

Algorithmic Game Theory

Summer Term 2024

Exercises 1

15–19/04/2024

Problem 1.

Consider the games **Battle of the Partners** as well as **Prisoner's Dilemma** with the payoff tables displayed below. Although you already know the answers to the questions below from the lecture, work closely with the central definitions to answer them again.

- What strategy profiles provide Nash equilibria in both games?
- Is there a difference between the equilibria in both games?
- Are the equilibria Pareto optimal in both games?
- Are there dominant strategies in both games?

Battle of the Partners:

(Cat, Dee)	Cinema	Dancing
Cinema	(10,7)	(2,2)
Dancing	(0,0)	(7,10)

Prisoner's Dilemma:

(Eli, Fyn)	Cooperate	Defect
Cooperate	(3,3)	(0,5)
Defect	(5,0)	(1,1)

Problem 2.

Consider the following two exercises.

- (a) Imagine that you are a fully rational decision maker facing a (one-shot) prisoner's dilemma. Explain why the outcome of the game will not be affected if you are allowed to meet and discuss with the other player before you make your move.
- (b) By definition, a non-cooperative game is a game in which the players are not able to form binding agreements. Why can we not just say that a non-cooperative game is a game in which the players do not actually cooperate?

Problem 3.

Consider the following 3×3 game.

$(\text{Player}_1, \text{Player}_2)$	l	c	r
T	(1,0)	(3,1)	(1,1)
M	(1,1)	(3,0)	(0,1)
B	(2,2)	(3,3)	(0,2)

- Identify all pairs of strategies where one strategy weakly dominates the other.
- Assume you are allowed to remove a weakly dominated strategy of some player. Do so, and repeat this process (of iterated elimination of weakly dominated strategies) until you find a single strategy pair of the original game.
- Find such an iterated elimination of weakly dominated strategies that results in a strategy pair other than the one found in (b), where both strategies, and the payoffs to the players, are different.
- What are the equilibria (in pure strategies) of the game?

Problem 4.

Consider the following three-player game in strategic form.

		Player ₃					
		L		R			
$(\text{Player}_1, \text{Player}_2)$		l	r	$(\text{Player}_1, \text{Player}_2)$		l	r
T		(3,4,4)	(1,3,3)	T		(4,0,5)	(0,1,6)
B		(8,1,4)	(2,0,6)	B		(5,1,3)	(1,2,5)

- Identify all pairs of strategies where one strategy strictly, or weakly, dominates the other.
- Apply iterated elimination of strictly dominated strategies to this game. What are the Nash equilibria of the game?