Exercise 9.1. Show that any Datalog program can be expressed as a safe Datalog program that is polynomial in size of the original program and given schema.

Exercise 9.2. Assume that the database uses a binary EDB predicate edge to store a directed graph. Try to express the following properties in semi-positive Datalog programs with a successor ordering, or explain why this is not possible.

1. The database contains an even number of elements.
2. The graph contains a node with two outgoing edges.
3. The graph is 3-colourable.
4. The graph is not connected (*).
5. The graph does not contain a node with two outgoing edges.
6. The graph is a chain.

Exercise 9.3. A Horn logic program is in propHorn2 if every rule it contains is of the form $H \leftarrow$ or $H \leftarrow B_1 \land B_2$.

It was claimed that entailment checking in propHorn2 is P-hard. To support this claim, explain how entailment in propositional Horn logic can be reduced to entailment in propHorn2. Argue how this reduction can be accomplished in logarithmic space.

Exercise 9.4. Prove that entailment checking in propositional Horn logic is P-hard.

Hint: Modify the ExpTime TM simulation from the lecture to simulate a PTime TM instead.

Exercise 9.5. Show that the following property cannot be expressed in Datalog:

The edge predicate has a proper cycle, i.e., a cycle that is not of the form $\text{edge}(a, a)$.

Can you express this property using …

1. … a successor ordering?
2. … atomic EDB negation?
3. … an equality predicate $\approx$ with the obvious semantics?
4. … an inequality predicate $\not\approx$ with the obvious semantics?