

Complexity Theory

Exercise 4: Time Complexity

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Exercise 4.1. Show that P is closed under union and star.

Exercise 4.2. Consider the problem **CLIQUE**:

Input: An undirected graph G and some $k \in \mathbb{N}$

Question: Does there exist a clique in G of size at least k ?

Suppose **CLIQUE** can be solved in time $T(n)$ for some $T: \mathbb{N} \rightarrow \mathbb{N}$ with $T(n) \geq n$ for all $n \in \mathbb{N}$. Furthermore, show that then the optimisation problem

Input: An undirected graph G

Compute: A clique in G of maximal size

can be computed in time $\mathcal{O}(n \cdot T(n))$. You can assume that T is monotone.

Exercise 4.3. Show that if a language L is NP-complete, then \bar{L} is coNP-complete.

Exercise 4.4. Show that if $P = NP$, then a polynomial-time algorithm exists that produces a satisfying assignment of a given satisfiable propositional formula.

Exercise 4.5. Show that finding paths of a given length in undirected graphs, i.e.,

$$\mathbf{PATH} = \{ \langle G, s, t, k \rangle \mid G \text{ contains a simple path from } s \text{ to } t \text{ of length } k \}$$

is NP-complete.