Exercise 3.1:
Consider the following program $P$:
\[
\begin{align*}
double(0,0). \\
double(s(X),s(s(Y))) & :- double(X,Y).
\end{align*}
\]

a) Give the Herbrand universe $HU_F$ and the Herbrand base $HB_{\Pi,F}$ determined by $P$.

b) Give two models of $P$.

c) Consider the following interpretations $I_1$ and $I_2$. For each case specify whether the given interpretation satisfies $P$ or not. Justify your answer.

- $I_1: D_{I_1} = \mathbb{N}, 0_{I_1} = 1, s(t)_{I_1} = 2 \times t_{I_1}, double_{I_1} = \{(a,a^2) \mid a \geq 1\}$

- $I_2: D_{I_2} = \mathbb{N}, 0_{I_2} = 0, s(t)_{I_2} = (2 \times t_{I_2}) + 1,
  \quad double_{I_2} = \{(0,0)\} \cup \{(a,a^2-a+1) \mid a \geq 1\}$

Exercise 3.2:
Consider the following program which specifies the descendant relation which is the relation of being a child of, or a child of a child of, or a child of a child of a child of, \ldots
\[
\begin{align*}
descend(X,Y) & :- \text{child}(X,Y). \\
descend(X,Y) & :- \text{child}(X,Z), descend(Z,Y).
\end{align*}
\]

With the input database
\[
\begin{align*}
\text{child}(\text{anne}, \text{bridget}). \\
\text{child}(\text{bridget}, \text{caroline}). \\
\text{child}(\text{caroline}, \text{donna}). \\
\text{child}(\text{donna}, \text{emily}).
\end{align*}
\]

Give the search tree for the query: \texttt{- descend(anne, donna)}. 

Exercise 3.3:
Consider the program from Exercise 3.2. What happens if we change the order of the rules and goals. What is the result of the queries – descending(X,Y), descending(anne,emily), and descending(anne,bridget).

a) descending(X,Y) :- child(X,Z), descending(Z,Y).
   descending(X,Y) :- child(X,Y).

b) descending(X,Y) :- descending(Z,Y), child(X,Z).
   descending(X,Y) :- child(X,Y).

c) descending(X,Y) :- child(X,Y).
   descending(X,Y) :- descending(Z,Y), child(X,Z).

Exercise 3.4:
Consider the following program for addition.

```prolog
add(0,Y,Y).
add(s(X),Y,s(Z)) :- add(X,Y,Z).
```

Give the search tree and instantiations for the query:

?- add(s(s(s(0))), s(s(0)), R).