Exercise Sheet 9: Semi-Positive Datalog

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Exercise 9.1. Show that any Datalog program can be expressed as a safe Datalog program that is polynomial in size of the original program and given schema.

Exercise 9.2. Assume that the database uses a binary EDB predicate *edge* to store a directed graph. Try to express the following properties in semi-positive Datalog programs with a successor ordering, or explain why this is not possible.

1. The database contains an even number of elements.
2. The graph contains a node with two outgoing edges.
3. The graph is 3-colourable.
4. The graph is *not* connected (*).
5. The graph does not contain a node with two outgoing edges.
6. The graph is a chain.

Exercise 9.3. A Horn logic program is in *propHorn2* if every rule it contains is of the form $H \leftarrow$ or $H \leftarrow B_1 \wedge B_2$.

It was claimed that entailment checking in *propHorn2* is P-hard. To support this claim, explain how entailment in propositional Horn logic can be reduced to entailment in *propHorn2*. Argue how this reduction can be accomplished in logarithmic space.

Exercise 9.4. Prove that entailment checking in propositional Horn logic is P-hard.

Hint:

Modify the ExpTime TM simulation from the lecture to simulate a PTime TM instead.

Exercise 9.5. Show that the following property cannot be expressed in Datalog:

The *edge* predicate has a *proper* cycle, i.e., a cycle that is not of the form *edge*(a, a).

Can you express this property using …

1. … a successor ordering?
2. … atomic EDB negation?
3. … an equality predicate $\approx$ with the obvious semantics?
4. … an inequality predicate $\not\approx$ with the obvious semantics?