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# Strong Equivalence for Argumentation Semantics Based on Conflict-Free Sets

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- Argumentation is a dynamic reasoning process.
- During the process the participants come up with new arguments.
  - Which effects causes additional information wrt. a semantics?
  - Which information does not contribute to the results?

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- Two AFs $F$ and $G$ are strongly equivalent (wrt. a semantics $\sigma$) iff $F \cup H$ and $G \cup H$ have the same $\sigma$-extensions for each AF $H$.
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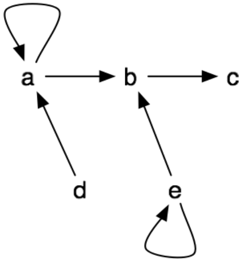
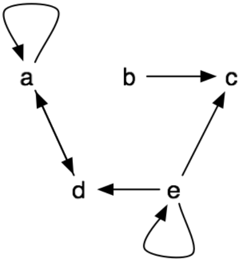
  - One can savely replace an AF by a strongly equivalent one without changing its extensions.
- In a negotiation between two agents: SE allows to characterize situations where the two agents have an equivalent view of the world which is moreover robust to additional information.

# Motivation ctd.



## Example



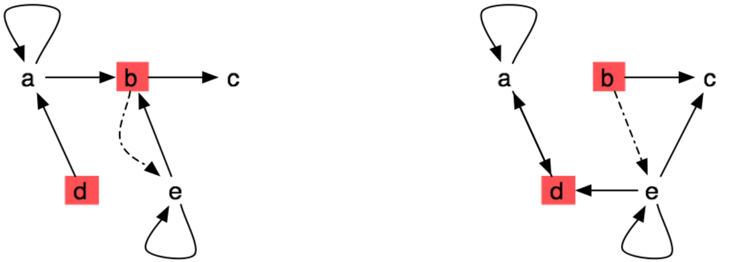




- AFs $F$ and $G$ are equivalent (wrt. stable semantics).

## Example


- $stable(F \cup H) = stable(G \cup H) = \{\{b, d\}\}$.

## Example




- We identify the stable kernel of a framework $F = (A, R)$ which removes redundant attacks:
  - $F^{sk} = (A, R^{sk})$ where $R^{sk} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R\}$.

# Motivation ctd.

- Identification of redundant attacks is important in choosing an appropriate semantics.
- Strong equivalence has been analyzed for many semantics in [Oikarinen and Woltran, 2010].
- In this paper: naive, stage and *cf2* semantics.

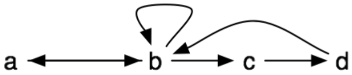
# Overview

## Argumentation Framework [Dung, 1995]

An argumentation framework ($AF$) is a pair $F = (A, R)$, where $A$ is a finite set of arguments and $R \subseteq A \times A$. Then $(a, b) \in R$ if $a$ attacks $b$.

## Example

$F = (A, R), A = \{a, b, c, d\}, R = \{(a, b), (b, a), (b, b), (b, c), (c, d), (d, b)\}$, directed graph

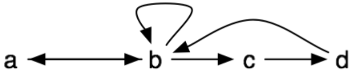


## Semantics for AFs

Let $F = (A, R)$ and $S \subseteq A$, we say

- $S$ is conflict-free in $F$, i.e. $S \in cf(F)$, if there are no $a, b \in S$, s.t. $(a, b) \in R$;
- $S$ is maximal conflict-free or naive, i.e. $S \in naive(F)$, if $S \in cf(F)$ and for each $T \in cf(F)$, $S \not\subset T$.

## Example



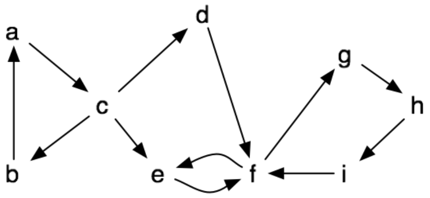

$$cf(F) = \{\emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}\}, naive(F) = \{\{a, c\}, \{a, d\}\}.$$

The *cf2* semantics is one of the SCC-recursive semantics introduced in [Baroni et al., 2005]

## Separation

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from $b$ to $a$. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of $F$.

## Example



# *cf2* Semantics

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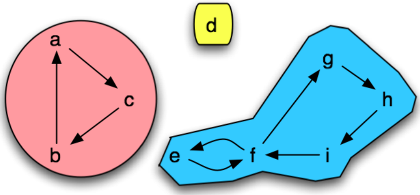

## Reachability

Let $F = (A, R)$ be an AF, $B$ a set of arguments, and $a, b \in A$. We say that $b$ is reachable in $F$ from $a$ modulo $B$, in symbols $a \Rightarrow^B_F b$, if there exists a path from $a$ to $b$ in $F|_B$.

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## Definition ($\Delta_{F,S}$)

For an AF $F = (A, R)$, $D \subseteq A$, and a set $S$ of arguments,

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow_F^{A \setminus D} b\},$$

and $\Delta_{F,S}$ be the least fixed-point of $\Delta_{F,S}(\emptyset)$.

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## *cf2* Extensions [Gaggl and Woltran, 2010]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a *cf2*-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([[F - \Delta_{F,S}]])$.

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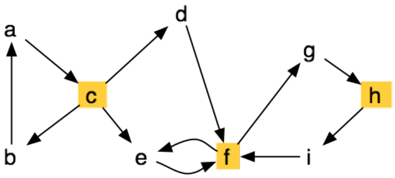
## Example

$S = \{c, f, h\}$ , $S \in cf(F)$.



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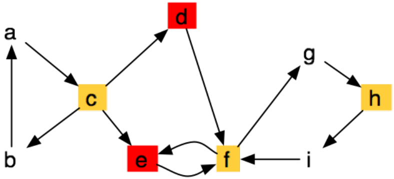

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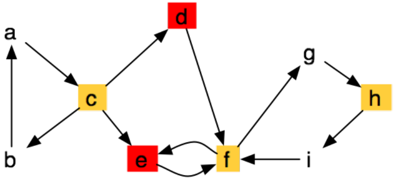

# *cf2* Semantics ctd.



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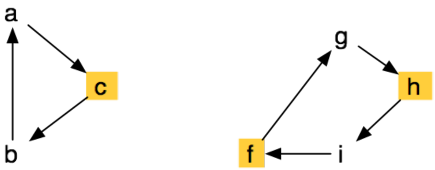
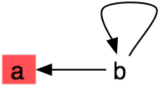


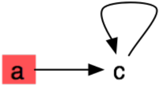
# Strong Equivalence (SE)

## Strong Equivalence [Oikarinen and Woltran, 2010]

Two AFs $F$ and $G$ are strongly equivalent to each other wrt. a semantics $\sigma$, in symbols $F \equiv_s^\sigma G$, iff for each AF $H$, $\sigma(F \cup H) = \sigma(G \cup H)$.

By definition $F \equiv_s^\sigma G$ implies $\sigma(F) = \sigma(G)$.

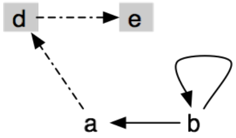


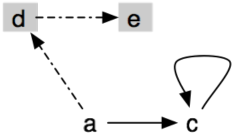




- $naive(F) = naive(G) = \{\{a\}\}$

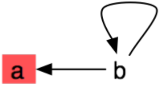
# SE wrt. *naive* Semantics

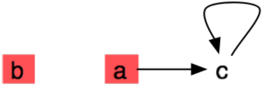





- $naive(F \cup H) = naive(G \cup H) = \{\{d\}, \{a, e\}\}$




- $naive(F \cup H) = naive(F) = \{\{a\}\}$ but
- $naive(G \cup H) = \{\{a, b\}\}$.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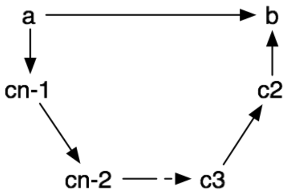
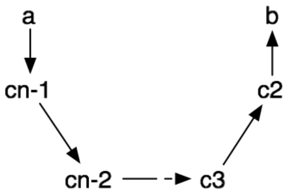
## Theorem

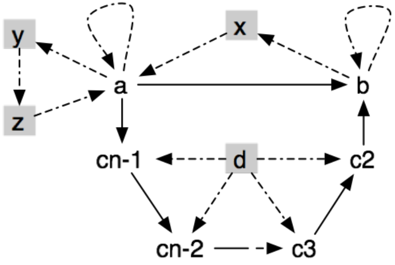
*The following statements are equivalent:*

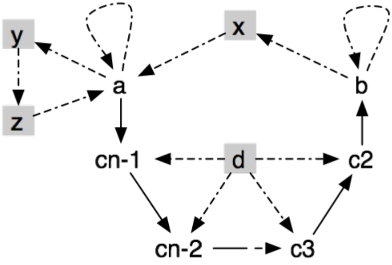
1. $F \equiv_s^{naive} G$;
2. $naive(F) = naive(G)$ *and* $A(F) = A(G)$;
3. $cf(F) = cf(G)$ *and* $A(F) = A(G)$.

# SE wrt. *cf2* Semantics



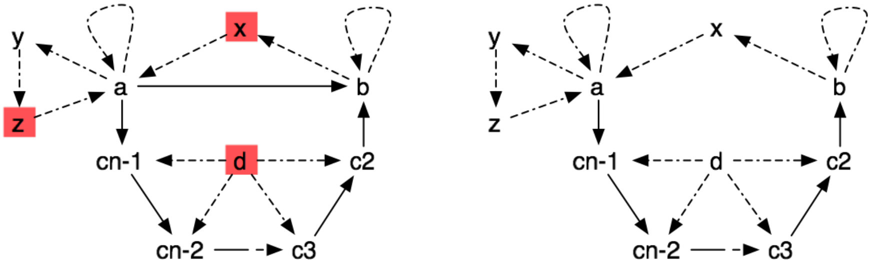


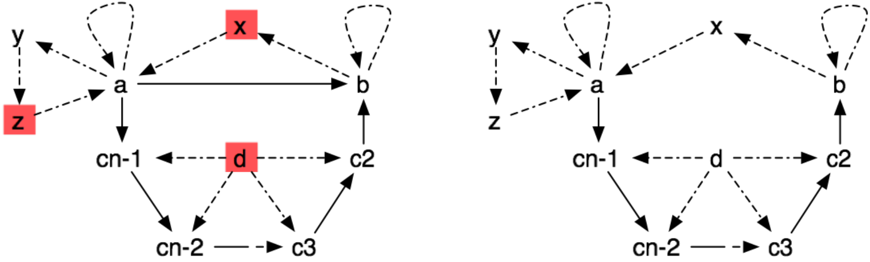

$$
\begin{aligned}
H \quad = \quad & (A \cup \{d, x, y, z\}, \\
& \{(a, a), (b, b), (b, x), (x, a), (a, y), (y, z), (z, a), \\
& (d, c) \mid c \in A \setminus \{a, b\}\}).
\end{aligned}
$$

# SE wrt. *cf2* Semantics






Let $E = \{d, x, z\}$, $E \in cf2(F \cup H)$ but $E \notin cf2(G \cup H)$.




Let $E = \{d, x, z\}$, $E \in cf2(F \cup H)$ but $E \notin cf2(G \cup H)$.

- No matter which AFs $F \neq G$, one can always construct an $H$ s.t. $cf2(F \cup H) \neq cf2(G \cup H)$;
- The missing attack in one AF leads to different SCCs and therefore to different $cf2$ extensions.

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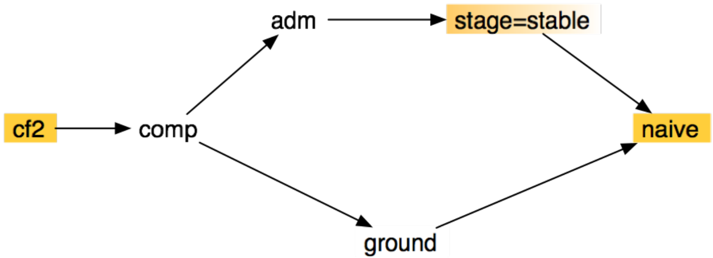
## Theorem

*For any AFs $F$ and $G$, $F \equiv_s^{\mathit{cf2}} G$ iff $F = G$.*

# Comparing Semantics wrt. SE



- We provide characterizations for strong equivalence wrt. stage, naive and *cf2* semantics.
- *cf2* semantics is the only one where no redundant attacks exist.
- *cf2* semantics treats self-loops in a more sensitive way than other semantics.
- We analyzed local and symmetric equivalence.

Baroni, P., Giacomin, M., and Guida, G. (2005).
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