: Time Complexity
November 14, 2018

**Exercise 4.1.** Show that $P$ is closed under union and star.

**Exercise 4.2.** Consider the problem CLIQUE:

- **Input:** An undirected graph $G$ and some $k \in \mathbb{N}$
- **Question:** Does there exists a clique in $G$ of size at least $k$?

Suppose CLIQUE can be solved in time $T(n)$ for some $T: \mathbb{N} \to \mathbb{N}$ with $T(n) \geq n$ for all $n \in \mathbb{N}$. Furthermore, show that then the optimisation problem

- **Input:** An undirected graph $G$
- **Compute:** A clique in $G$ of maximal size

can be computed in time $O(n \cdot T(n))$. You can assume that $T$ is monotone.

**Exercise 4.3.** Show that if a language $L$ is $NP$-complete, then $L$ is $coNP$-complete.

**Exercise 4.4.** Show that if $P = NP$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given satisfiable propositional formula.

**Exercise 4.5.** Show that finding paths of a given length in undirected graphs, i.e.,

$$\text{PATH} = \{ \langle G, s, t, k \rangle \mid G \text{ contains a simple path from } s \text{ to } t \text{ of length } k \}$$

is NP-complete.