TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science   Institute of Artificial Intelligence   Knowledge Representation and Reasoning

Hidden Units, Equivalences and Implications (Poster Presentation)

Norbert Manthey

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Mail to
Technische Universität Dresden
01062 Dresden

Office
Room 2006
Nöthnitzer Straße 46
01187 Dresden

Internet
http://www.wv.inf.tu-dresden.de

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Norbert Manthey

Knowledge Representation and Reasoning Group
Technische Universität Dresden, 01062 Dresden, Germany
norbert@janeway.inf.tu-dresden.de

Abstract. A new search space pruning technique, *hidden implications*, is introduced. Although they are not implied by a formula, adding them preserves satisfiability. This paper shows how *hidden implications* can be found and how they can be used to find *hidden units* or *hidden equivalences*.

1 Motivation

The motivation of this work is to find unit clauses that are not implied by the formula, but can be added without changing satisfiability. An example is the sudoku in Fig. 1 with focus on number *1* (bold numbers are given). Thin numbers represent possible placements. Let $f_{x,y}$ be the field in column x and row y. For $f_{9,1}$ and $f_{9,2}$ only *1* and *2* are possible. From rules like Boolean Constraint Propagation (BCP) and pure literals, no further deduction can be applied. Advanced reasoning (*Naked Pairs*[1]) leads to filling number *1* into $f_{8,9}$. The idea is the following: Assume we put a *1* into $f_{9,1}$, then in row 9 the only valid place for *1* is $f_{8,9}$. The same statement holds, if we decide to not put *1* into $f_{9,1}$. *Hidden Implications*, the counterpart in SAT, that is a combination of probing [1] and finding autarkies, are introduced in this paper and reach the same deduction on CNF.

9					**2**			*1*	*1*
8	**1**								**3**
7				**1**					**4**
6							**1**		
5			**1**			**2**			
4					**1**				**7**
3						**1**	**9**		**8**
2		*1*					**5**	*1*	1,2
1		*1*						**6**	1,2
	1	2	3	4	5	6	7	8	9

Fig. 1. *Hidden autarky* sudoku

2 Hidden Implicants

Hidden units, hidden equivalences and *hidden implications* are formulas that can be added to a formula F without changing the satisfiability. In the sequel, we briefly introduce the background of hidden formulas. Let F be a formula in CNF represented as set of clauses, a clause be set of literals and J be a (partial) interpretation. Interpretations are represented by the set of literals

[1] see http://www.sudokuoftheday.com/pages/techniques-1.php

which they map to $\top$. The reduct of a formula F with respect to an interpretation J is written as $F|_J$ and is computed by removing all satisfied clauses and all unsatisfied literals from F. A interpretation J is called *autarky* wrt. F, if $E|_J \subseteq F$ for all subsets E of F [2]. Altering J s.t. it models a literal l: $J \models l$, is denoted by $J[l]$. If necessary, the complement of l is removed from J. Boolean Constraint Propagation BCP applied to F given J is denoted by $BCP(F, J)$ and will be used as set that stores all literals, for which the unit rule was applicable. In the following *hidden implications* (HI) are introduced, which can be turned into *hidden units* (HU) or *hidden equivalences* (HE) by applying probing. HI can also partially simulate "high level autarkies", a case mentioned by Mate Soos[2], by using HE. The construction of such an *hidden implications* is based on the Shannon Expansion. Given a literal x, let $F_x := F|_x \wedge x$ then $F \equiv F_x \vee F_{\overline{x}}$.

Definition 1. *The hidden implication $HI(F) := (x \to J_x) \wedge (\overline{x} \to J_{\overline{x}})$ of F is the conjunction of $x \to J_x$ and $\overline{x} \to J_{\overline{x}}$, where J_x and $J_{\overline{x}}$ are autarkies for F_x and $F_{\overline{x}}$, respectively.*

Theorem 2. *Adding hidden implications $HI(F)$ to a formula $F' := F \wedge HI(F)$ keeps the satisfiability of F.*

If Theorem 2.1 of [1] is applied to $F \wedge HI(F)$, a HU l can be found, if there exists a literal l, for which $l \in J_x$ and $l \in J_{\overline{x}}$ holds. Similarily, HEs $x \equiv l$ are obtained, if there exists a literal l with $l \in J_x$ and $\overline{l} \in J_{\overline{x}}$. Note, that for the second step of probing, both implied literals and hidden implicants can be combined.

Example 3. Let $F := \{\{1, 2\}, \{3, 4\}, \{\overline{1}, \overline{3}\}, \{\overline{2}, \overline{4}\}, \{5, 6, 7\}, \{8, 9, 10\}, \{11, 12, 13\}, \{\overline{1}, \overline{5}\}, \{\overline{3}, \overline{5}\}, \{\overline{1}, \overline{8}\}, \{\overline{3}, \overline{8}\}, \{\overline{5}, \overline{11}\}, \{\overline{8}, \overline{11}\}, \{\overline{9}, \overline{10}, \overline{12}, \overline{13}\}\}$, then $J_1 := BCP(F, 1) = \{\overline{2}, \overline{3}, 4, \overline{5}, \overline{8}\}$ and $J_{\overline{1}} := BCP(F, \overline{1}) = \{2, 3, \overline{4}, \overline{5}, \overline{8}\}$. For both $F|_{J_1}$ and $F|_{J_{\overline{1}}}$ literal 11 is pure, and thus $HI(F) := \{1 \to 11, \overline{1} \to 11\}$, resulting in $HU(F) := \{11\}$. Notice, that this unit clause is not necessary, because $J := \{1, 2, 3, \overline{4}, \overline{5}, \overline{8}, 9, \overline{11}, 12, \overline{13}\}$ also satisfies F. This example almost corresponds to the sudoku in Fig. 1. Variable 1 is $f_{9,1} = 1$, and variables 11 is $f_{8,9} = 1$. The claues in the example encode the relation between these two positions.

Since detecting autarkies is an $\mathcal{NP}$-hard problen, a polynomial subcategory can also be used for detection, for example *pure literals.* Current limited experiments showed that HIs occur in application benchmarks without altering the solving performance significantly. As future work, more extensive experiments and more advanced reasoning will be analyzed to improve the reasoning.

References

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[2] see `http://www.msoos.org/2011/07/note-to-self-higher-level-autarkies/`.