Tractable Diversity

Scalable Multiperspective Ontology Management via Standpoint $\mathscr{EL}$

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International Center for Computational Logic

TECHNISCHE UNIVERSITÄT DRESDEN

Motivation

Multiperspective Knowledge Management

Motivation: Knowledge Integration

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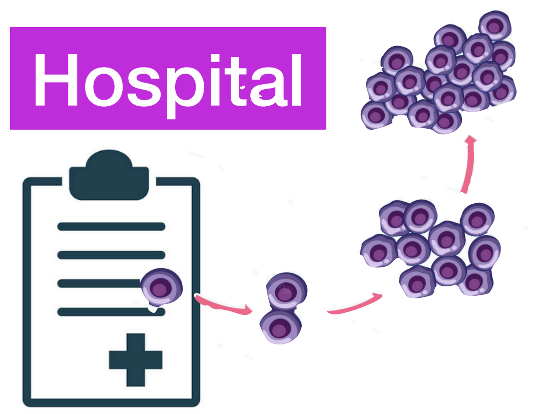
SNOMED CT

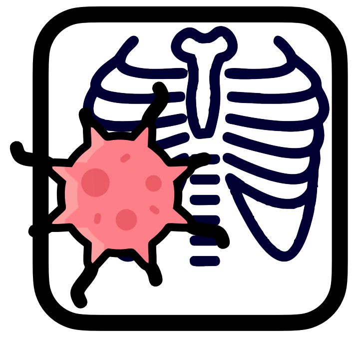
Hospital



WIKIDATA

yago select knowledge

Private Lab

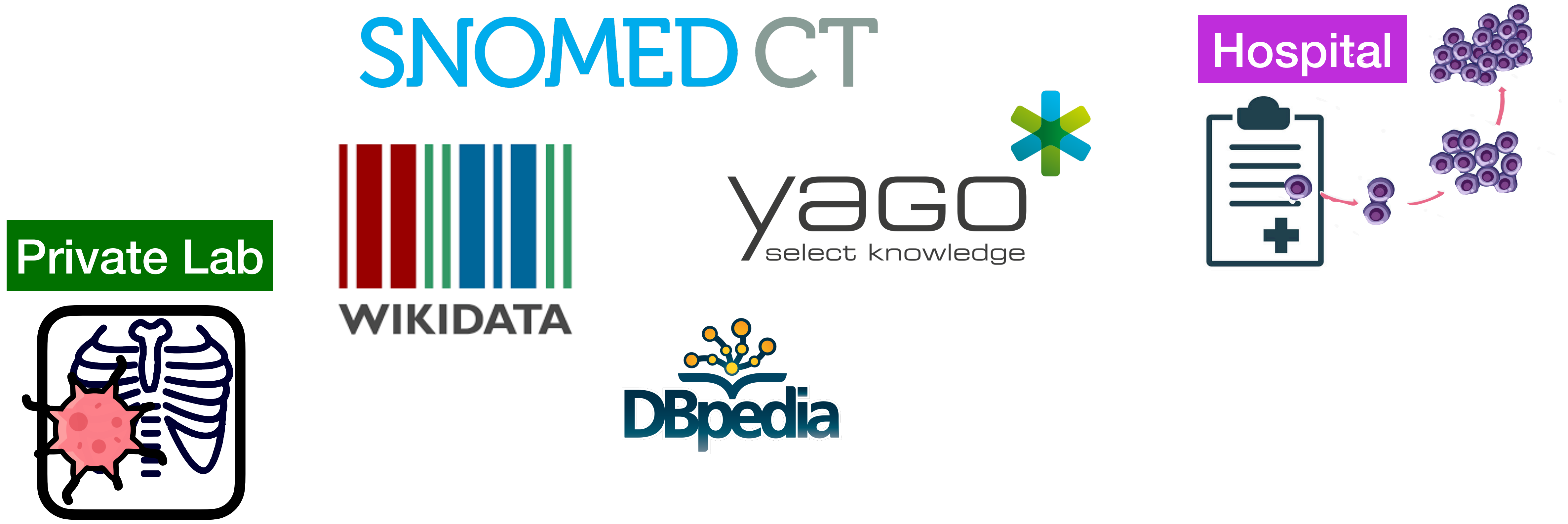


DBpedia

Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available



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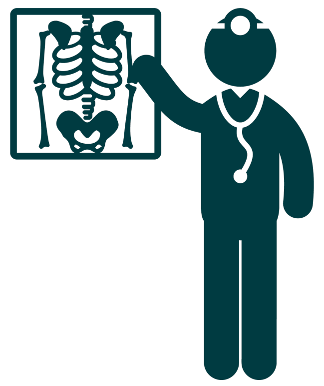

Tumor(t)
patientPart(p, t)

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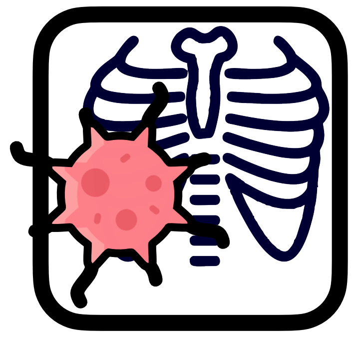

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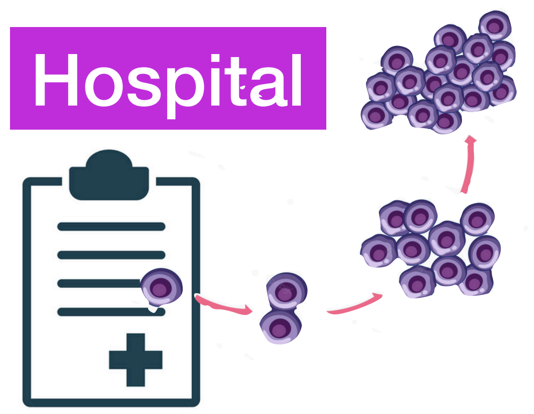

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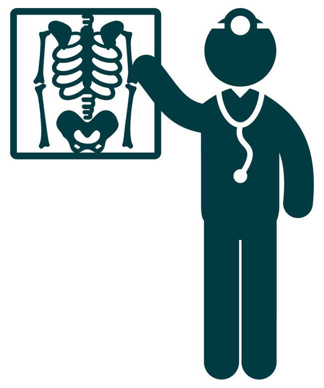

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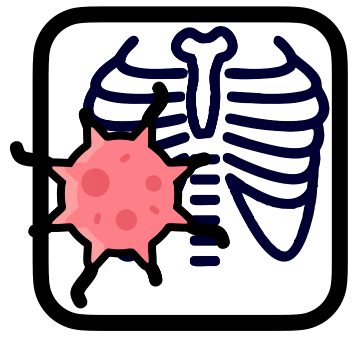

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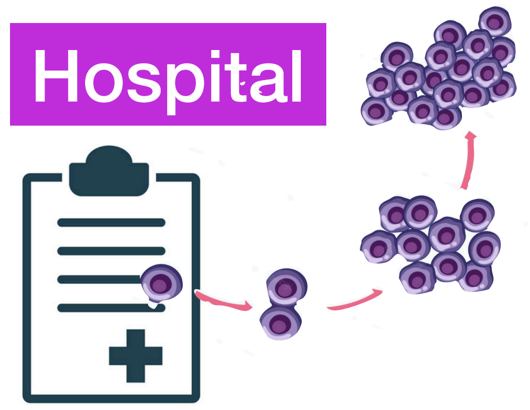

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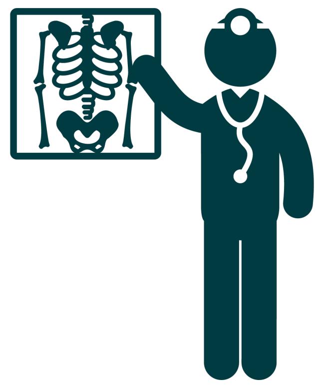

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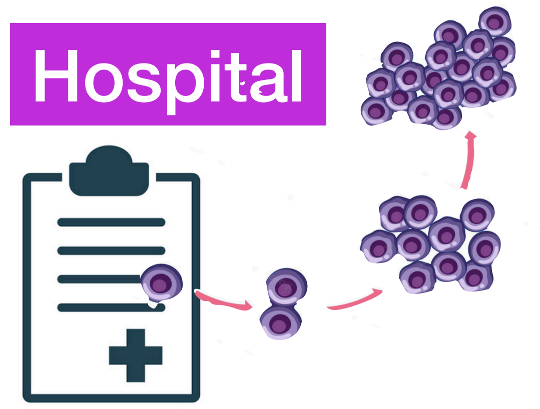

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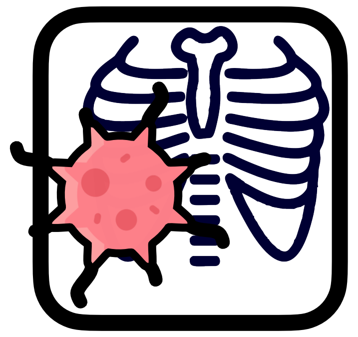

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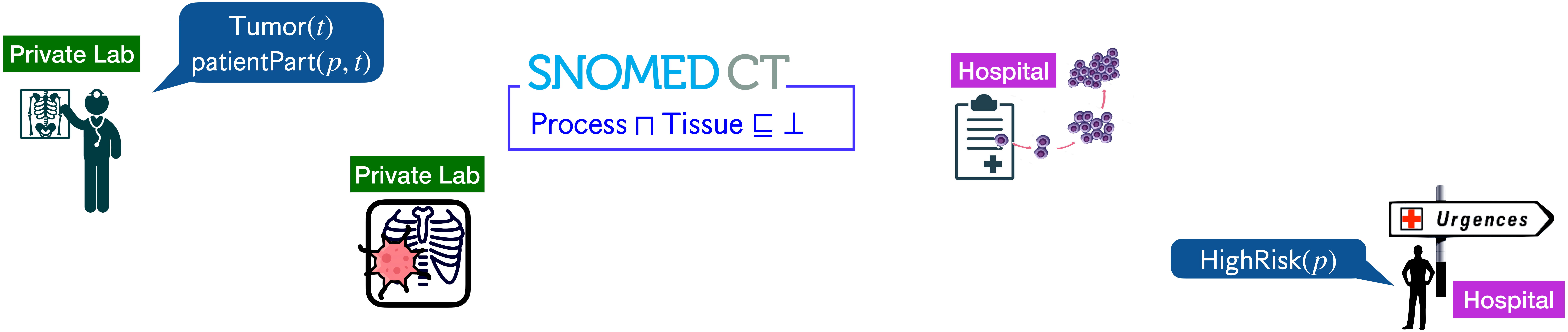

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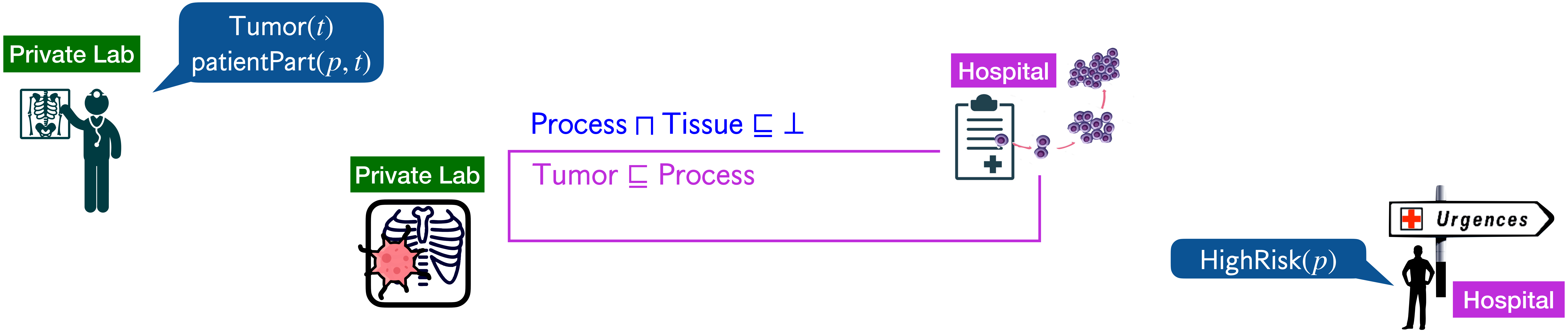

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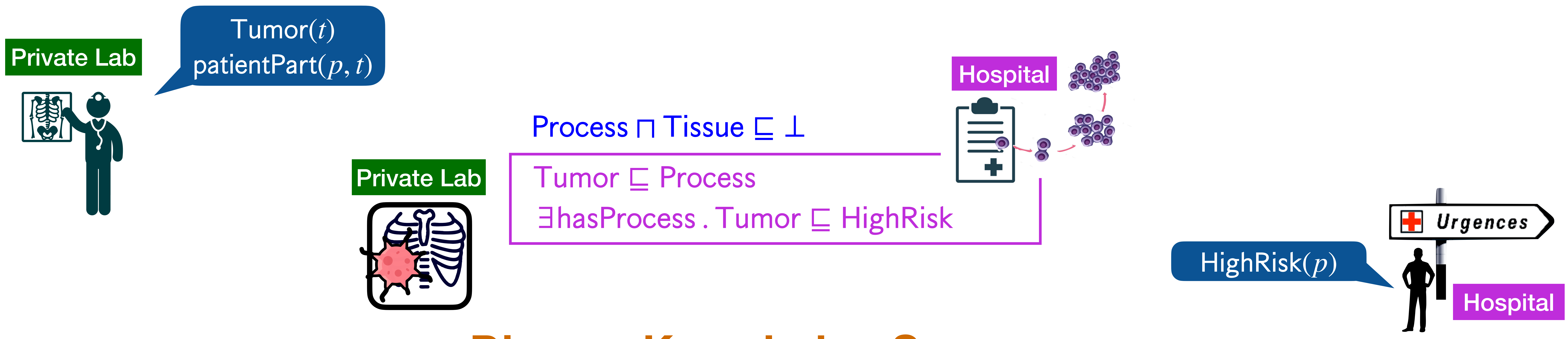

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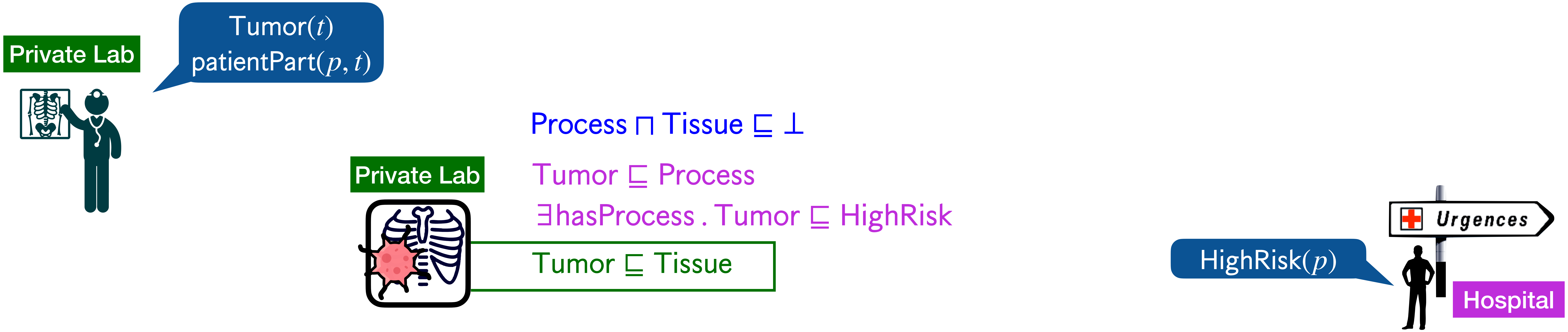

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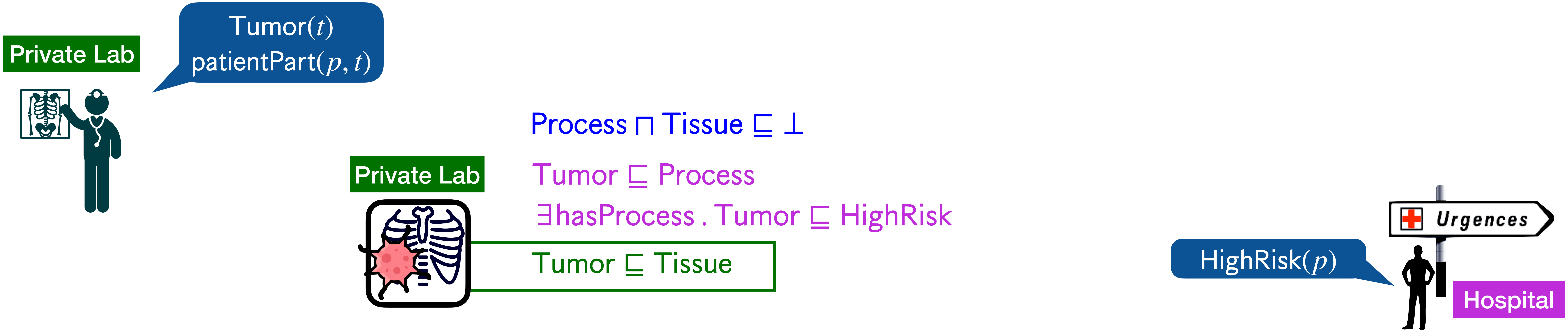

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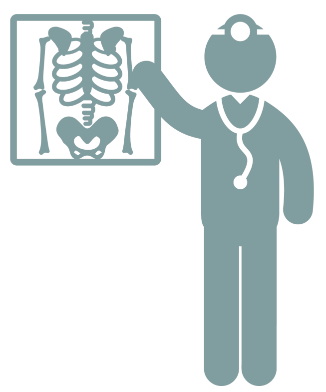

$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

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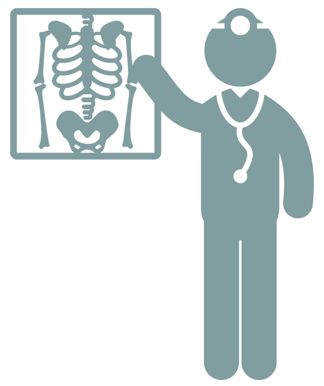

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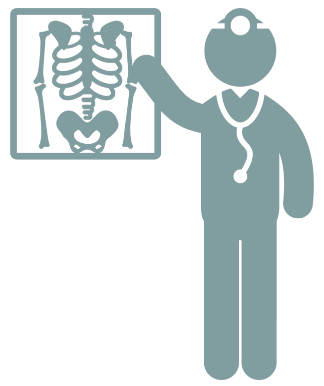

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$\mathsf{Tumor}(t)$ → $\mathsf{Process}(t)$

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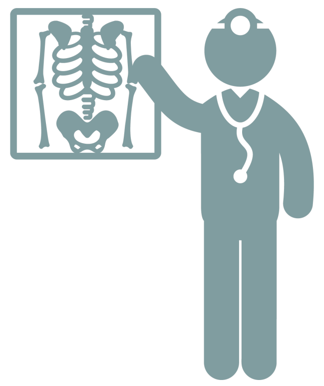

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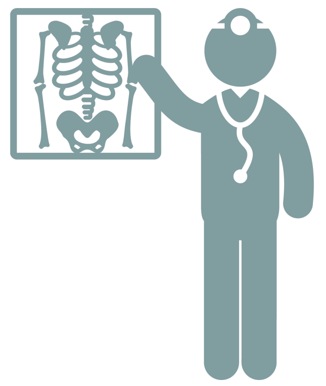

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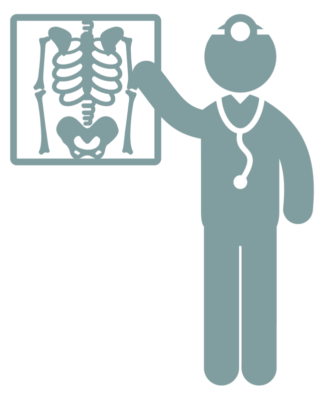

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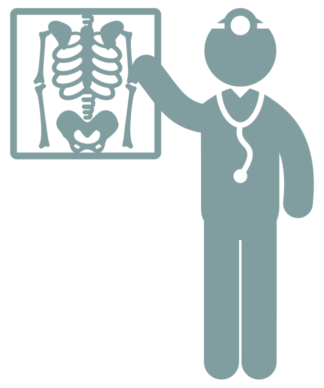

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Challenge: combining diverse (potentially conflicting) sources without weakening them

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$\square_e$ Unequivocal to e

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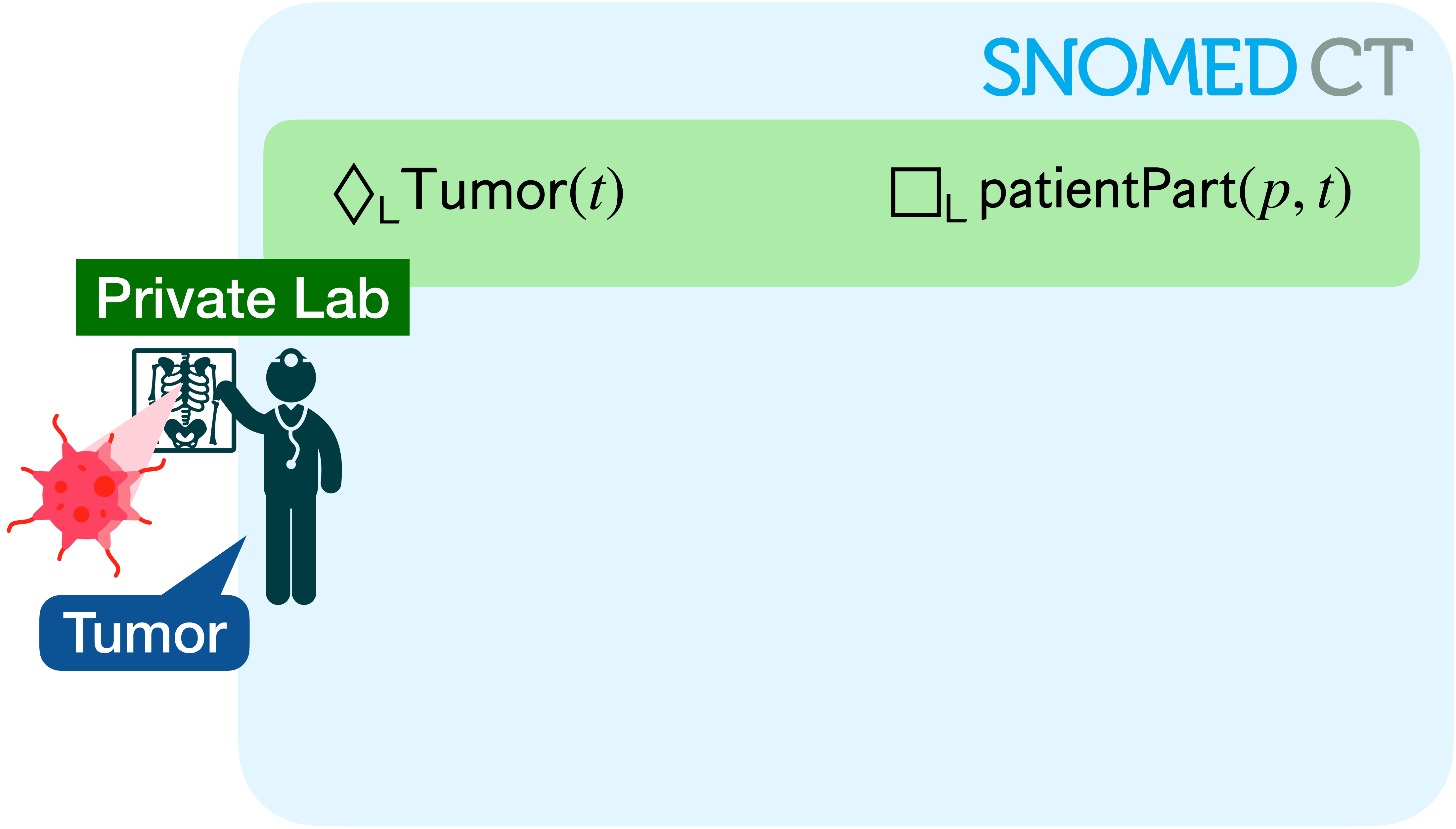

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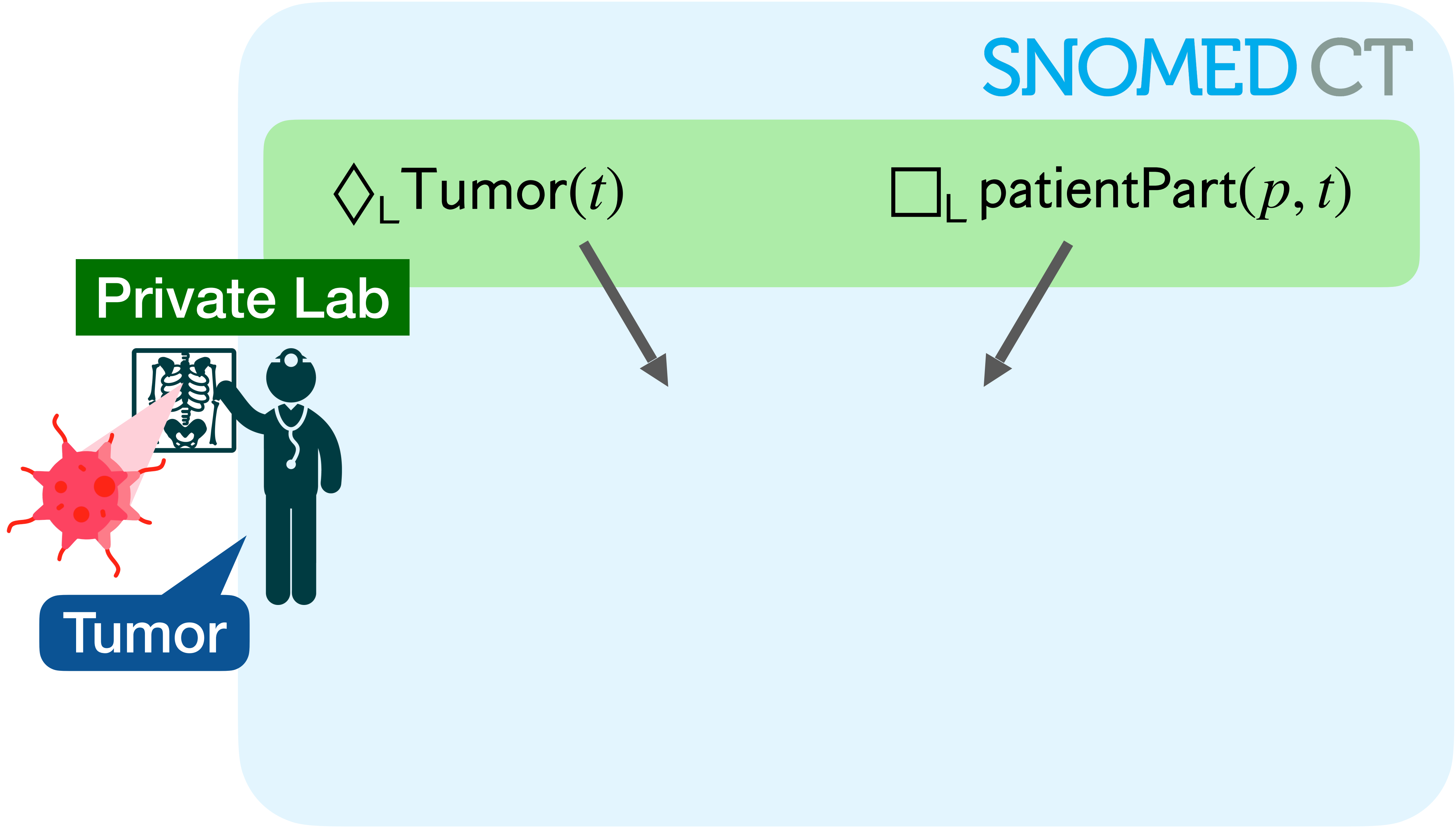

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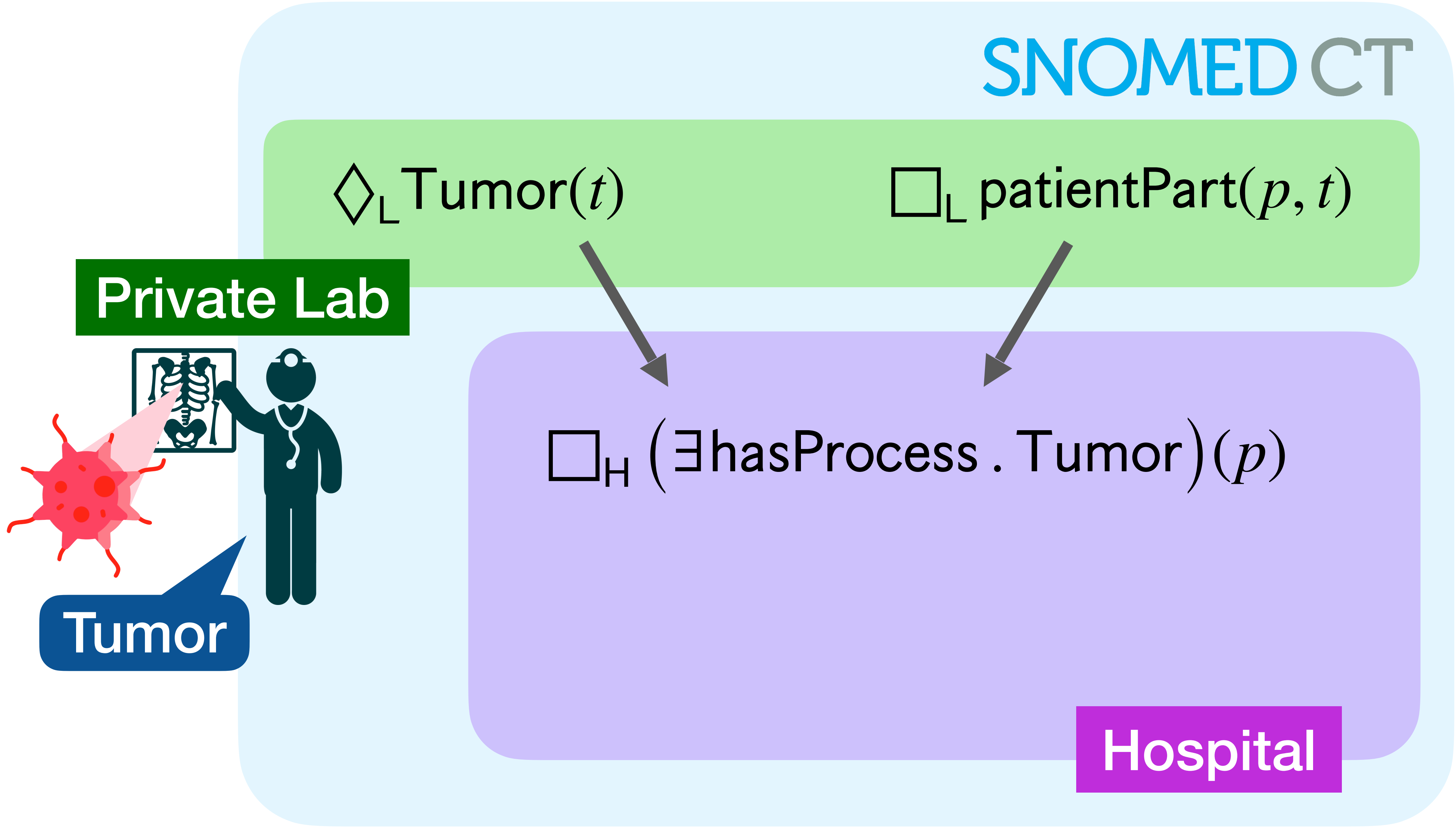

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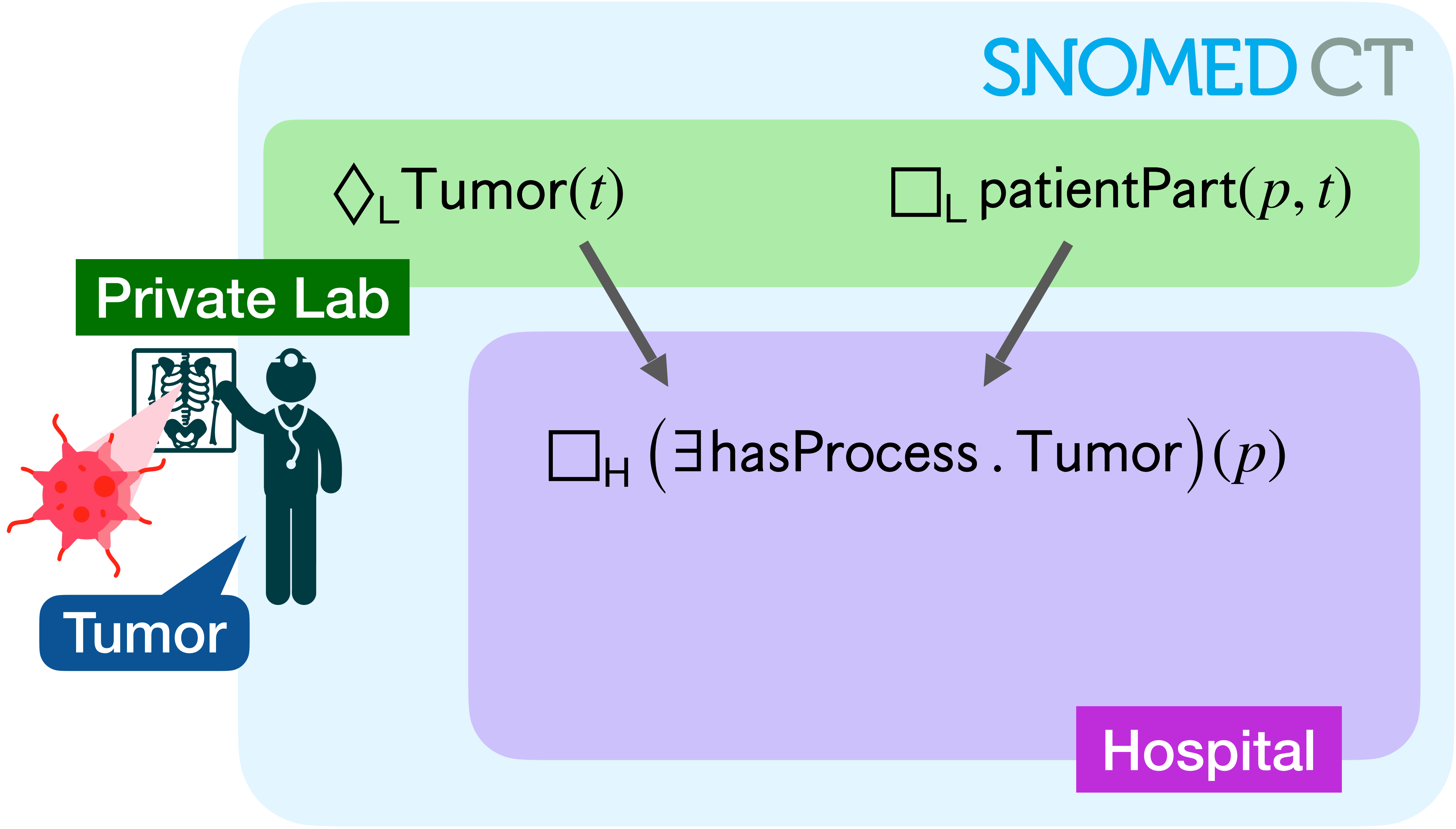

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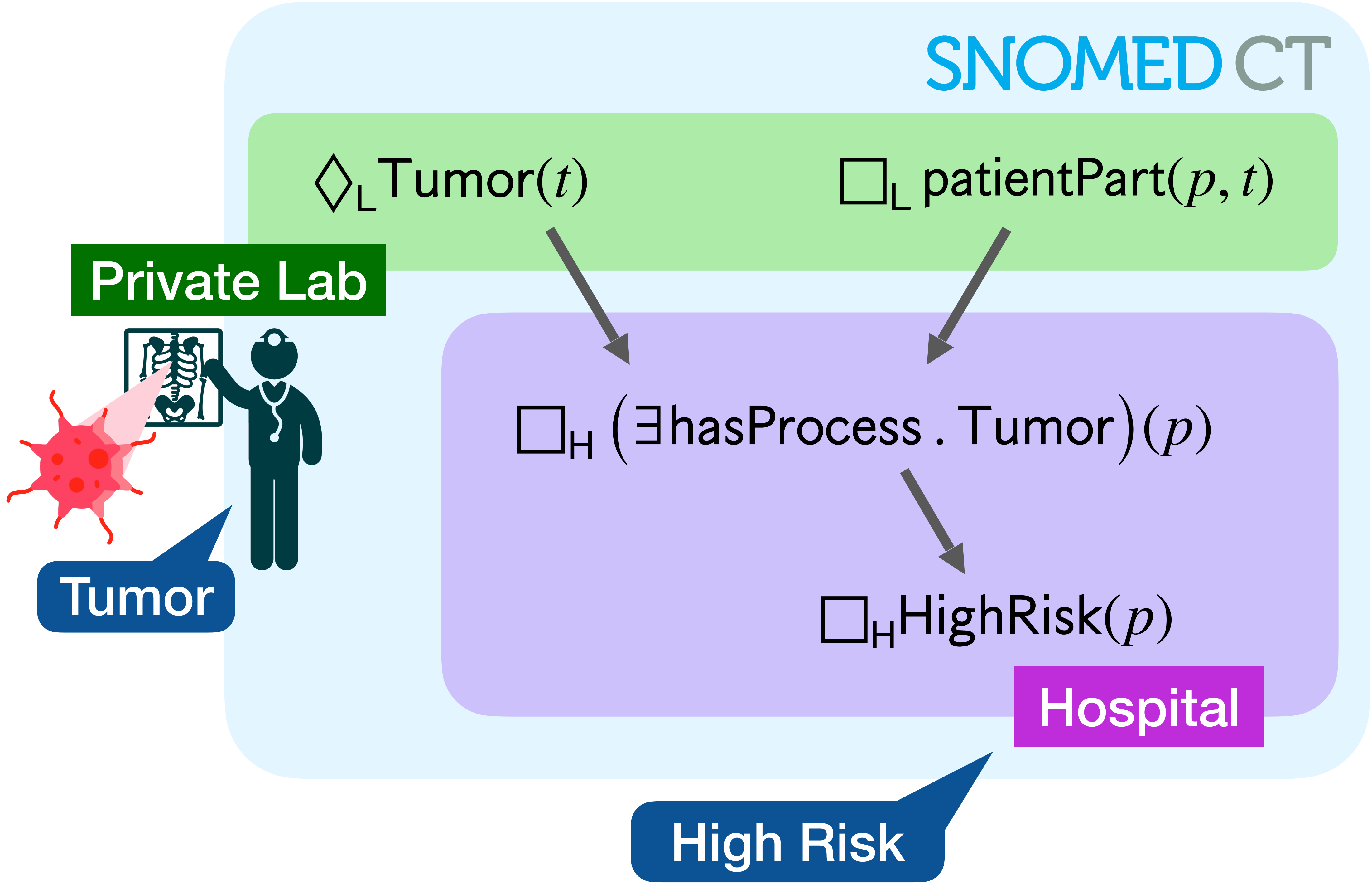

Standpoint *EL*

The description logic $\mathscr{EL}$

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Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concepts, roles and individuals

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Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

The description logic $\mathscr{EL}$

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Process ⊓ Tissue

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∃patientPart . Tumor

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$\mathsf{Process} \sqcap \mathsf{Tissue}$

$\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

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$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$$

With $A \in N_{\mathsf{C}}$, $r \in N_{\mathsf{R}}$

Tissue

Process ⊓ Tissue

∃patientPart . Tumor

The **set of axioms** includes:

- GCIs (general concept inclusions): $C \sqsubseteq D$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concepts, roles and individuals

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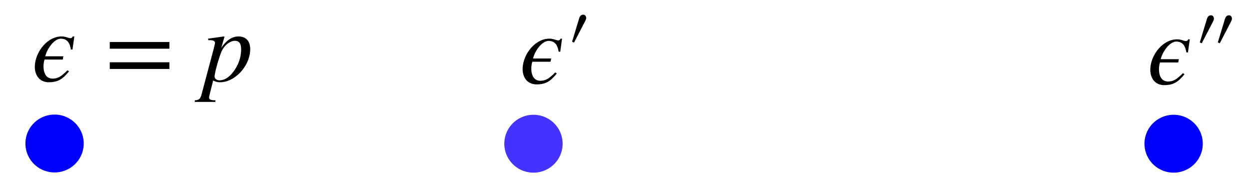

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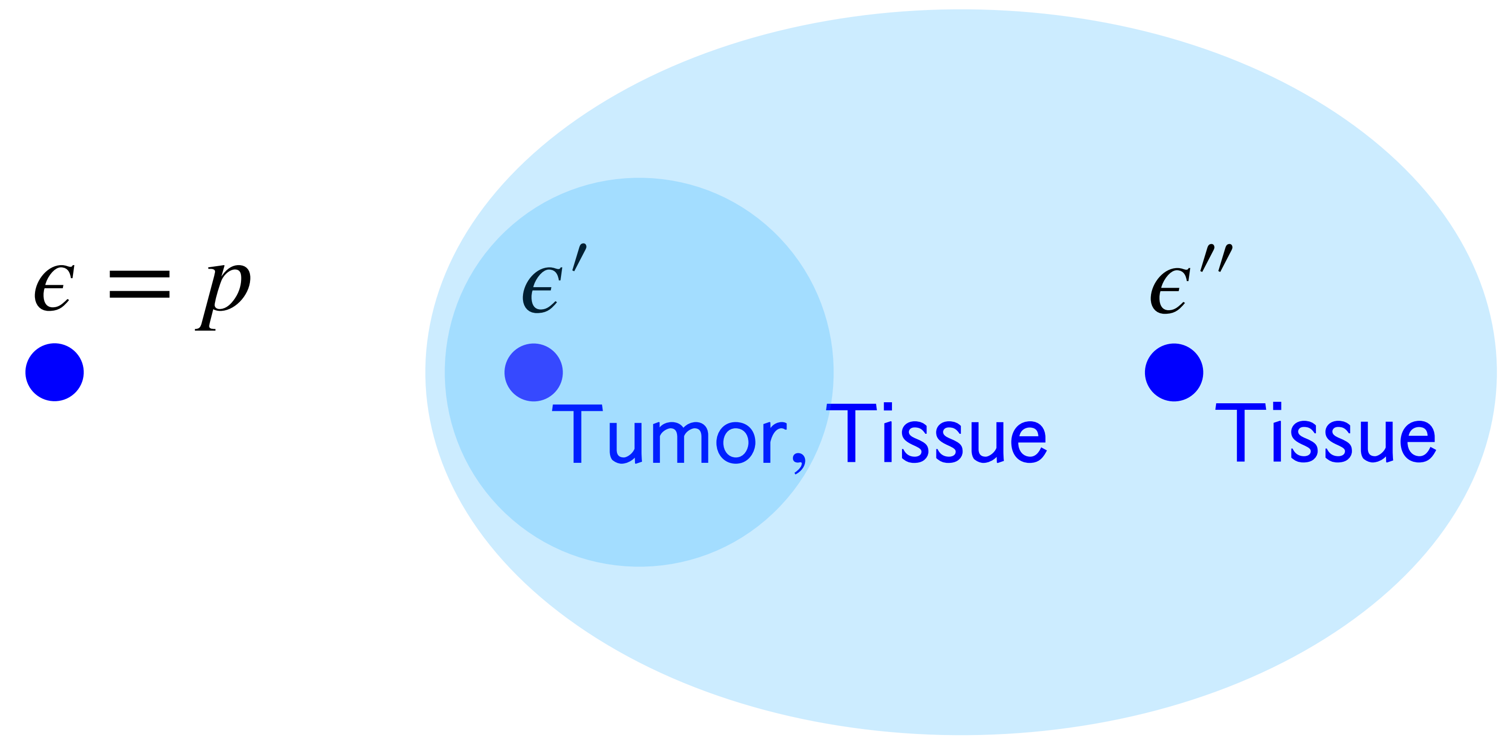

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Towards Standpoint-$\mathscr{EL}$

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Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

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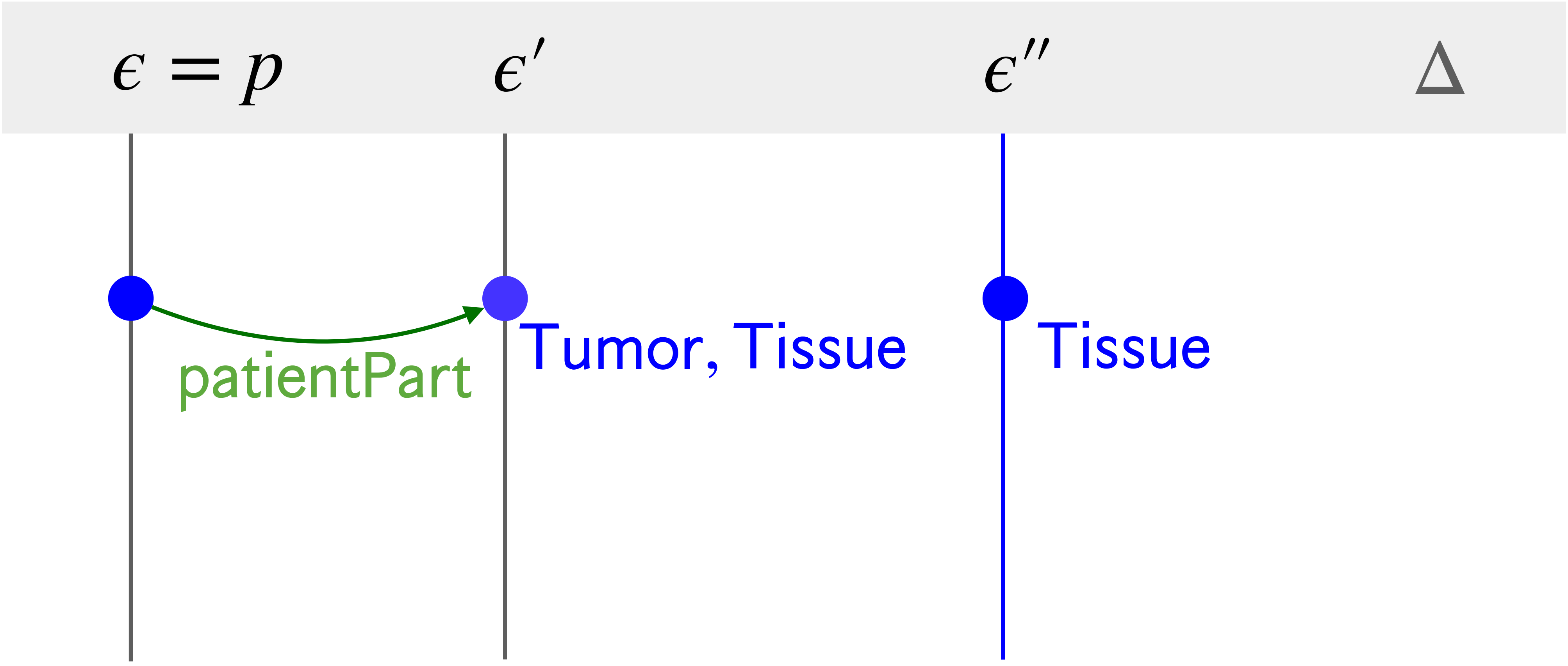

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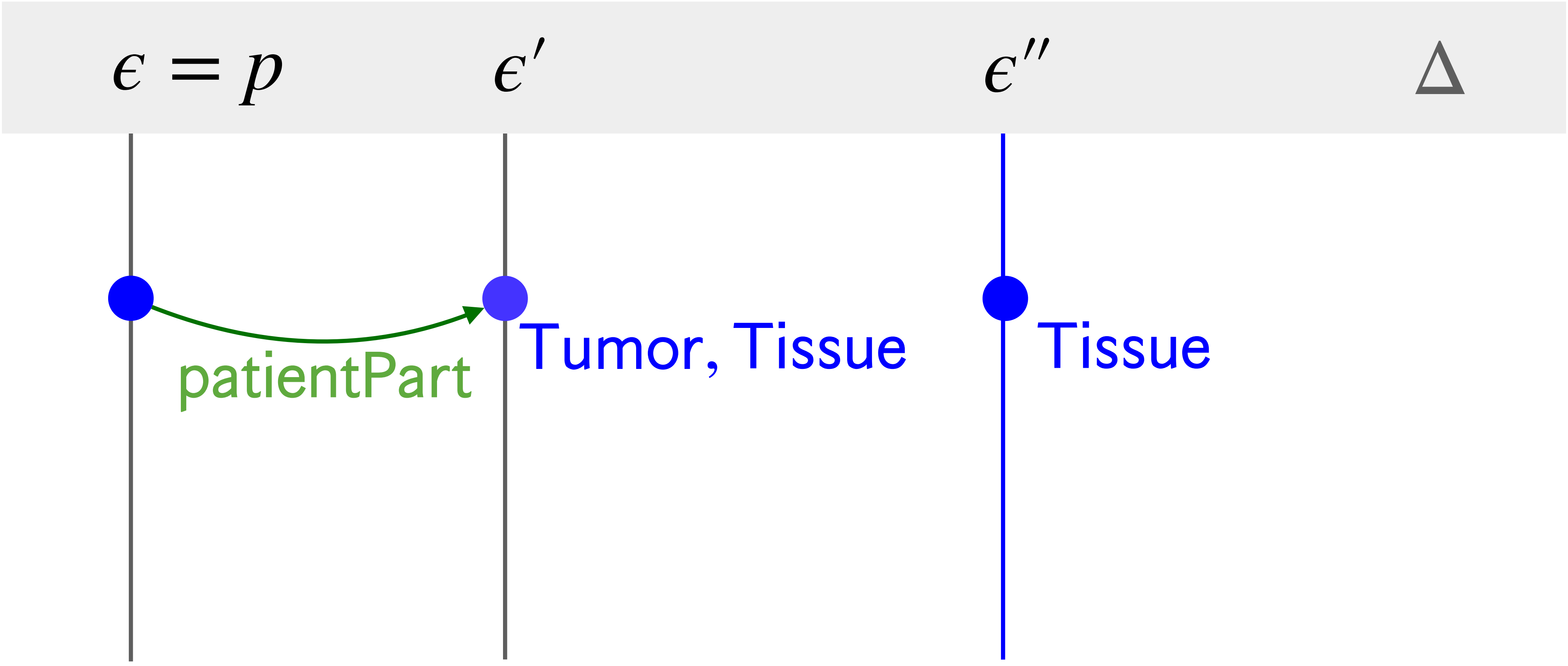

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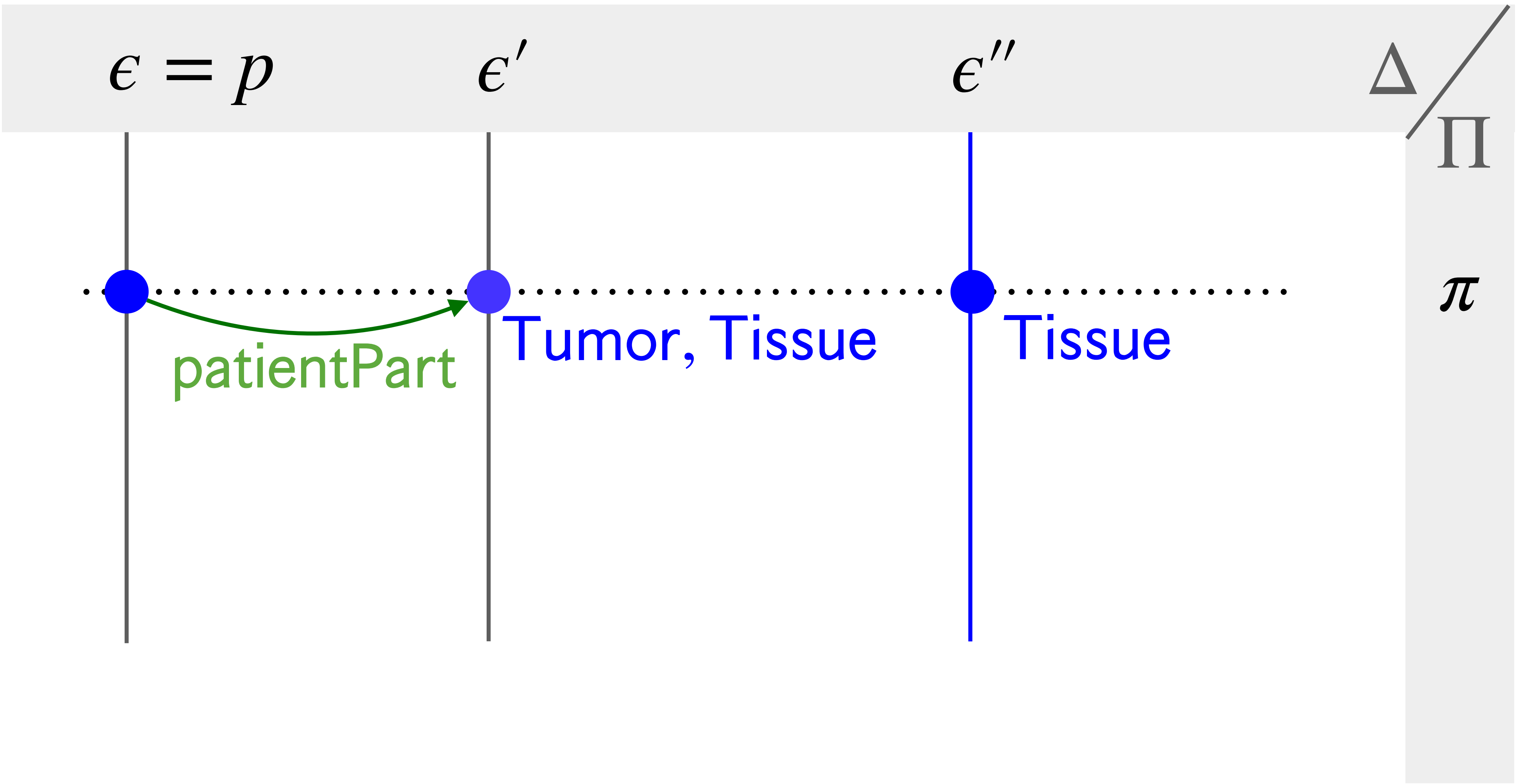

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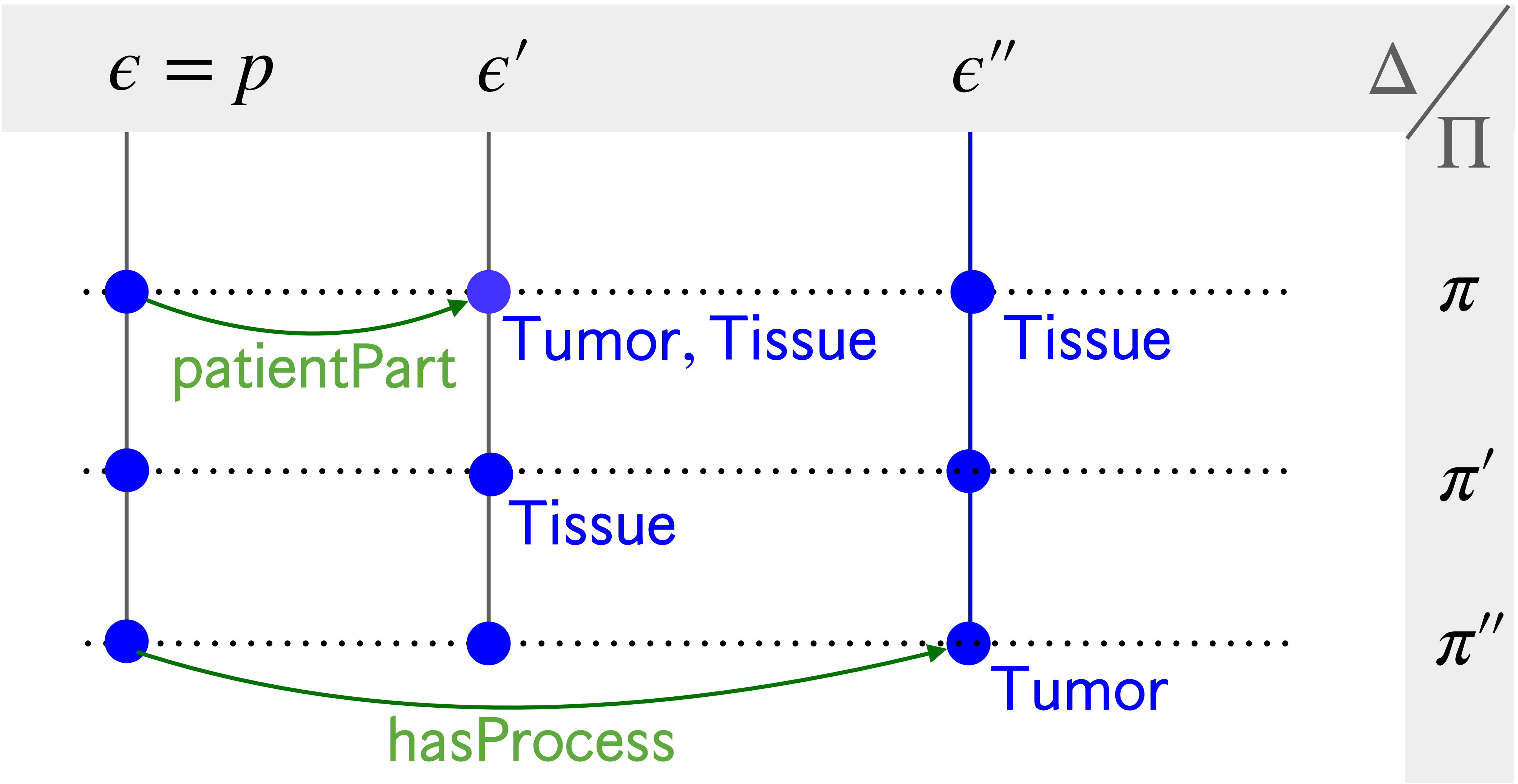

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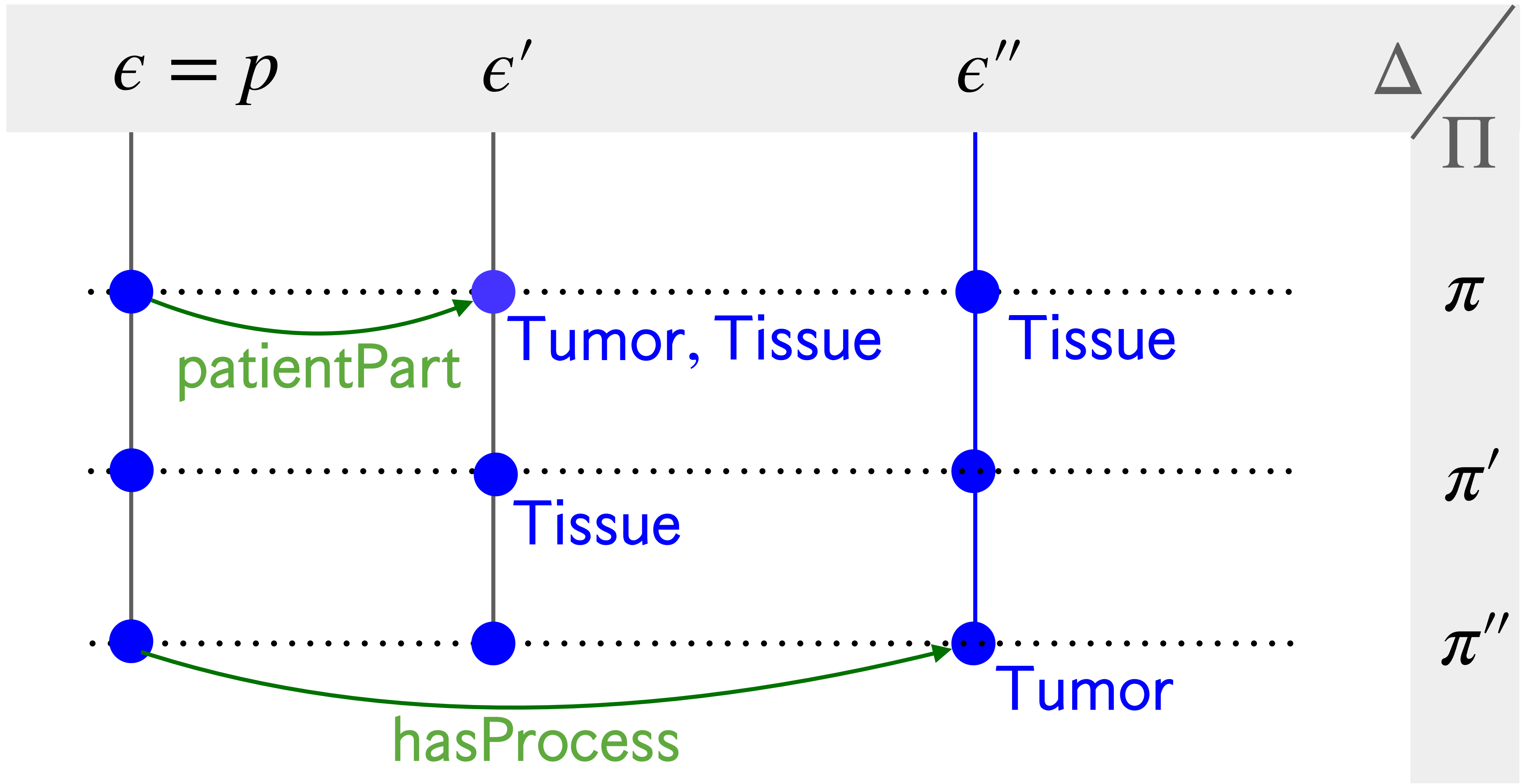

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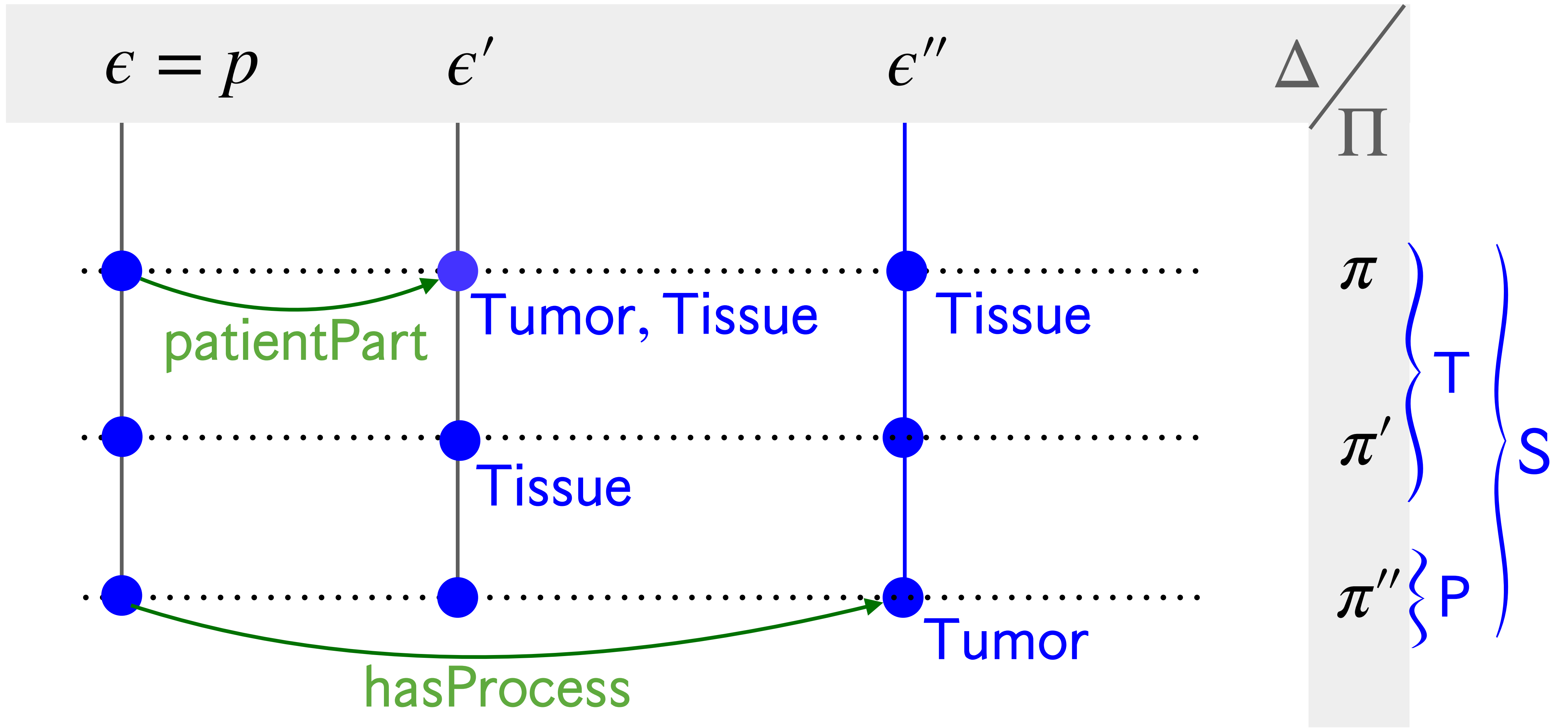

Complexity and Automated Reasoning

Tractable Reasoning in $\mathbb{S}_{\mathscr{EL}}$

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Many sentential fragments of FOL (including DLs) enhanced with SL preserve the complexity of the fragment.

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Lucía Gómez Álvarez, Sebastian Rudolph, Hannes Straß; (*ISWC 2022*)

But monodic modal extensions of DLs usually lead to a blowup in complexity.

Main contributions:

- ➡ Complexity of the satisfiability of Standpoint-$\mathscr{EL}$ $\rightarrow$ PTime
- ➡ A (complexity-optimal) tableau-based algorithm
- ➡ Tractability is easily lost:
  - Empty standpoints $\rightarrow$ NP-hard
  - Rigid roles $\rightarrow$ CoNP-hard

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  - Nominal Concepts → ExpTime-hard

Conclusions and Future Work

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- ➡ Tractability is preserved for self, role-chains and boolean combinations of axioms
- ➡ Datalog-based prototypical implementation
- ➡ Towards conceptual modelling with standpoints for knowledge integration challenges

The end.

Tableau Algorithm for $\mathbb{S}_{\mathscr{EL}}$

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Example KB:

$\Box_{op}$ Good $\sqsubseteq$ Great

$\Box_{op}$ Good(tom)

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Example KB:

$\Box_{\mathsf{op}}\,\mathsf{Good} \sqsubseteq \mathsf{Great}$

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Model: $\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$

Tableau Algorithm for $\mathbb{S}_{\mathscr{EL}}$

- The tableau generates a set of constraint systems (CS).

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A constraint associates a variable v to:

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- A concept C $\longrightarrow$ the element ϵ is in the set of C at π

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- A variable v of the CS for ϵ corresponds to some precisification π.

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Local labelling (LL) rule:

$\mathbf{R}_{\preceq}$ If $\{x:\mathsf{s},\ x':\mathsf{s}\preceq\mathsf{s}'\} \subseteq S$ but $(x:\mathsf{s}') \notin S$, then set $S := S \cup \{x:\mathsf{s}'\}$.

Local content (LC) rules:

$\mathbf{R}_{\sqcap}$ If $\{x:C,\ x:D\} \subseteq S$, $(x:C \sqcap D) \notin S$ and $C \sqcap D \in \mathsf{C}_{\mathcal{K}}$, then set $S := S \cup \{x:C \sqcap D\}$.

$\mathbf{R}_{\sqsubseteq}$ If $\{x:C,\ x:C \sqsubseteq D\} \subseteq S$ but $(x:D) \notin S$, then set $S := S \cup \{x:D\}$.

$\mathbf{R}_{\Box}$ If $\{x:\Box_{\mathsf{s}}\Phi,\ x':\mathsf{s}\} \subseteq S$ but $(x':\Phi) \notin S$, then set $S := S \cup \{x':\Phi\}$.

$\mathbf{R}_{g}$ If $(x:\mathbf{G}) \in S$ but $(x':\mathbf{G}) \notin S$, then set $S := S \cup \{x':\mathbf{G}\}$.

$\mathbf{R}_{a}$ If $\{x:a,\ x:C(a)\} \subseteq S$ but $(x:C) \notin S$, then set $S := S \cup \{x:C\}$.

$\mathbf{R}_{\Diamond}$ If $(x:\Diamond_{\mathsf{s}}C) \in S$ and $\{x':\mathsf{s},\ x':C\} \nsubseteq S$ for all x' in S, then create a fresh variable x' and set $S := S \cup \{x':C,\ x':\mathsf{s},\ x':*,\ x':\top\}$.

Global non-generating (GN) rules:

$\mathbf{R}_{\downarrow}$ If $(x:C) \in \mathcal{S}(\varepsilon)$, $\langle \varepsilon', x', \varepsilon, x, R\rangle \in \mathcal{R}$, and $\exists R.C \in \mathsf{C}_{\mathcal{K}}$, but $(x':\exists R.C) \notin \mathcal{S}(\varepsilon')$, then set $\mathcal{S}(\varepsilon') := \mathcal{S}(\varepsilon') \cup \{x':\exists R.C\}$.

$\mathbf{R}_{r}$ If $\{x:a,\ x:R(a,b)\} \subseteq \mathcal{S}(\varepsilon)$ and $(x':b) \in \mathcal{S}(\varepsilon')$, but $\langle \varepsilon, x, \varepsilon', x, R\rangle \notin \mathcal{R}$, then set $\mathcal{S}(\varepsilon') := \mathcal{S}(\varepsilon') \cup \{x:\top\} \cup \{x:\mathsf{s} \mid \mathsf{s} \in \mathsf{st}_{\varepsilon}(x)\}$ and $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon, x, \varepsilon', x, R\rangle\}$.

$\mathbf{R}_{r'}$ If $\{x:b,\ x:R(a,b)\} \subseteq \mathcal{S}(\varepsilon)$ and $(x':a) \in \mathcal{S}(\varepsilon')$, but $\langle \varepsilon', x, \varepsilon, x, R\rangle \notin \mathcal{R}$, then set $\mathcal{S}(\varepsilon') := \mathcal{S}(\varepsilon') \cup \{x:\top\} \cup \{x:\mathsf{s} \mid \mathsf{s} \in \mathsf{st}_{\varepsilon}(x)\}$ and $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon', x, \varepsilon, x, R\rangle\}$.

$\mathbf{R}_{\exists'}$ If $(x:\exists R.C) \in \mathcal{S}(\varepsilon)$, $(C, \mathsf{st}_{\varepsilon}(x), x') \in \mathcal{L}(\varepsilon')$ with $\varepsilon \neq \varepsilon'$ or $x = x'$, but $\langle \varepsilon, x, \varepsilon'', x'', R\rangle \notin \mathcal{R}$ whenever $(C, \mathsf{st}_{\varepsilon}(x), x'') \in \mathcal{L}(\varepsilon'')$ and $\varepsilon \neq \varepsilon''$ or $x = x''$, then set $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon, x, \varepsilon', x', R\rangle\}$.

Global generating (GG) rule:

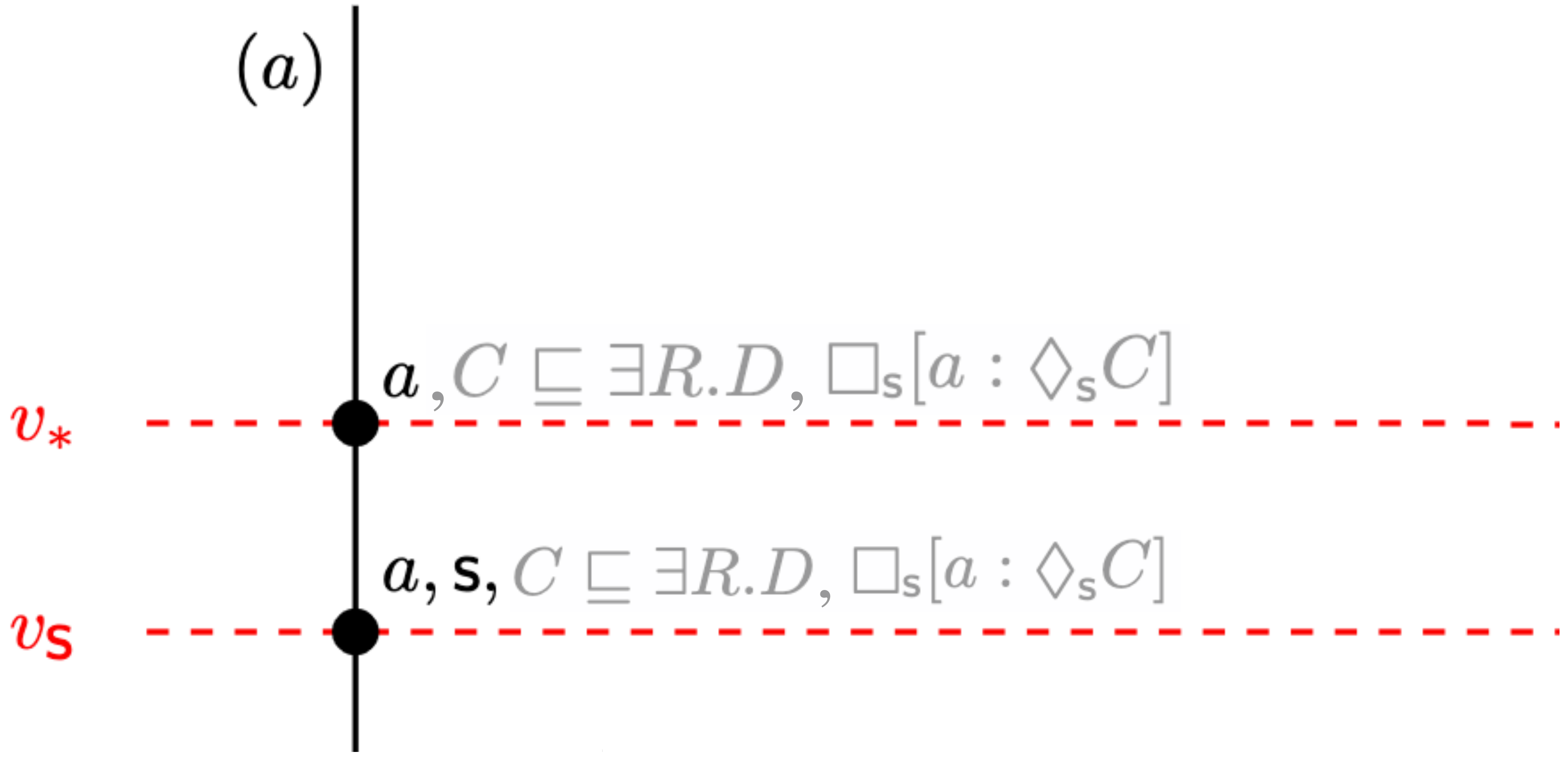
$\mathbf{R}_{\exists}$ If$(x:\exists R.C) \in \mathcal{S}(\varepsilon)$, but $\langle \varepsilon, x, \varepsilon'', x'', R\rangle \notin \mathcal{R}$ whenever $(C, \mathsf{st}_{\varepsilon}(x), x'') \in \mathcal{L}(\varepsilon'')$ and $\varepsilon \neq \varepsilon''$ or $x = x''$, then create ε' and a fresh variable x', and then set $\mathcal{L}(\varepsilon') := \{(C, \mathsf{st}_{\varepsilon}(x), x')\}$, $\mathcal{S}(\varepsilon') := S_0^{\mathcal{K}} \cup \{x':C,\ x':\top\} \cup \{x':\mathsf{s} \mid \mathsf{s} \in \mathsf{st}_{\varepsilon}(x)\}$, $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon, x, \varepsilon', x', R\rangle\}$.

Tableau Algorithm for $\mathbb{S}_{\mathscr{EL}}$

Example:

$C \sqsubseteq \exists R.D$

$\Box_{\mathsf{s}}[a : \Diamond_{\mathsf{s}} C]$



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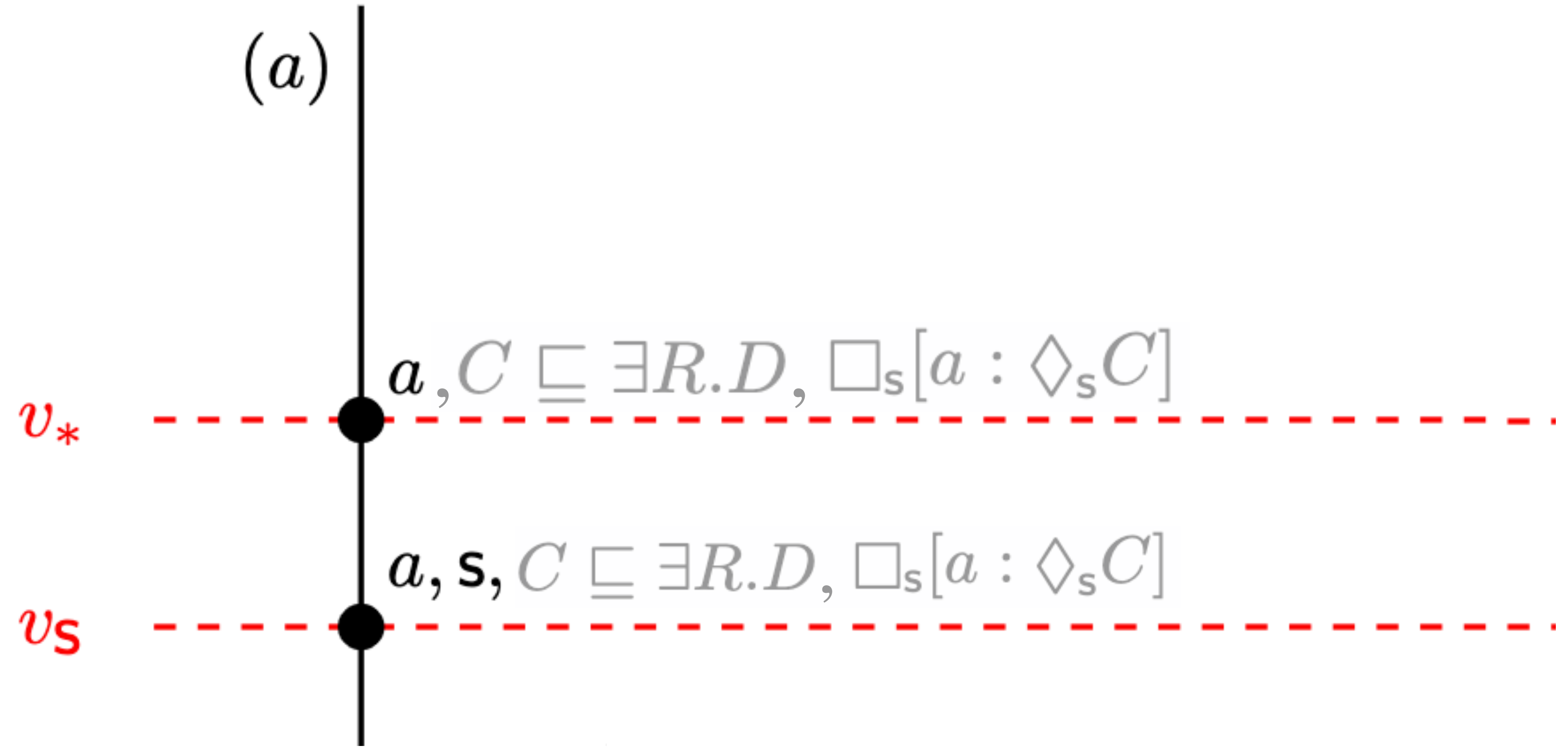

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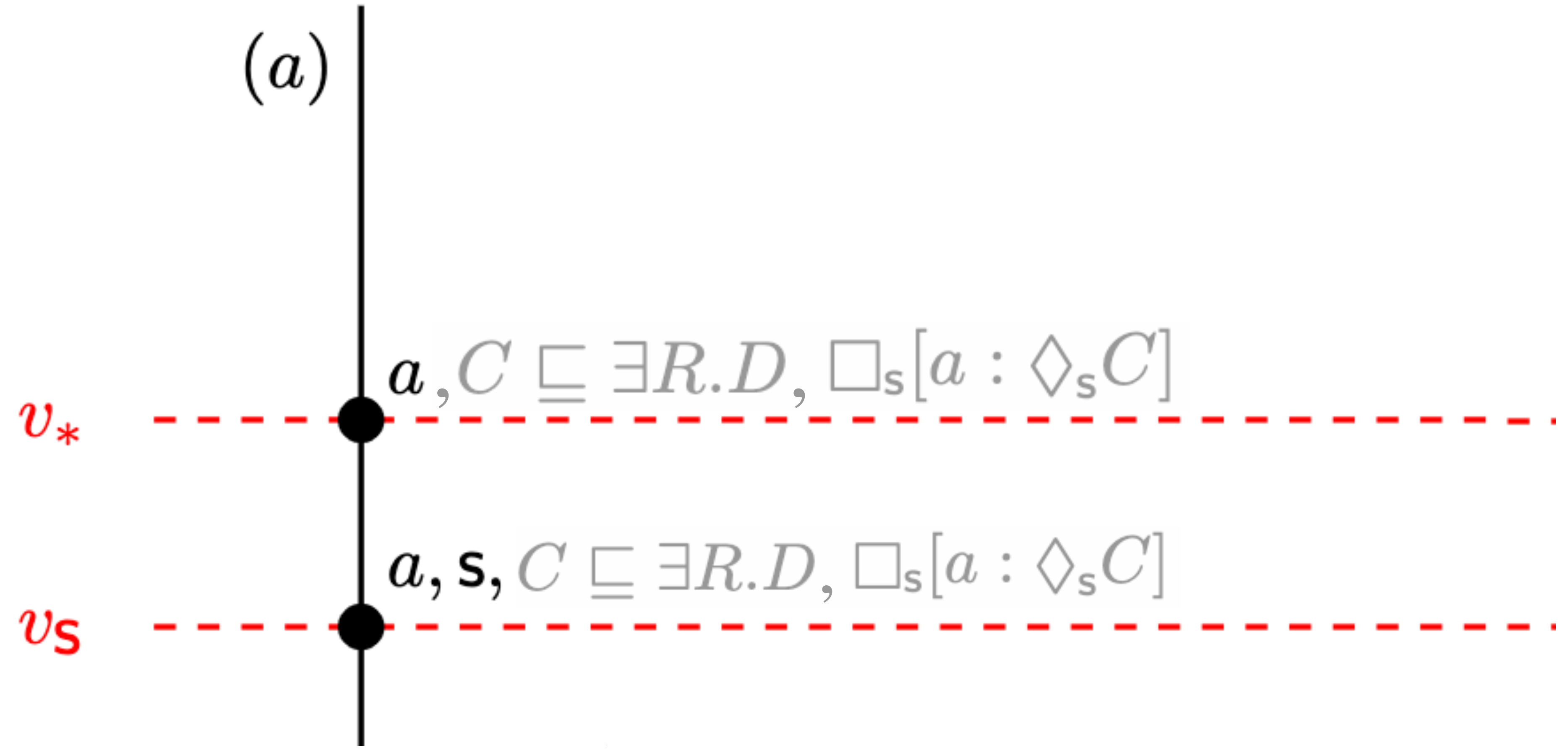

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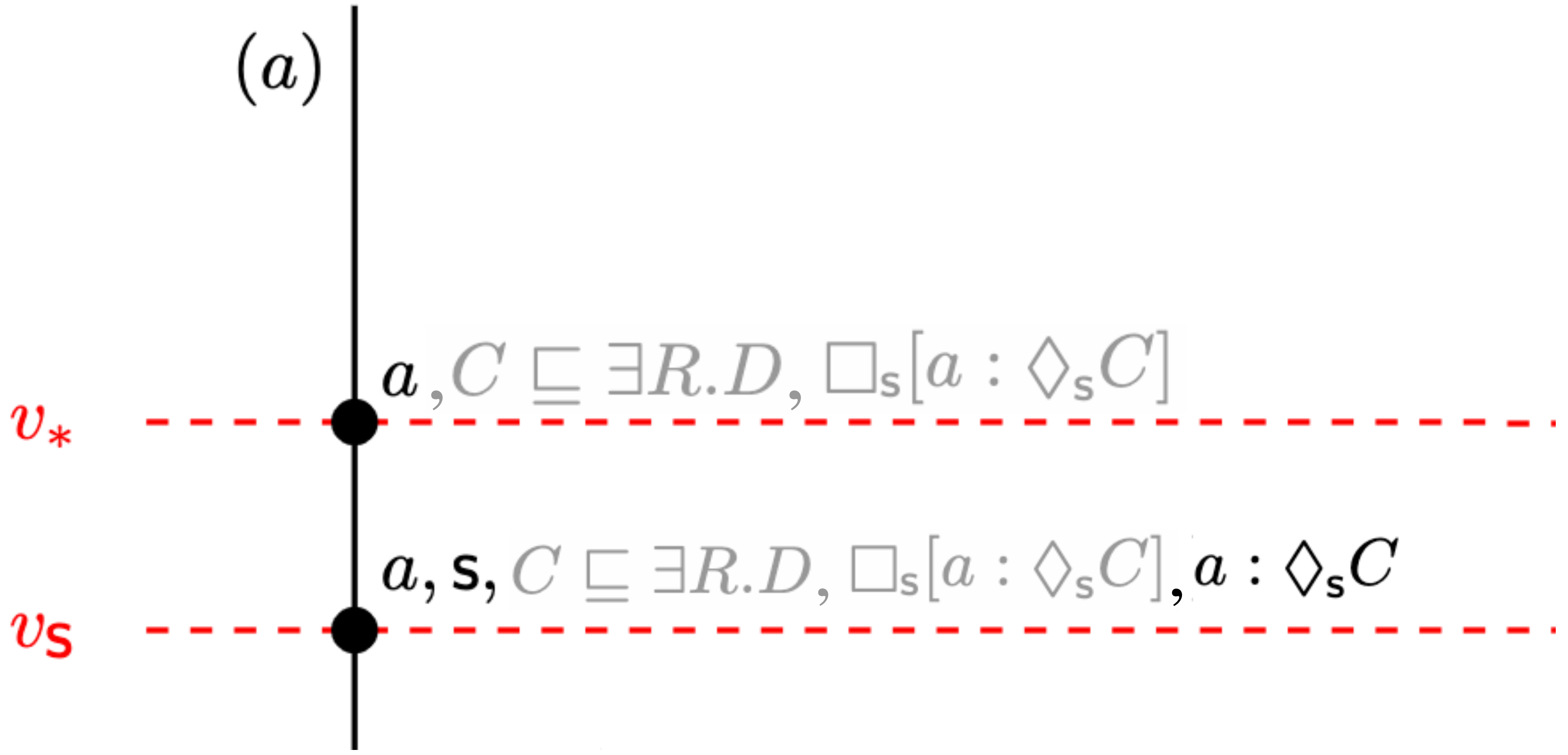

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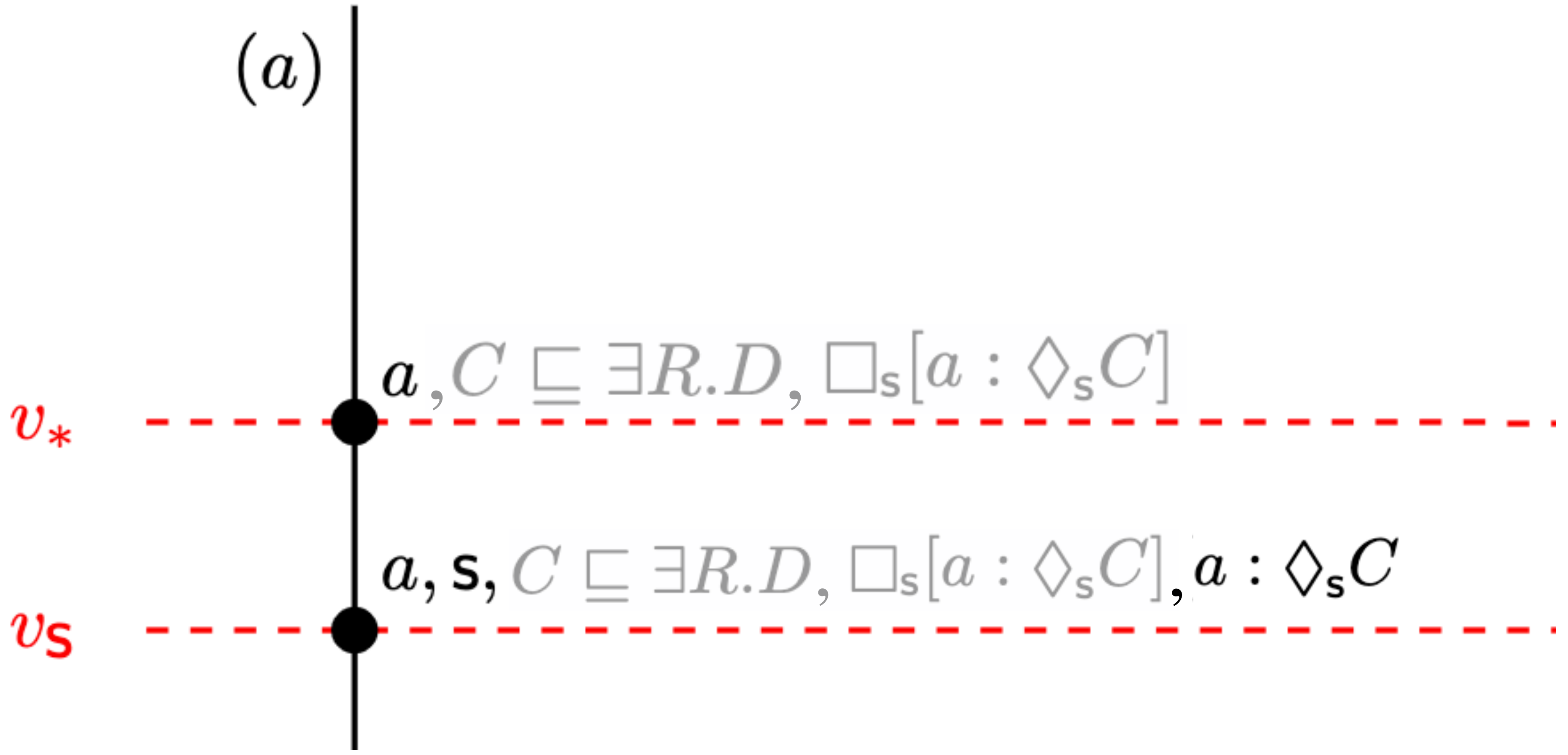
$\square_{\mathsf{s}}[a : \lozenge_{\mathsf{s}} C]$



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Example:

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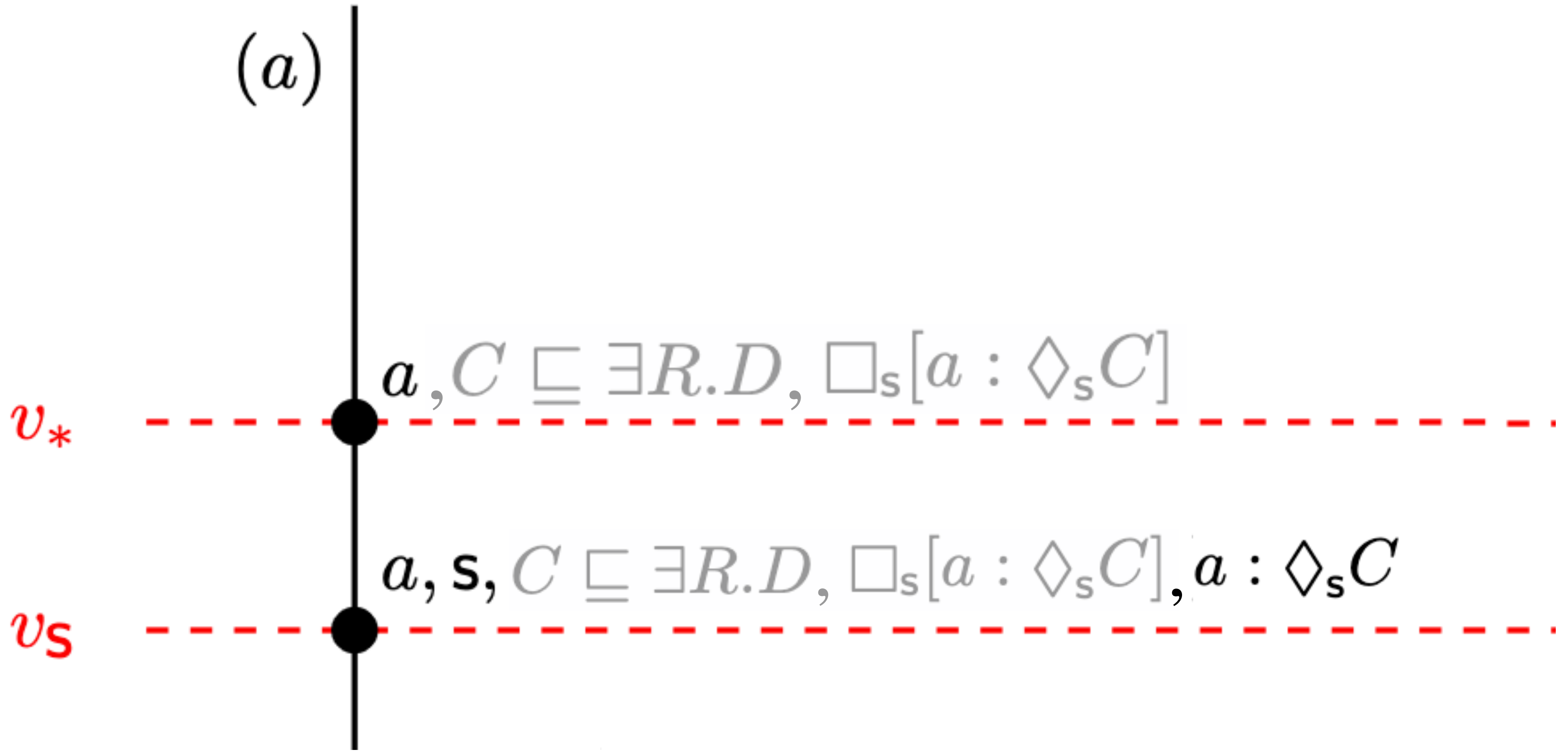
$\mathbf{R}_a$ If $\{x : a,\ x : C(a)\} \subseteq S$ but $(x : C) \notin S$, then set $S := S \cup \{x : C\}$.



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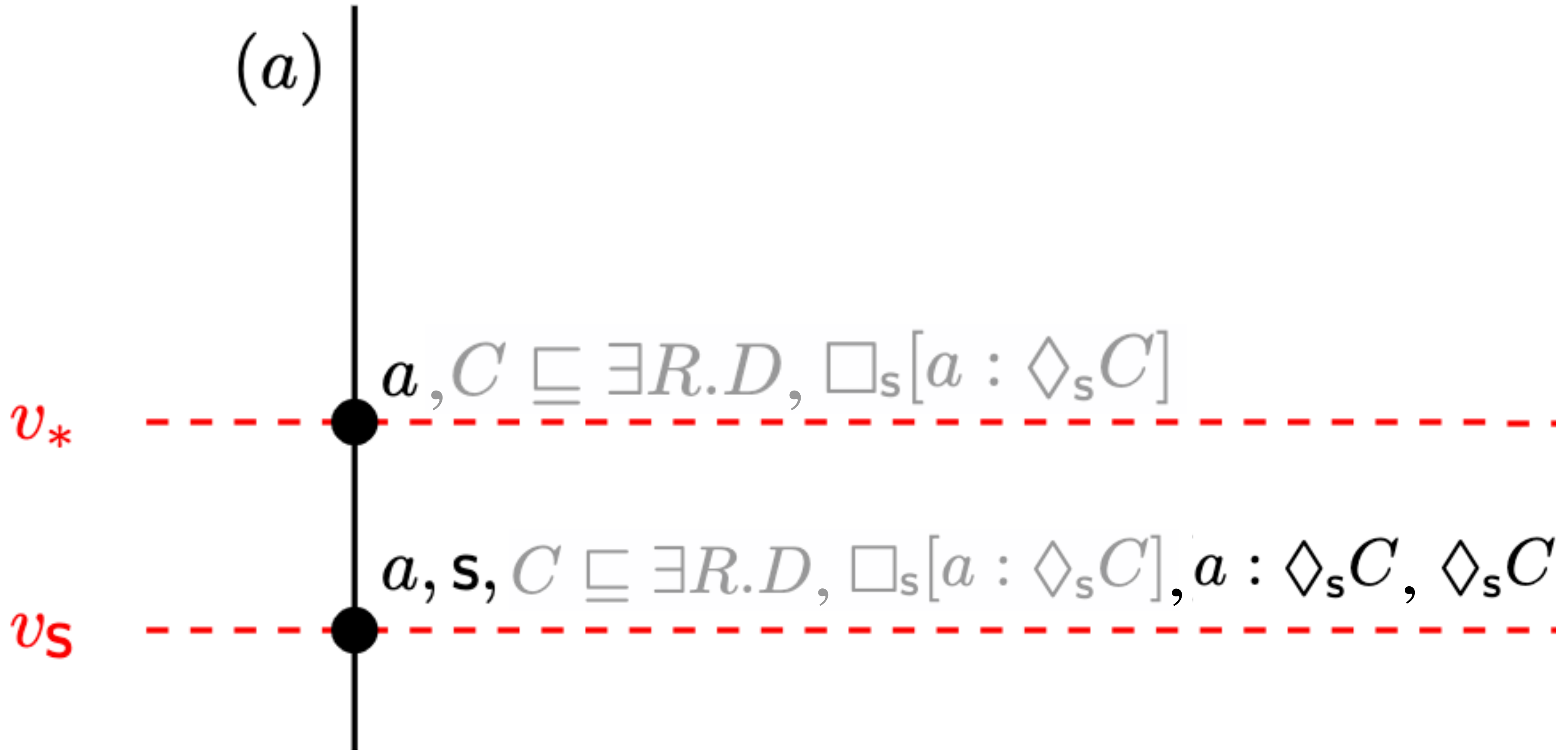

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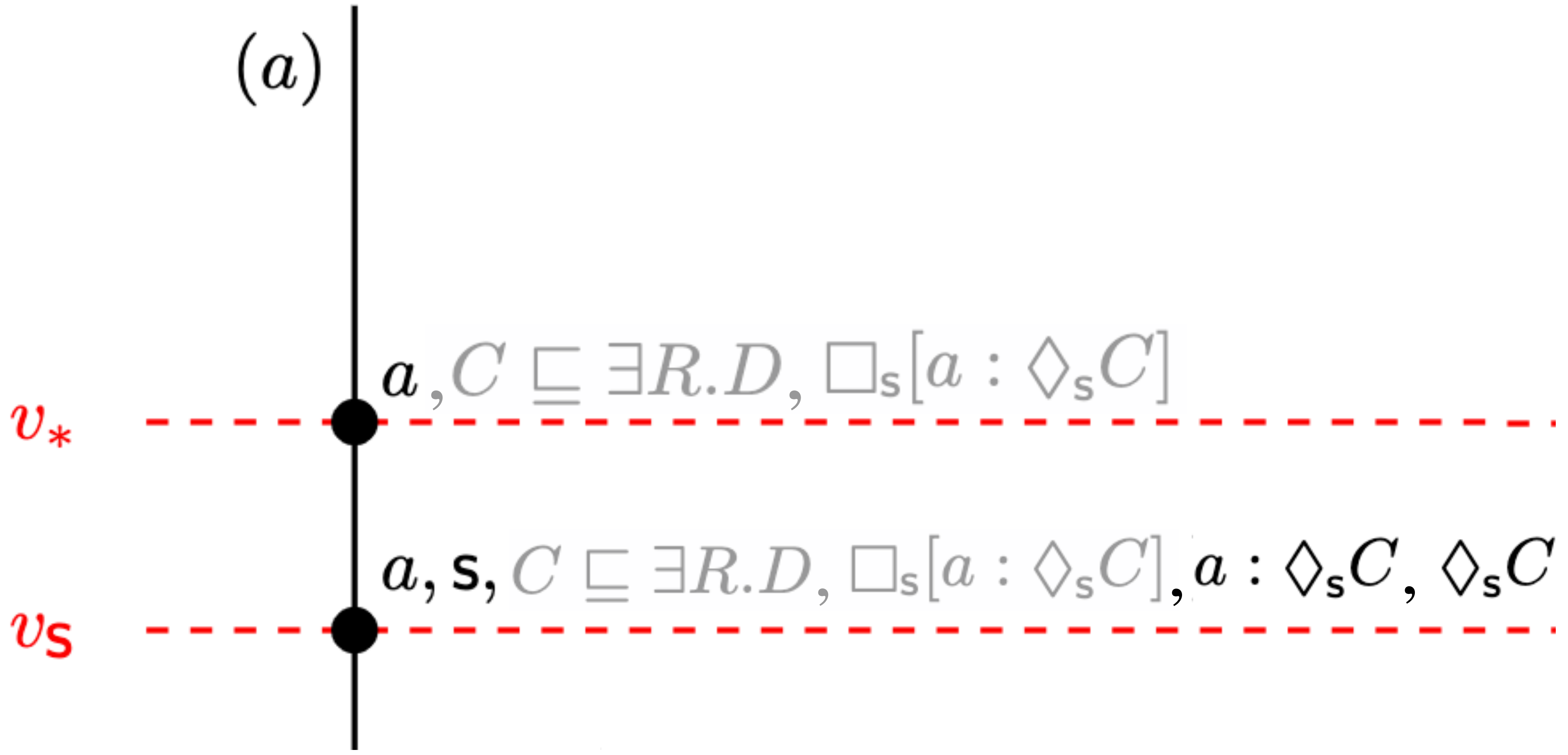

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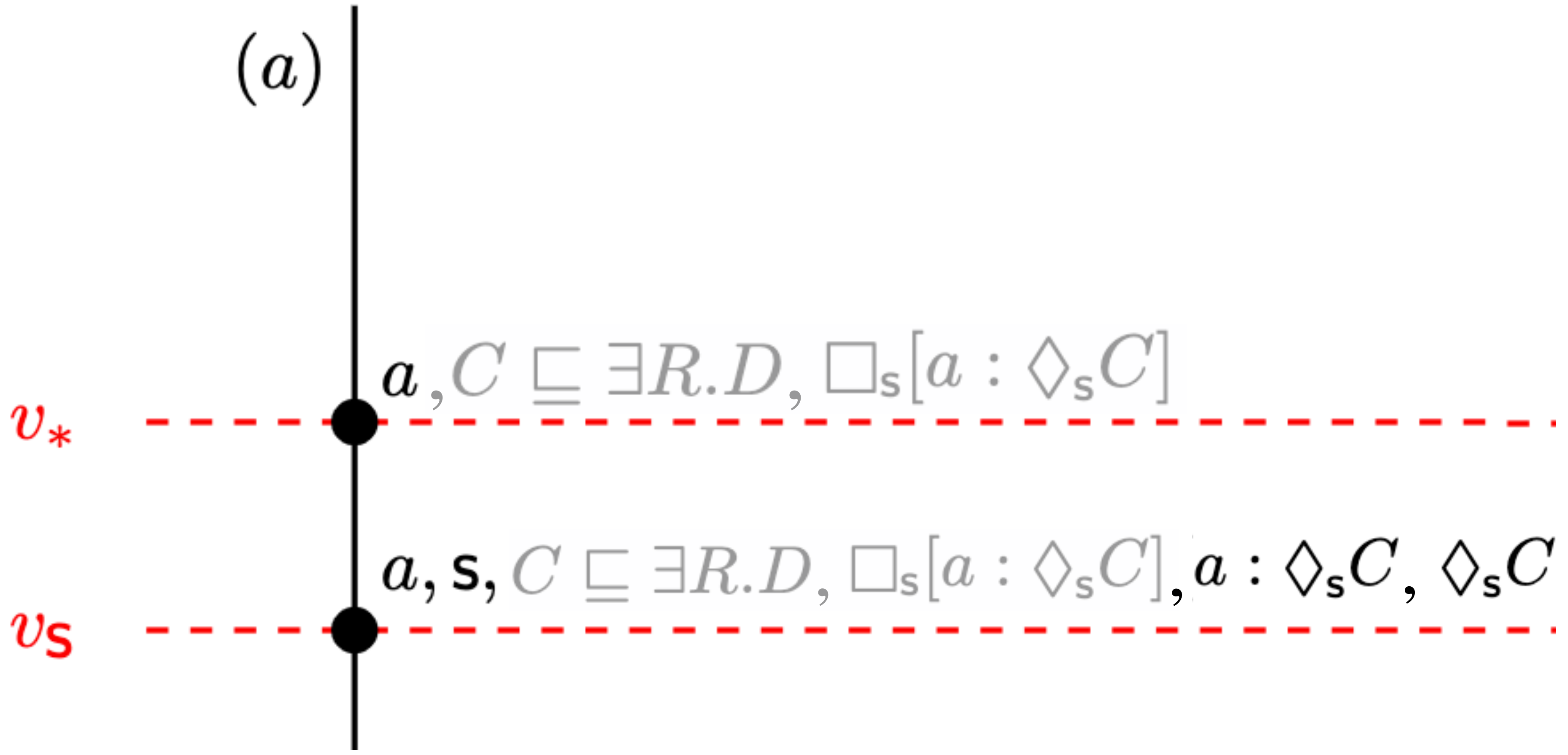

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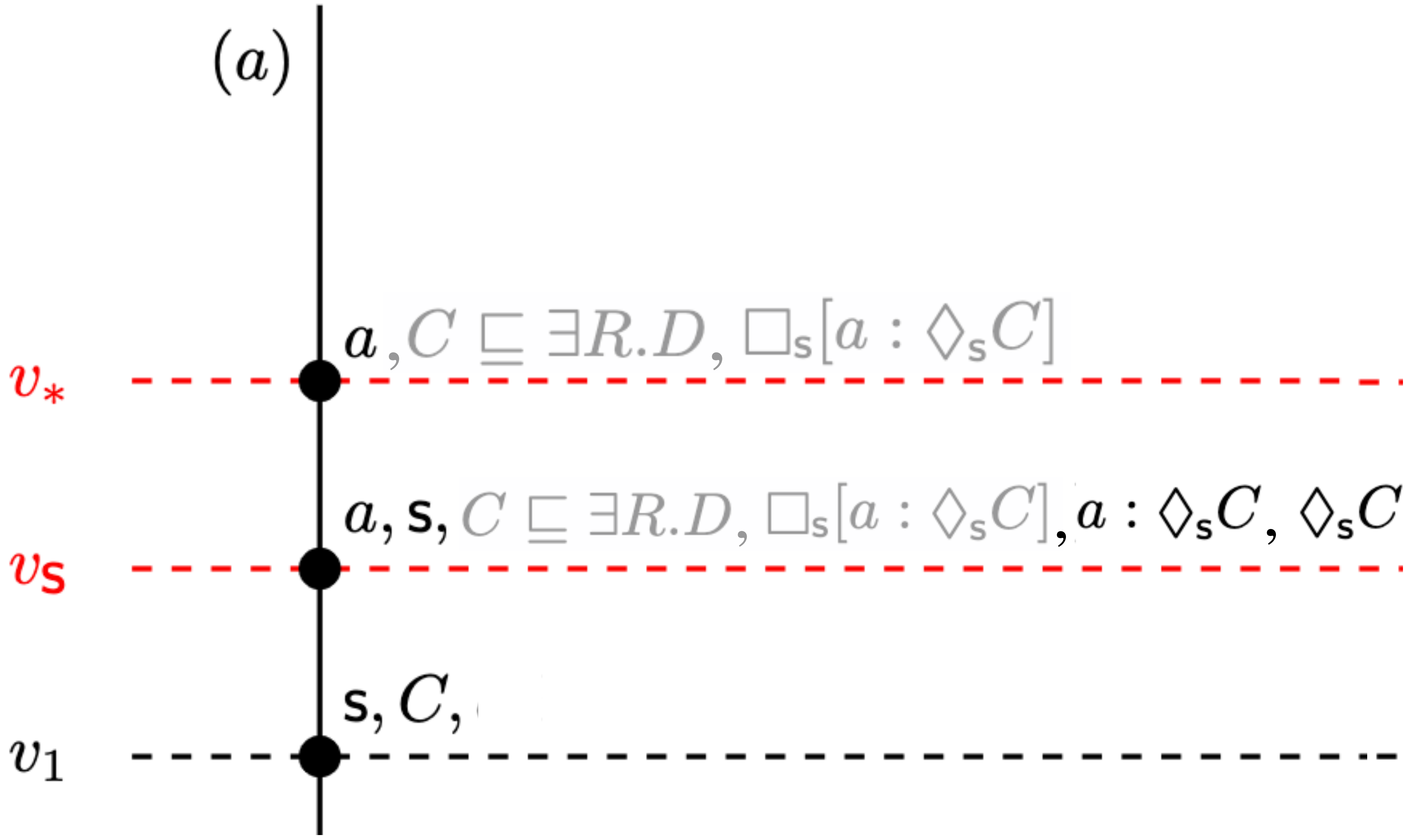

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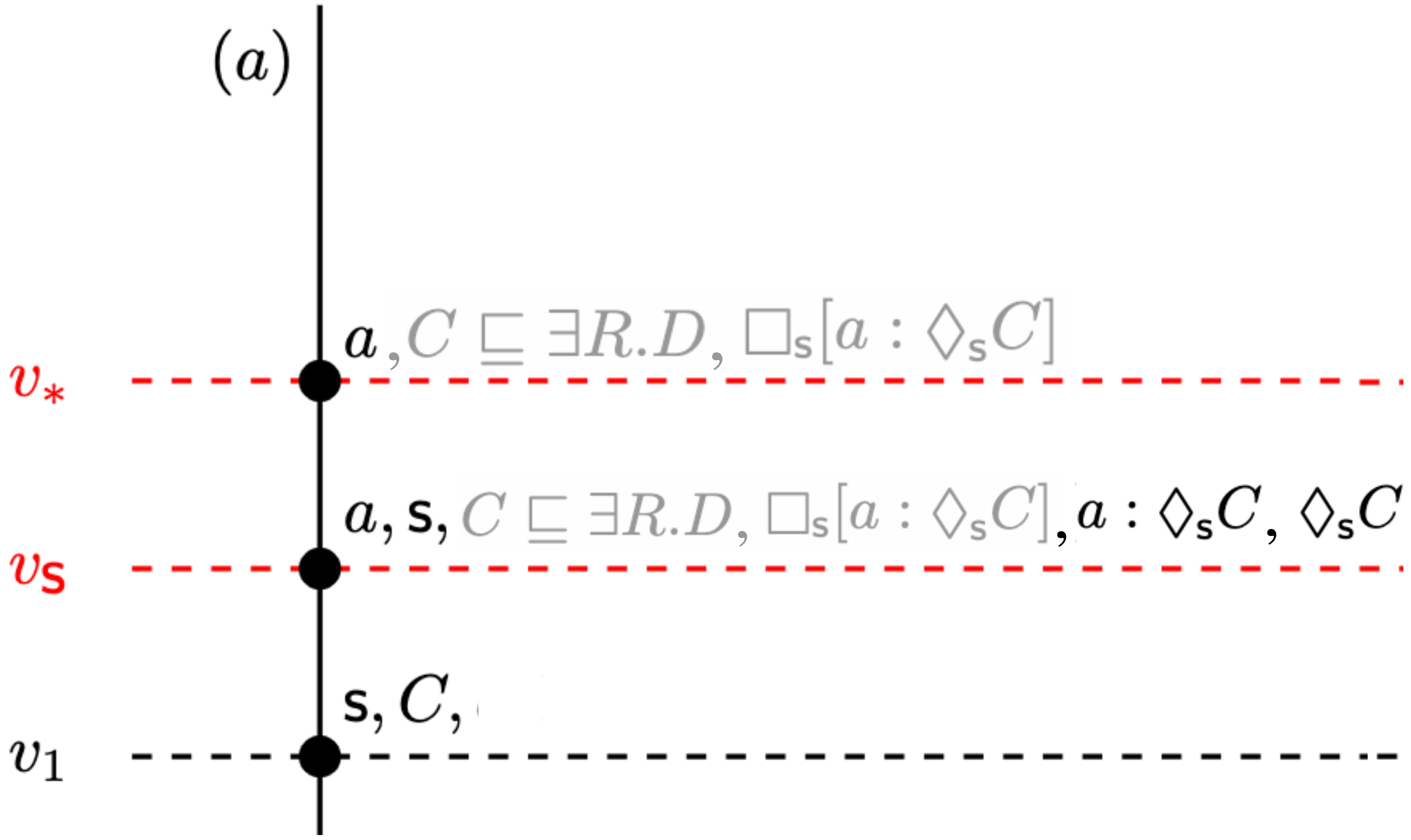

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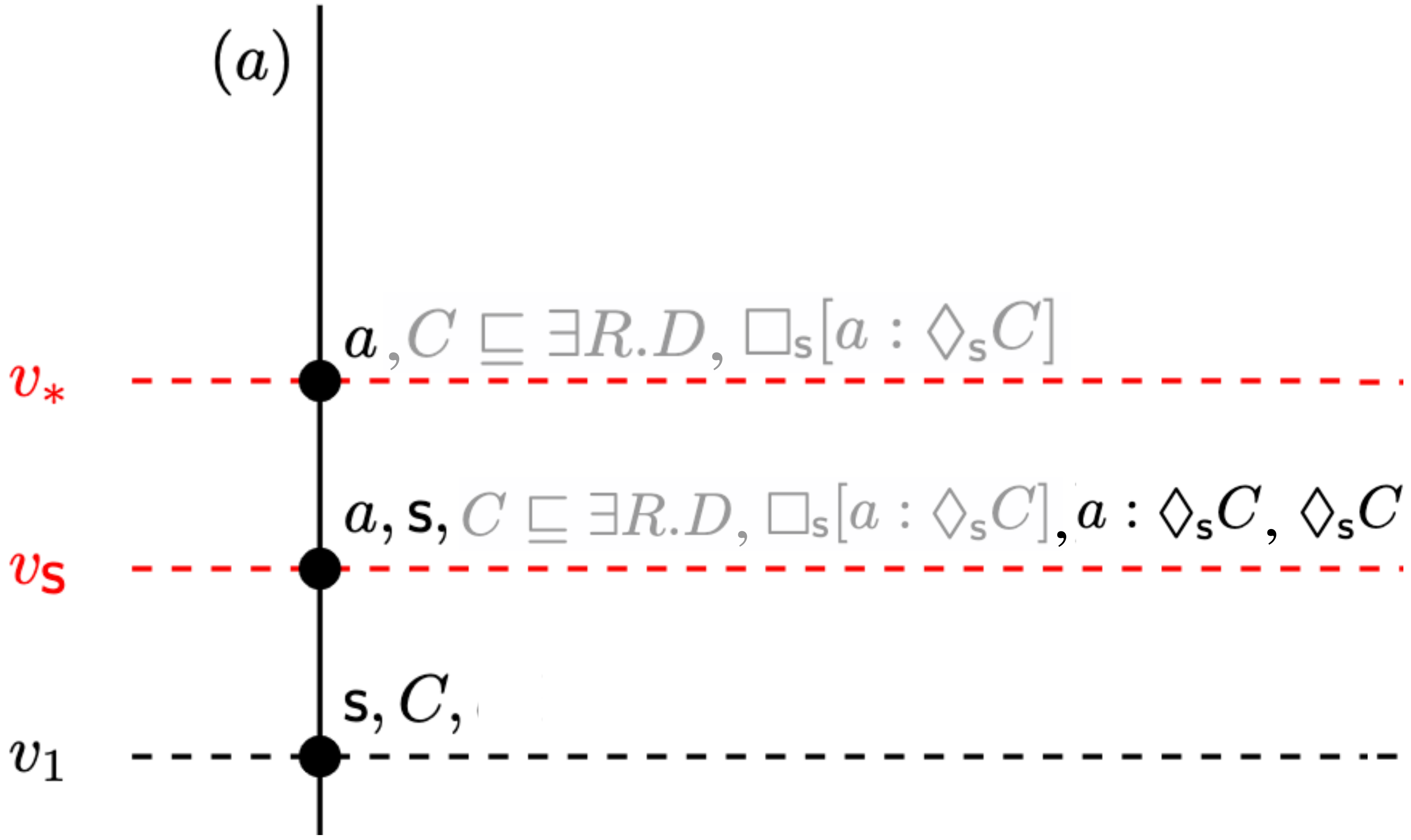

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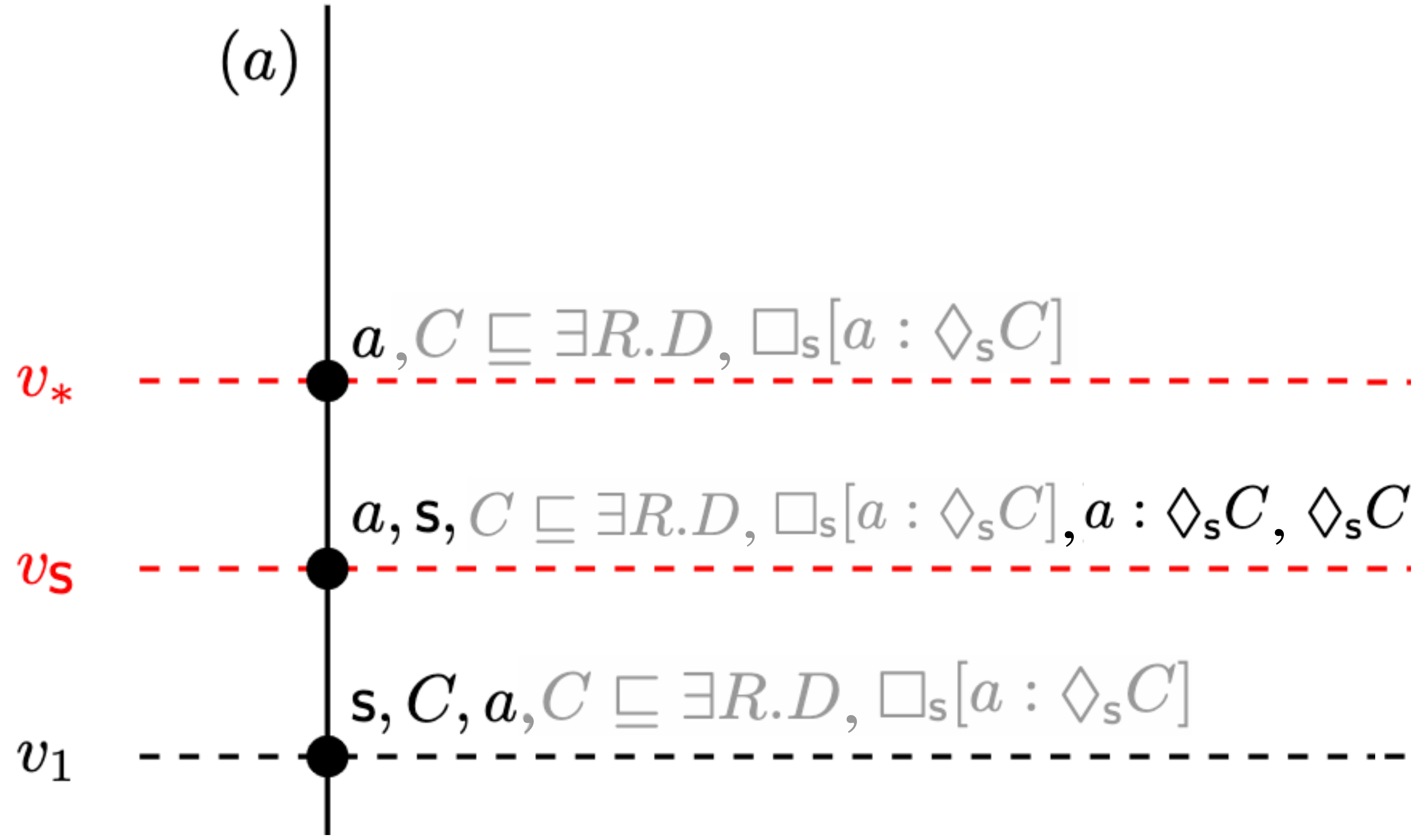

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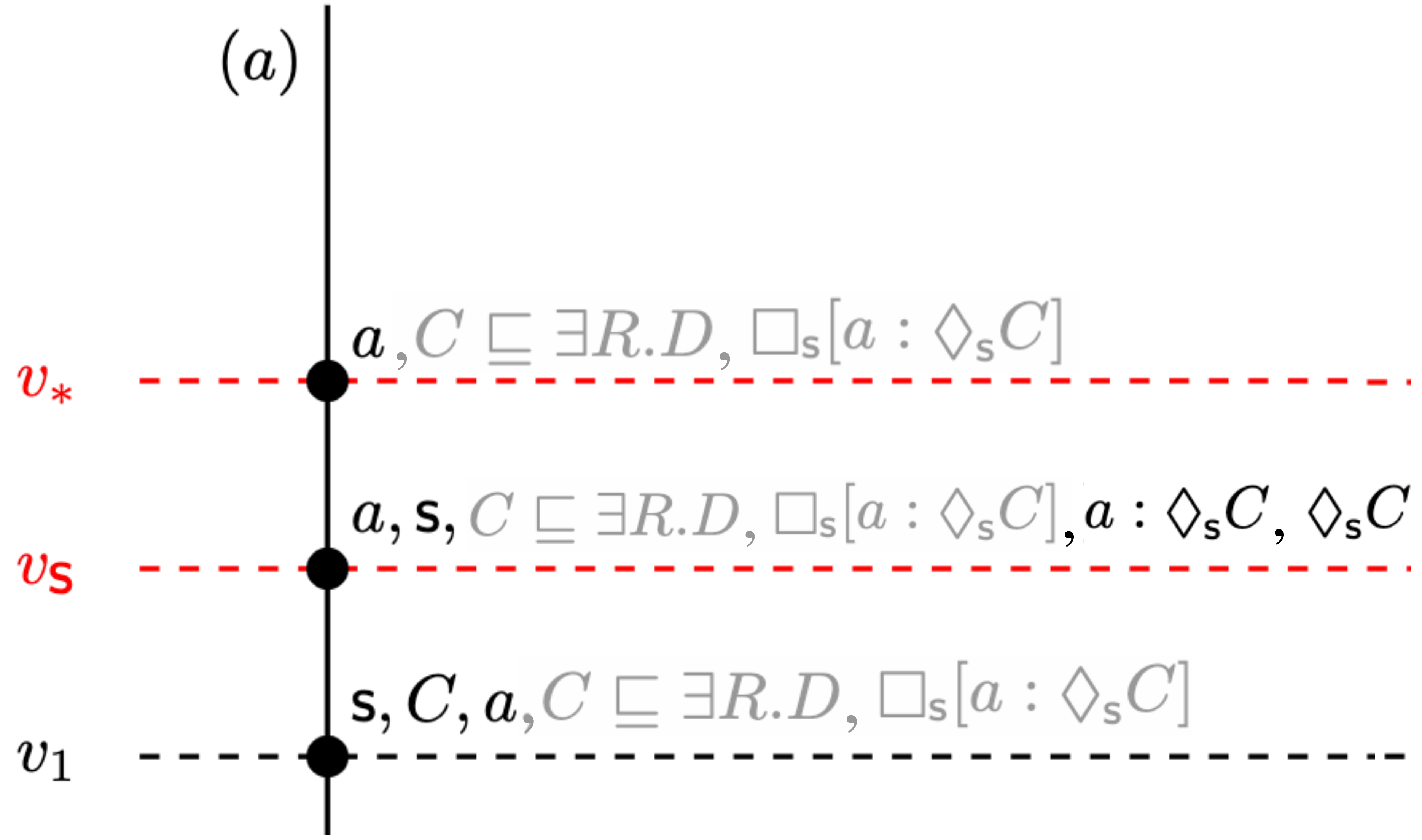

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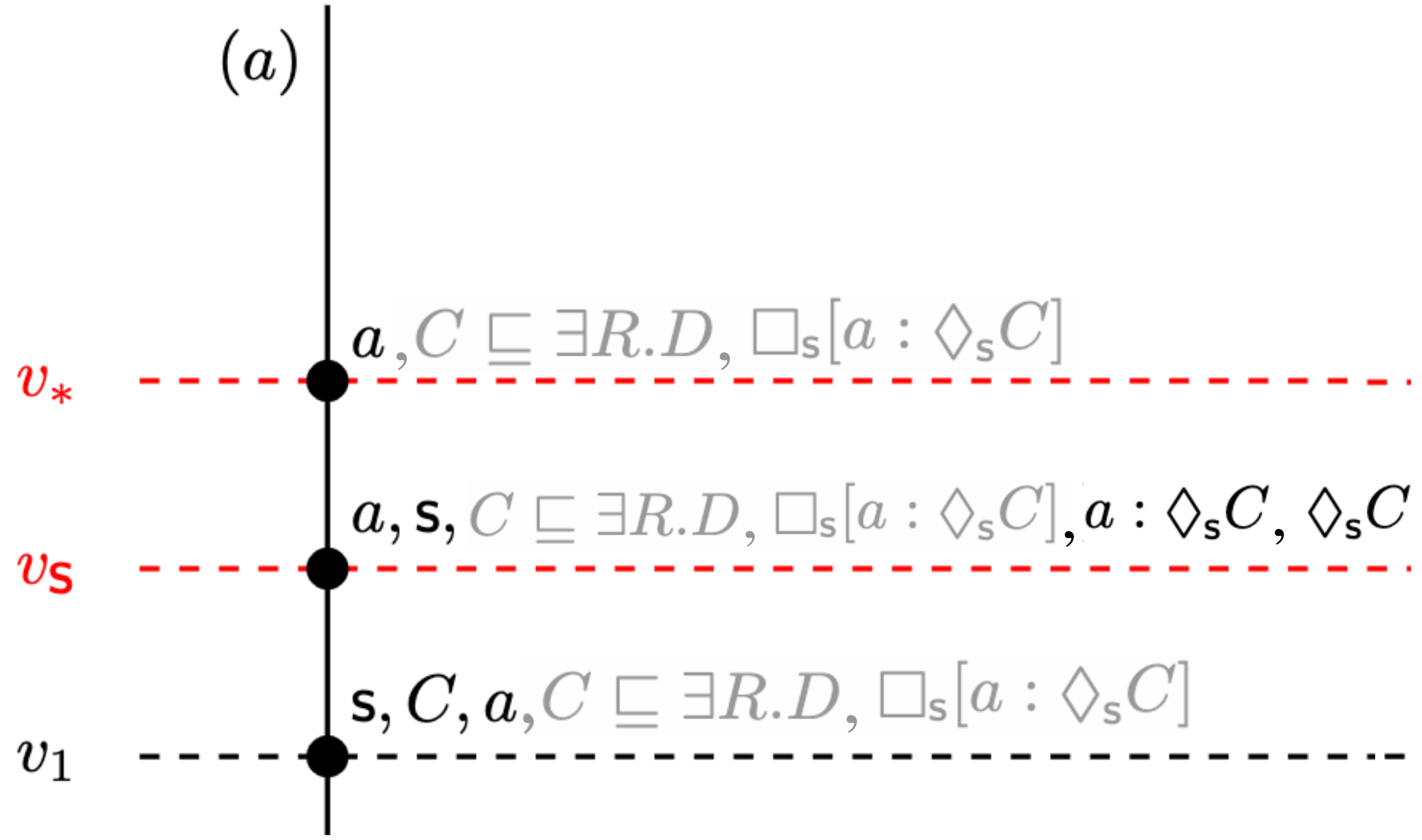

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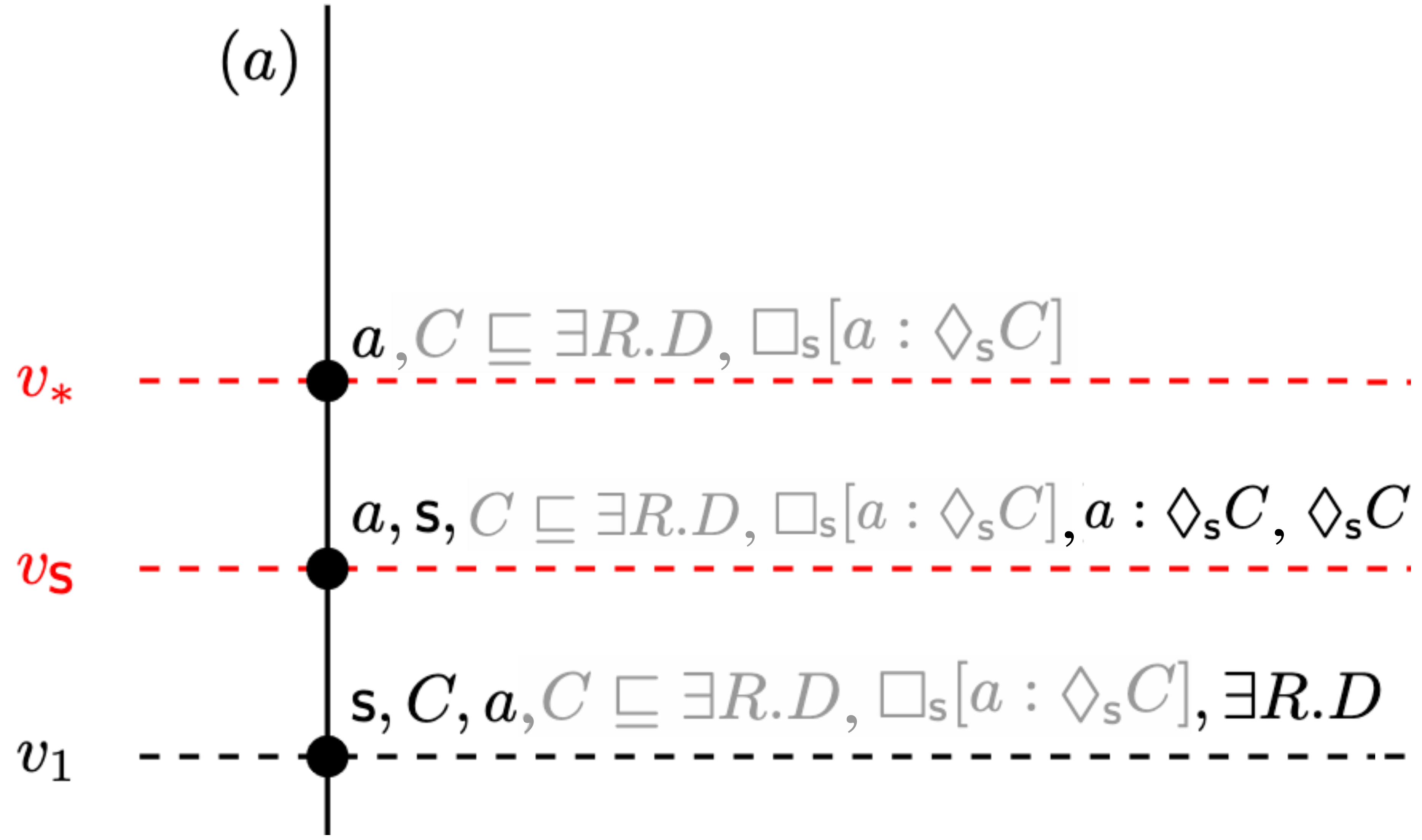

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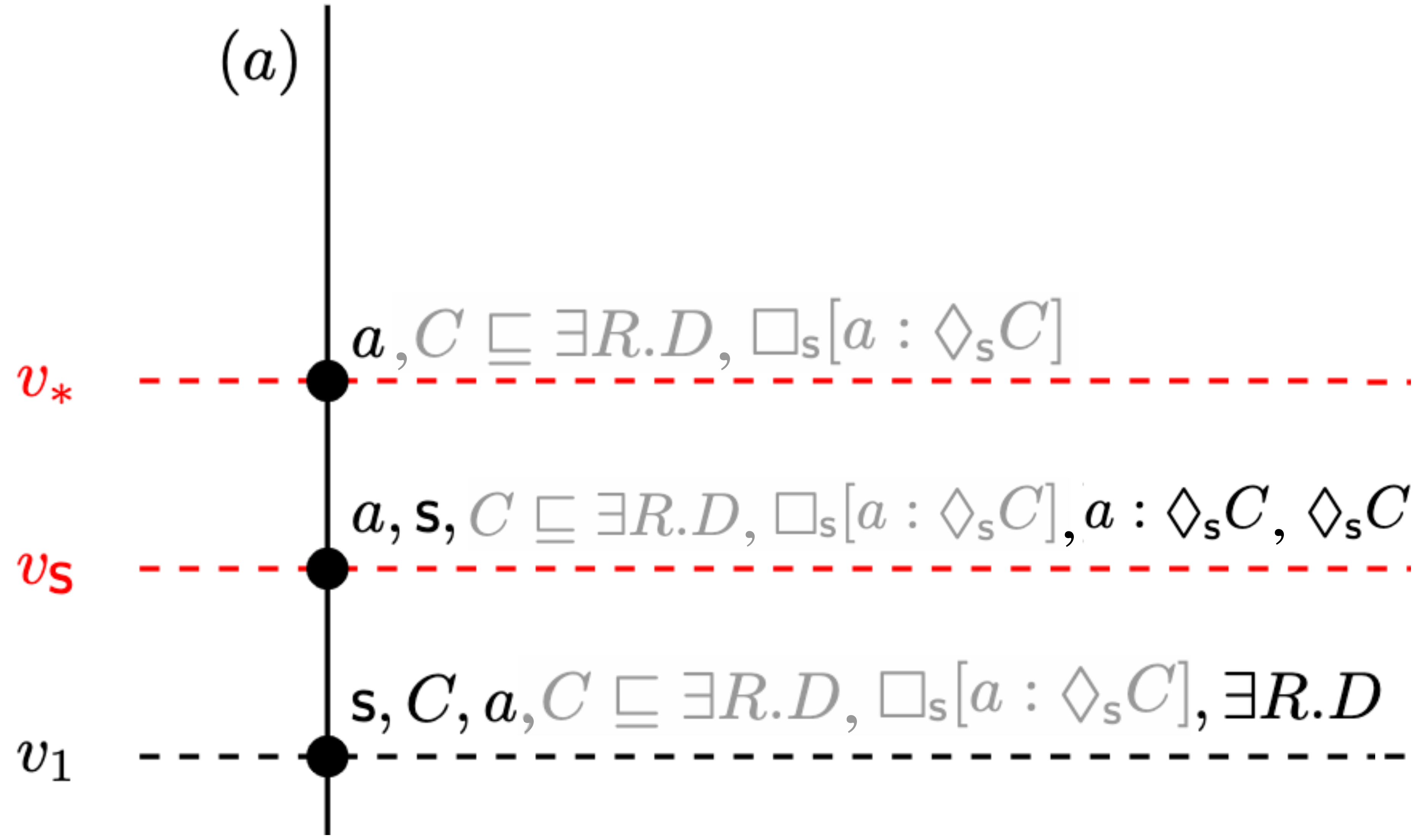

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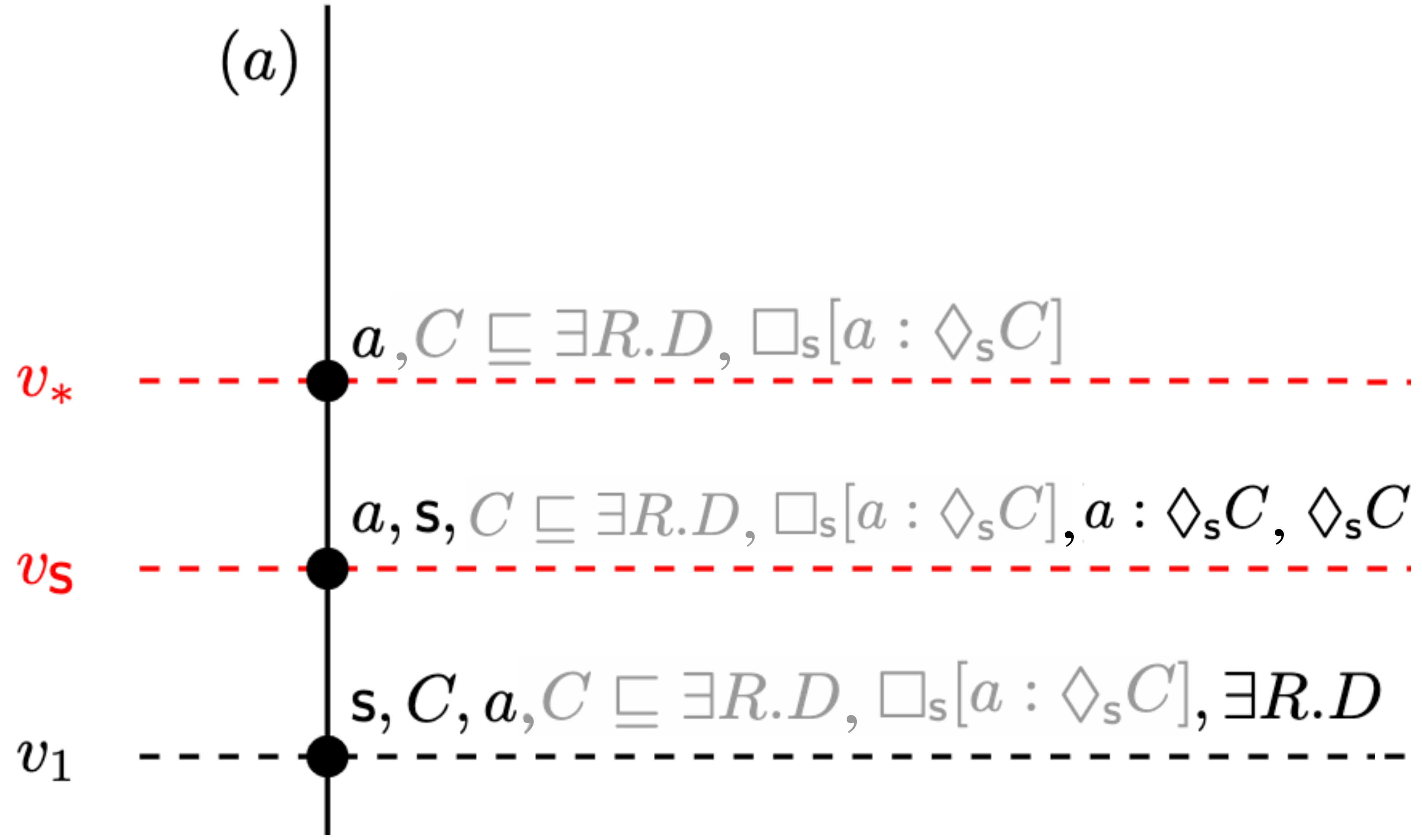

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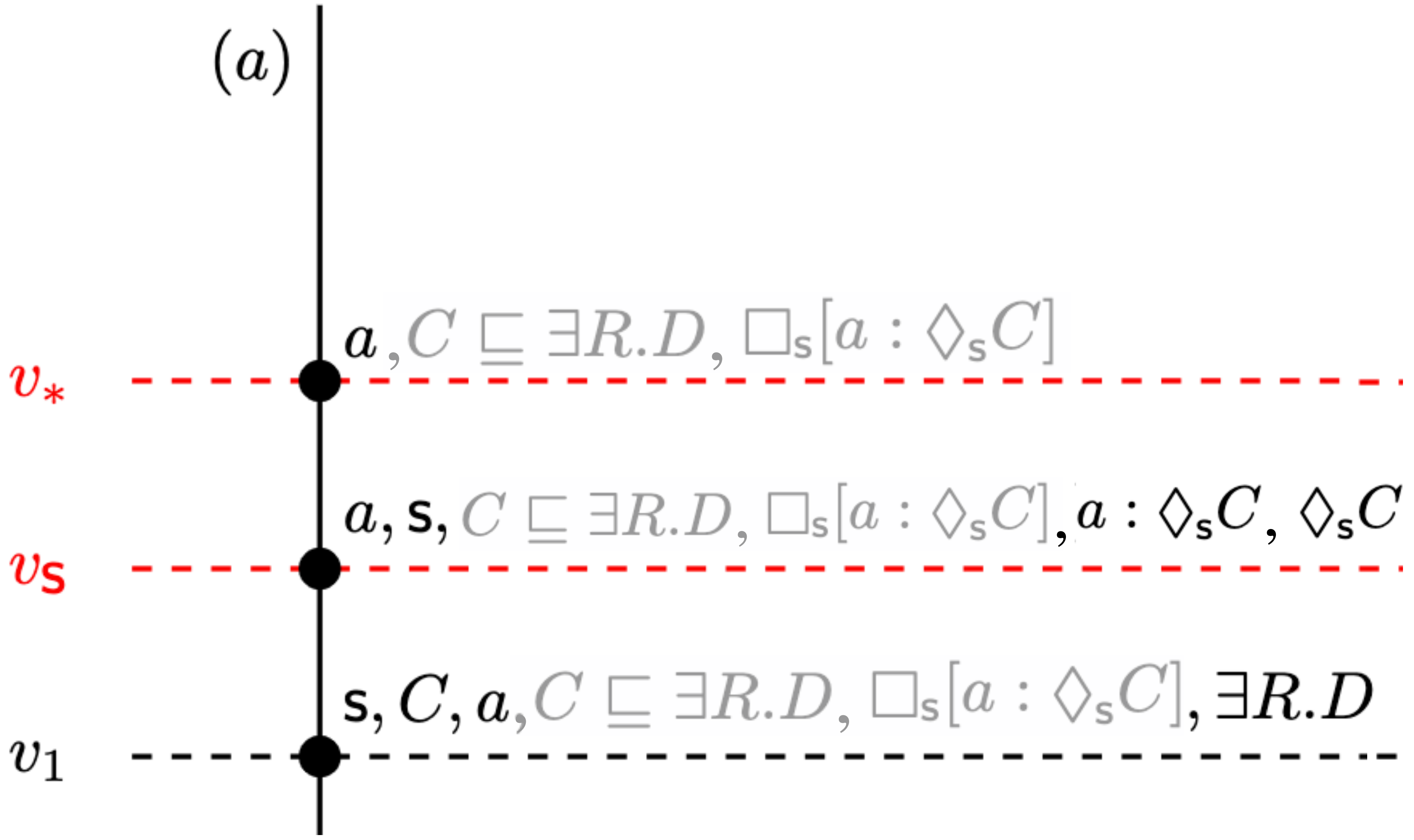

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$\langle \varepsilon, x, \varepsilon'', x'', R\rangle \notin \mathcal{R}$ whenever $(C, \mathsf{st}_\varepsilon(x), x'') \in \mathcal{L}(\varepsilon'')$ and $\varepsilon \neq \varepsilon''$ or $x = x''$,
then create ε' and a fresh variable x', and then set $\mathcal{L}(\varepsilon') := \{(C, \mathsf{st}_\varepsilon(x), x')\}$,
$\mathcal{S}(\varepsilon') := S_0^{\mathcal{K}} \cup \{x' : C,\ x' : \top\} \cup \{x' : \mathsf{s} \mid \mathsf{s} \in \mathsf{st}_\varepsilon(x)\}$, $\mathcal{R} := \mathcal{R} \cup \{\langle \varepsilon, x, \varepsilon', x', R\rangle\}$.

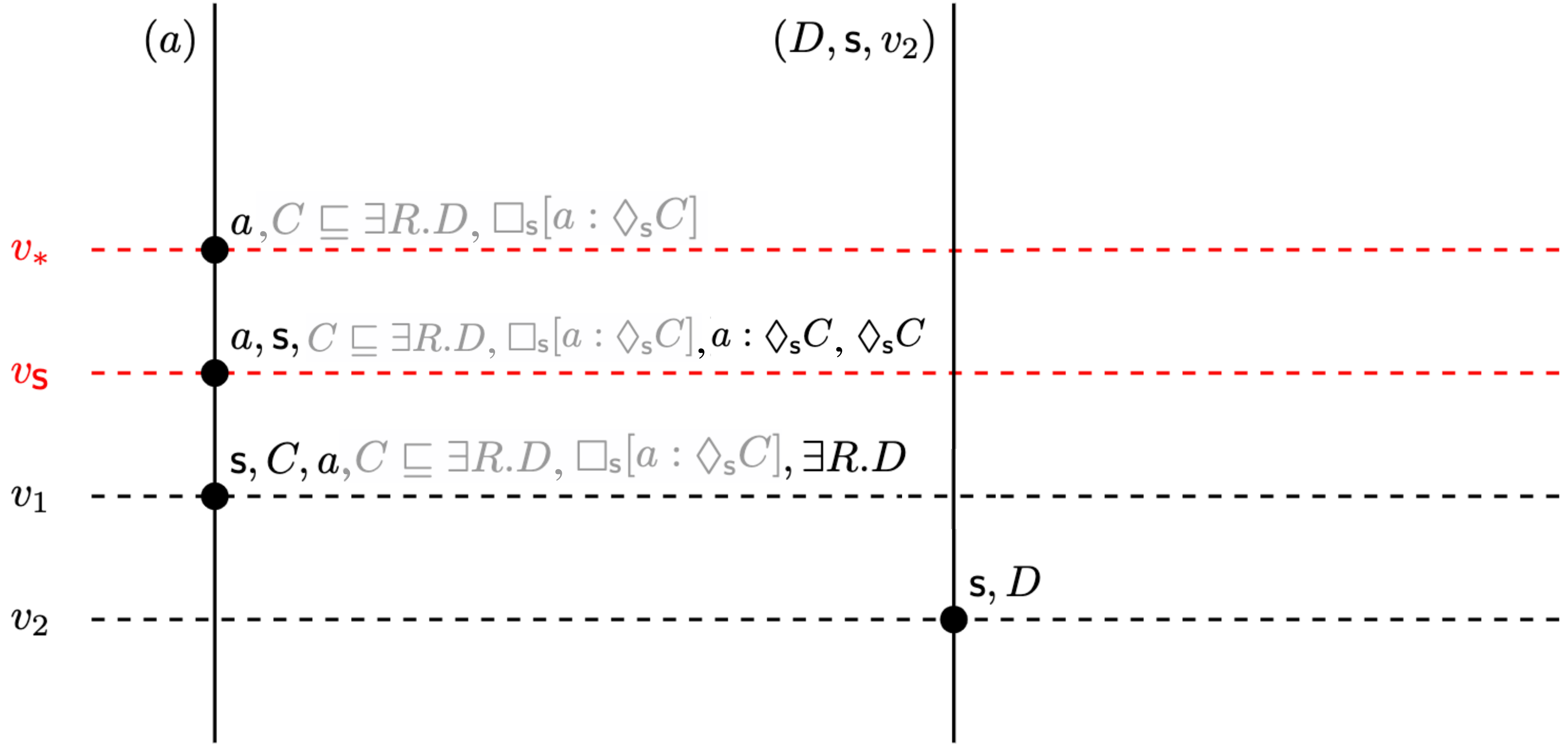

Tableau Algorithm for $\mathbb{S}_{\mathscr{EL}}$

Example:

$C \sqsubseteq \exists R.D$

$\Box_{\mathsf{s}}[a : \Diamond_{\mathsf{s}} C]$

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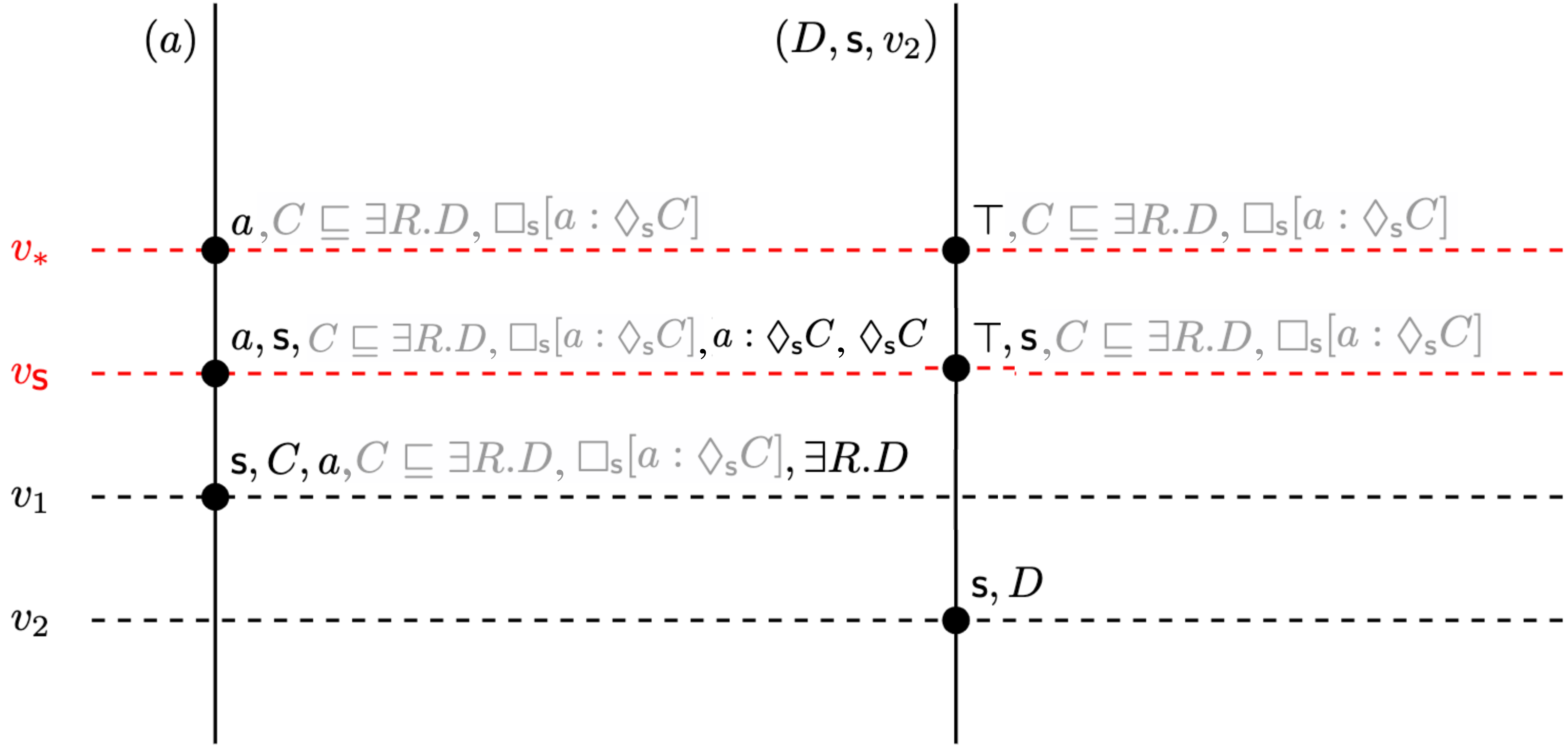

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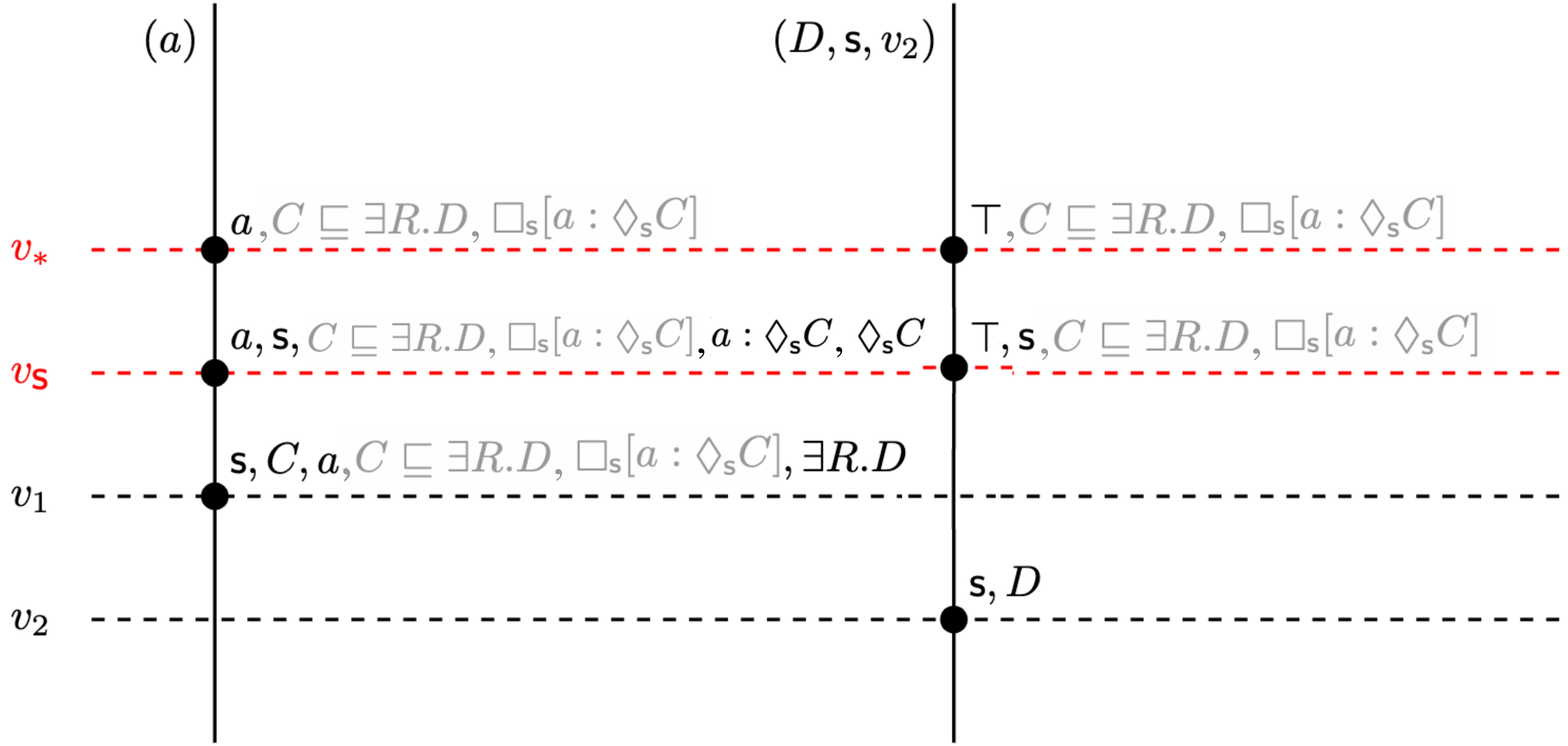

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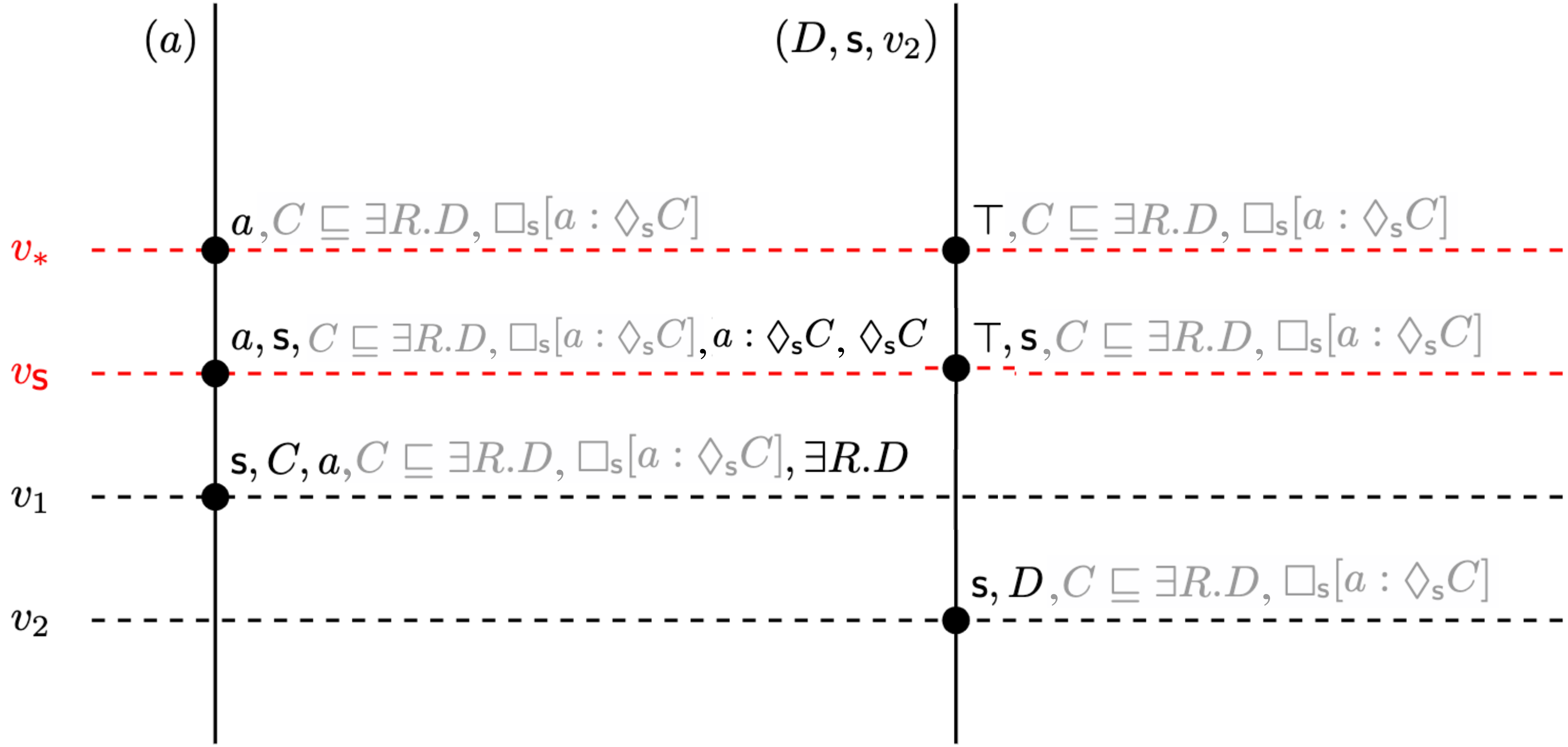

Models and Quasi-Models for $\mathbb{S}_{\mathscr{EL}}$

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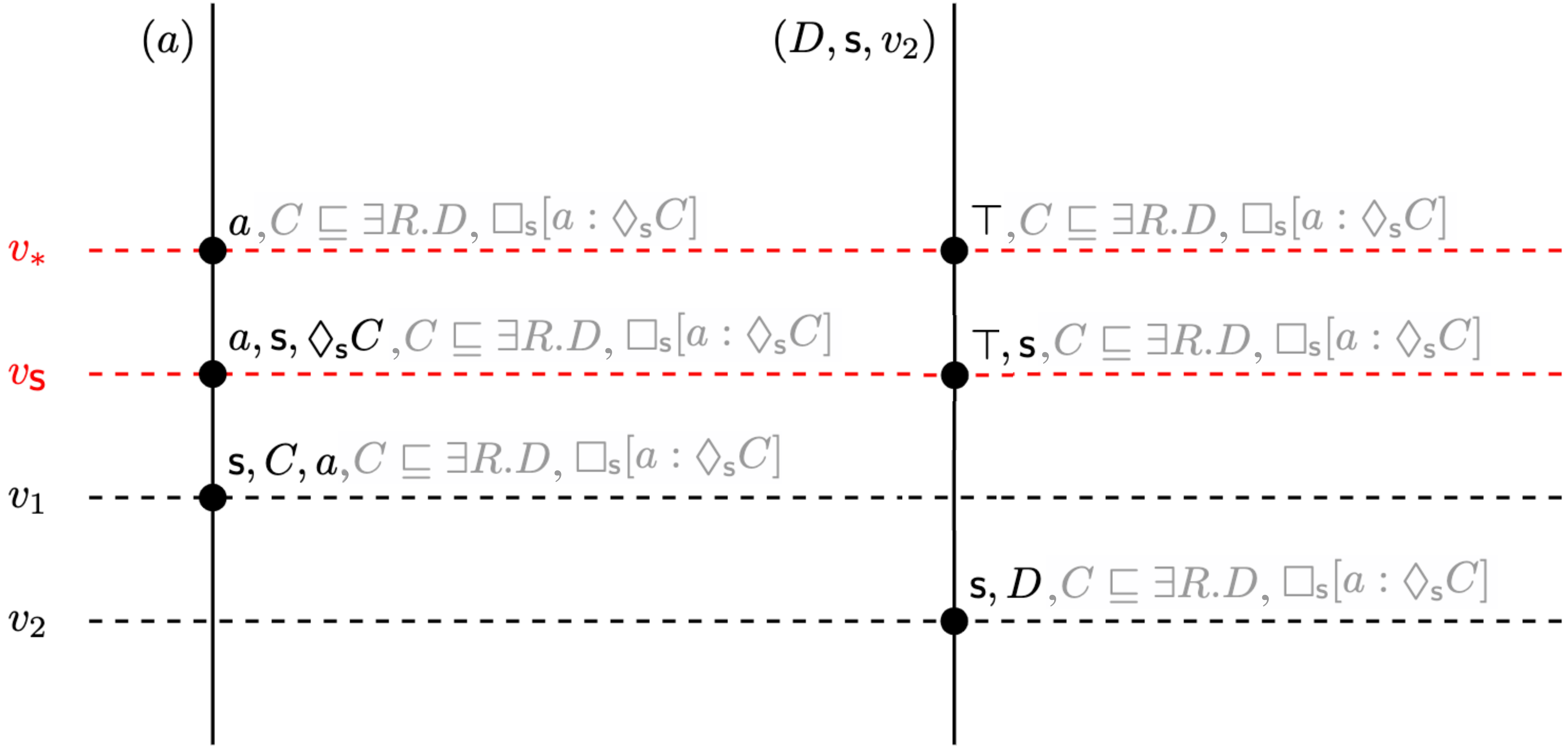

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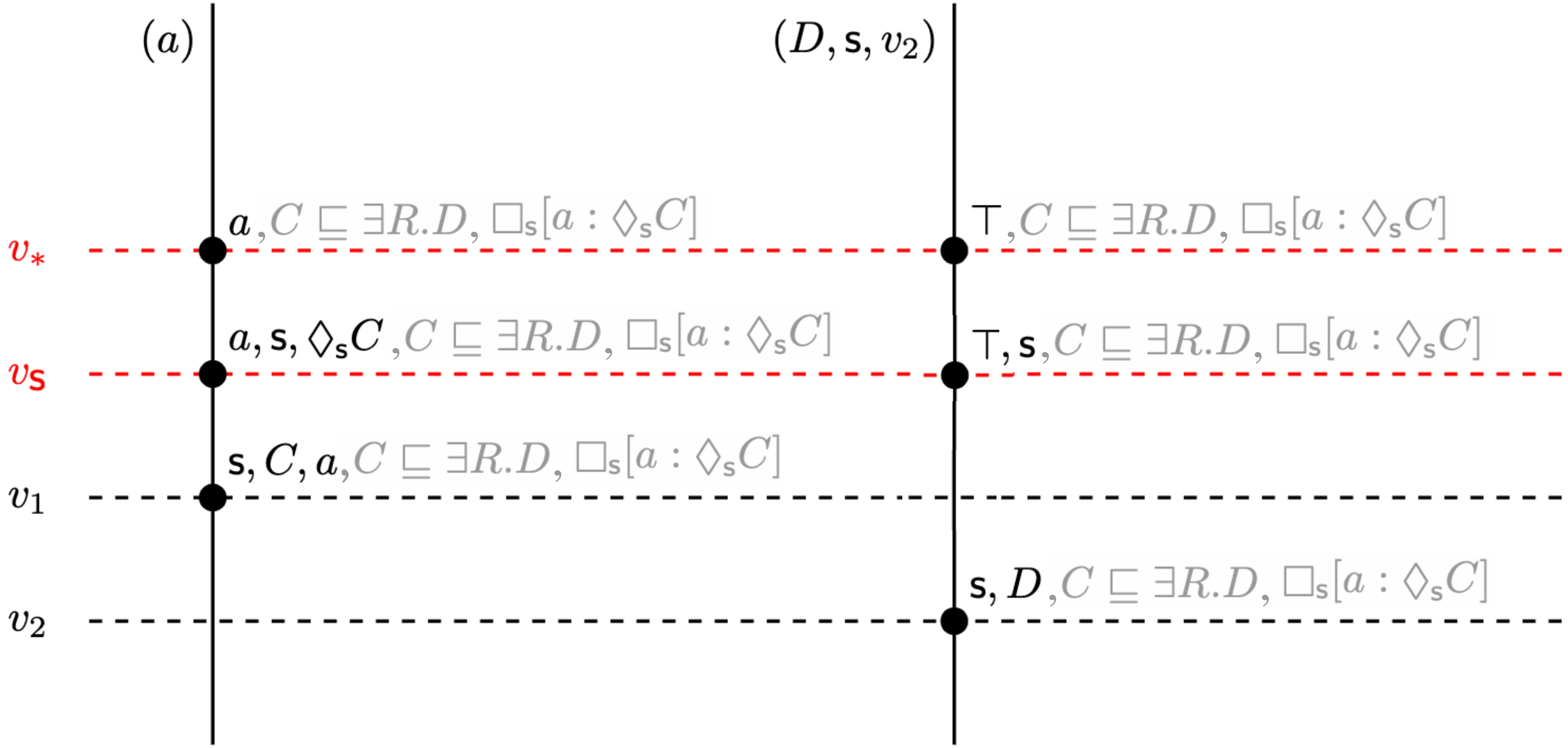

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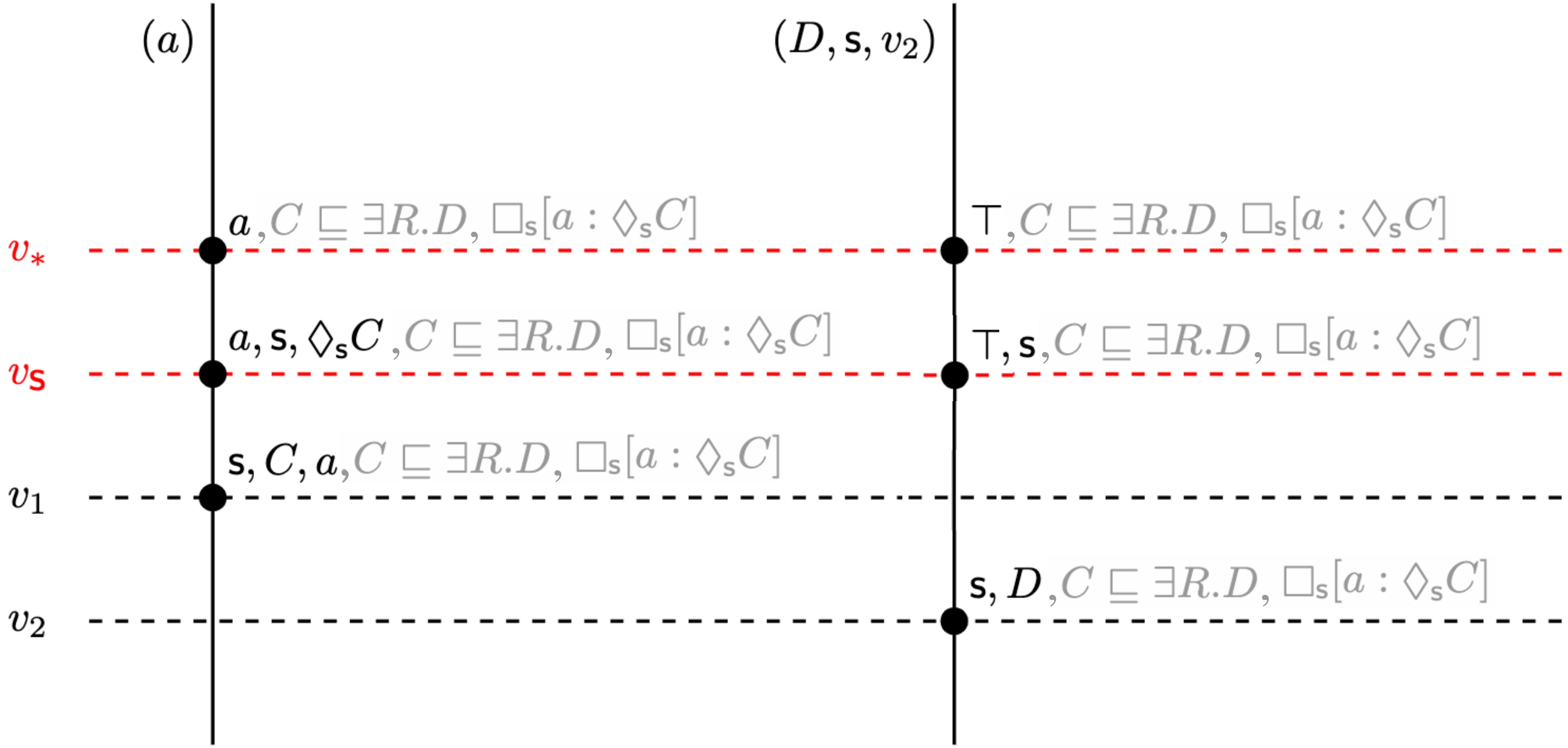

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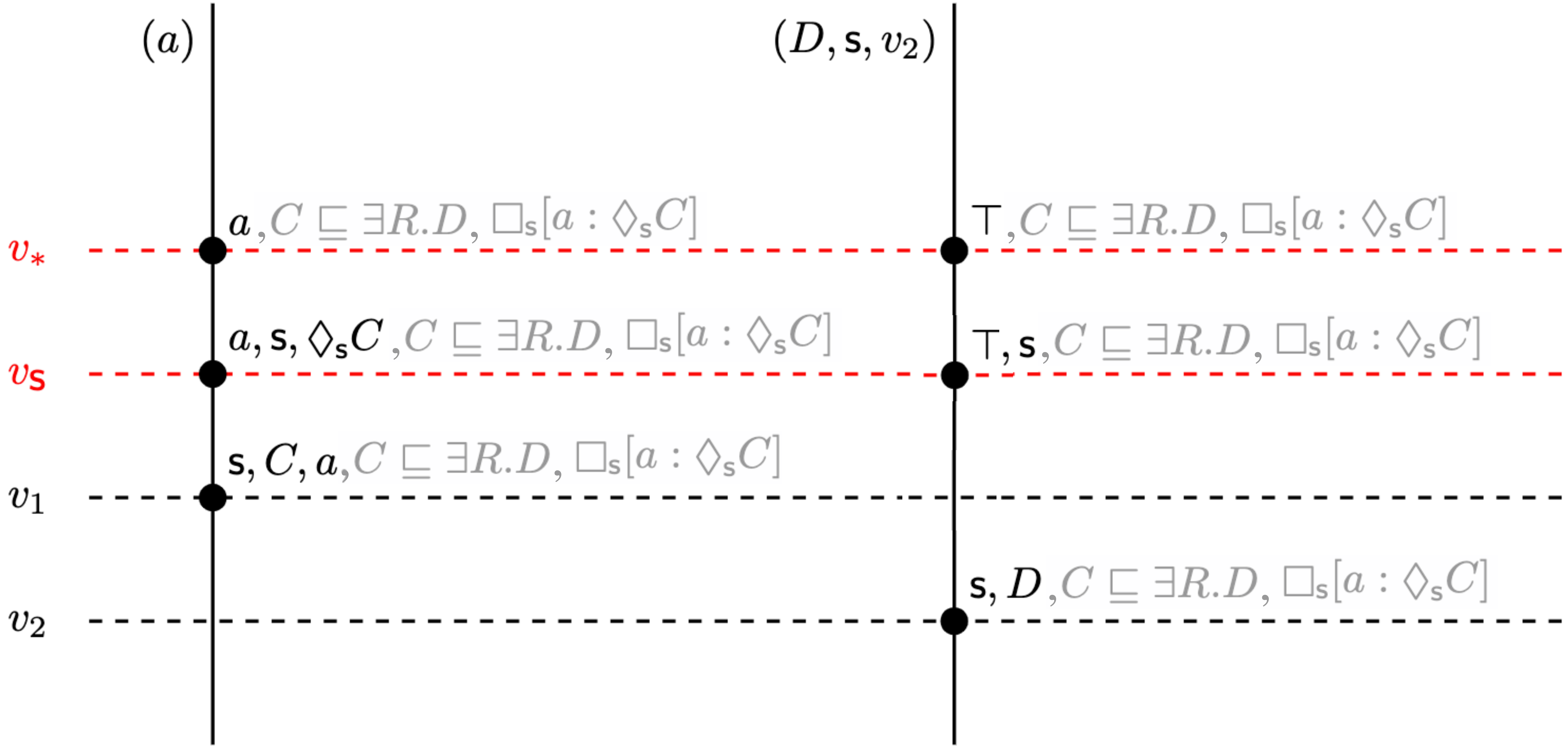

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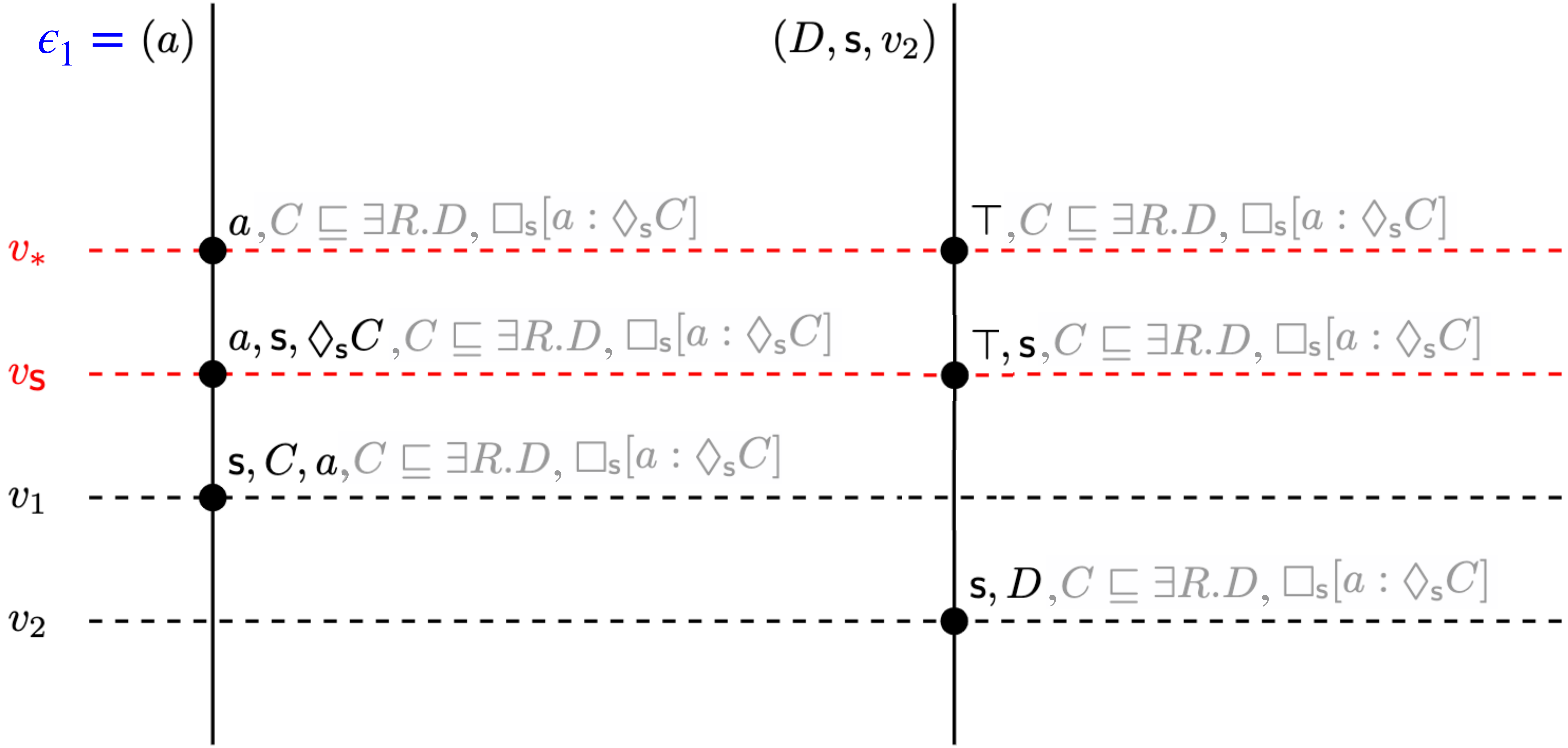

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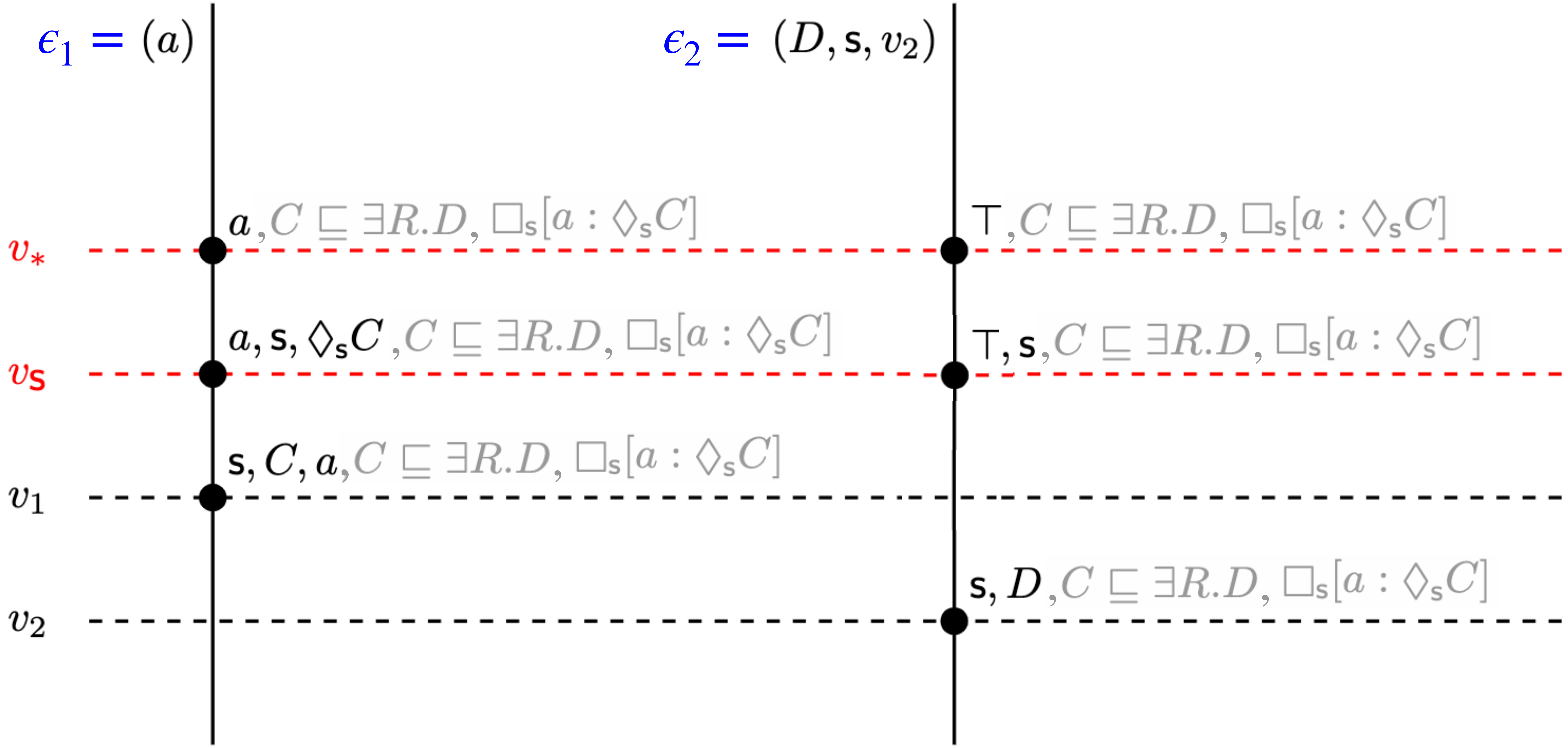

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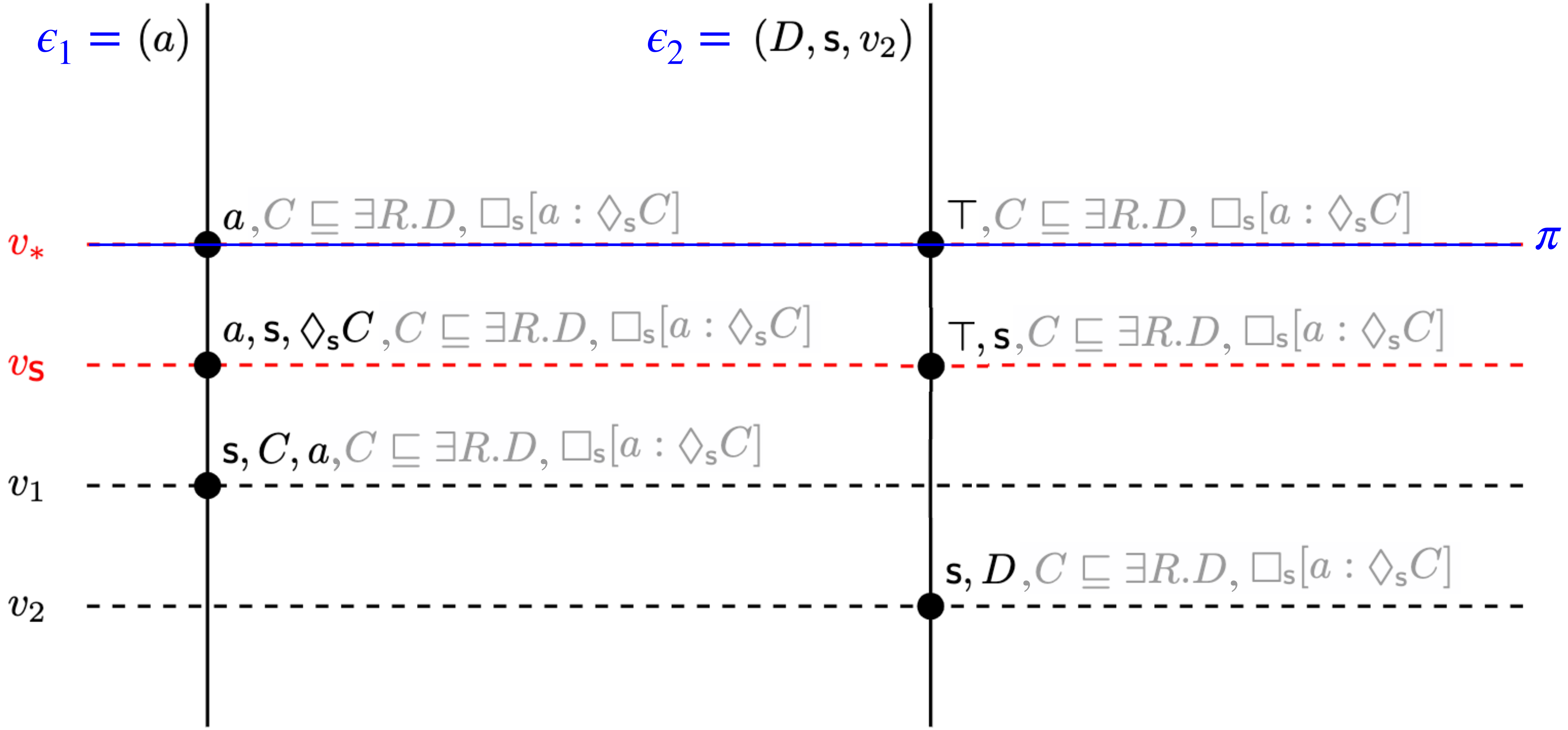

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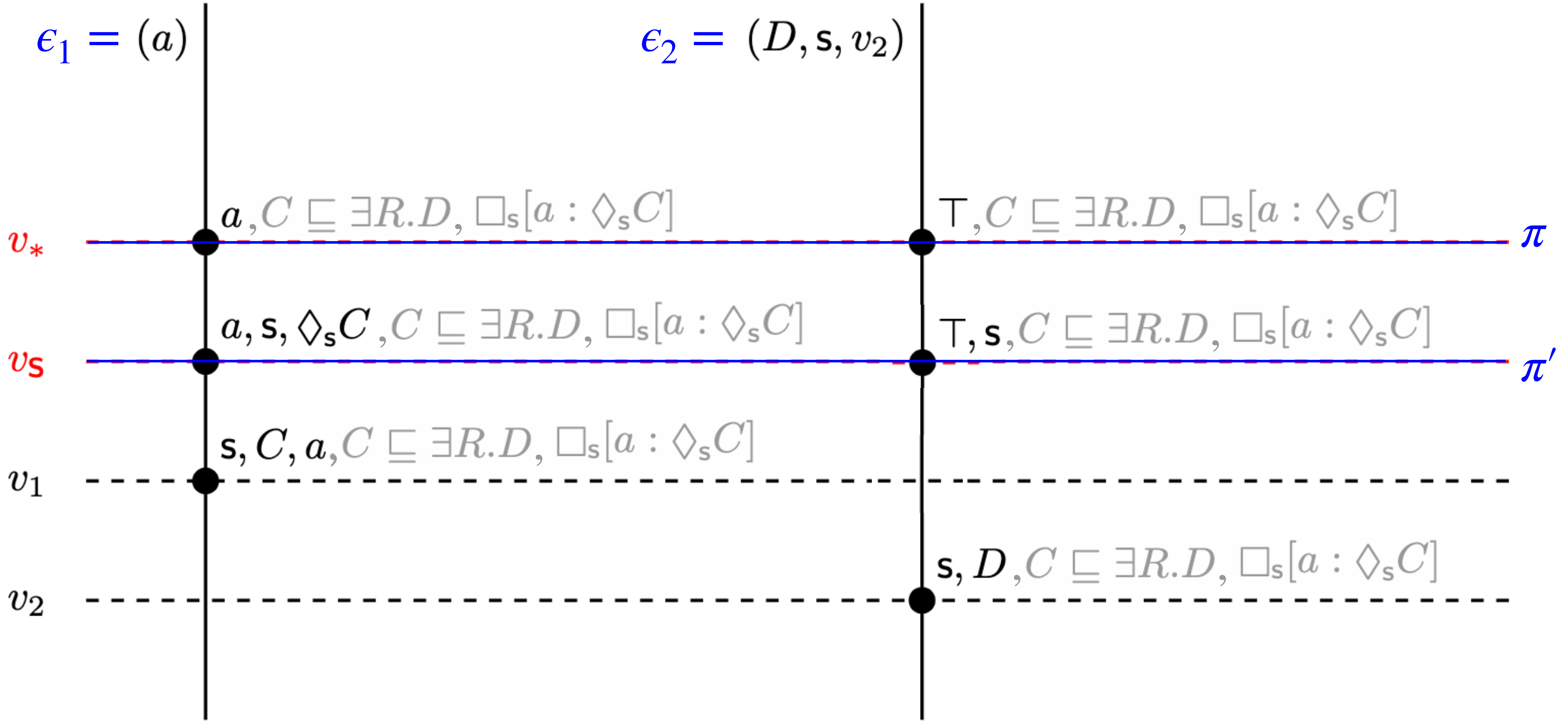

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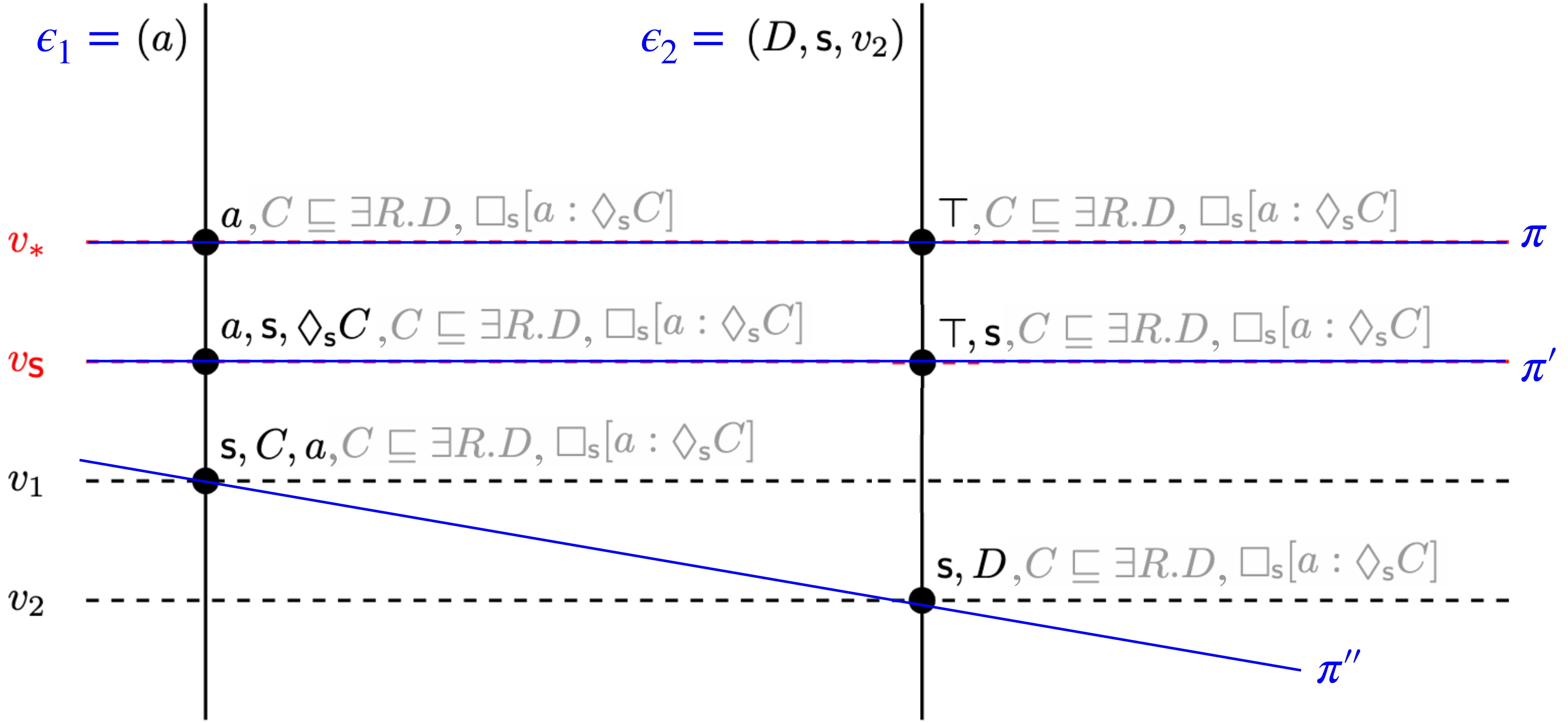

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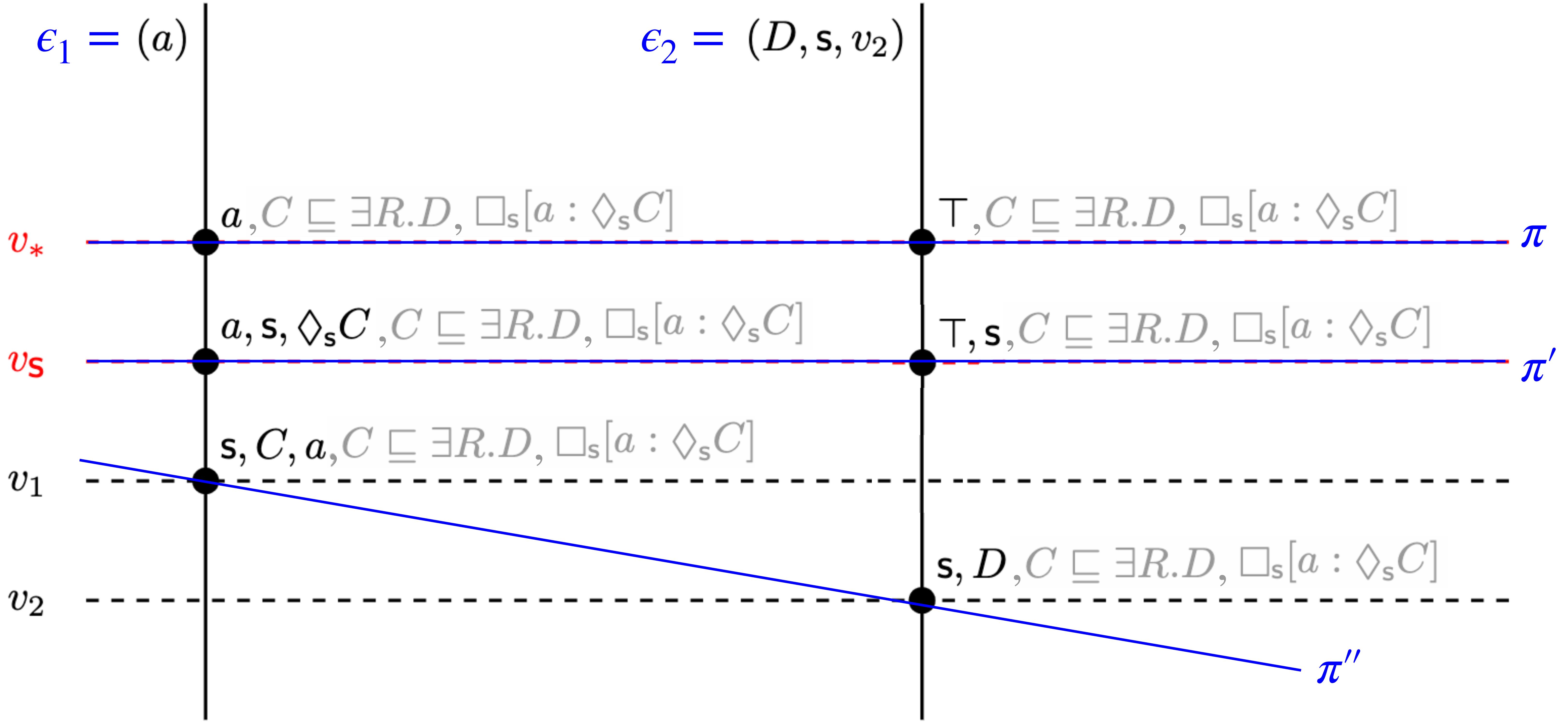

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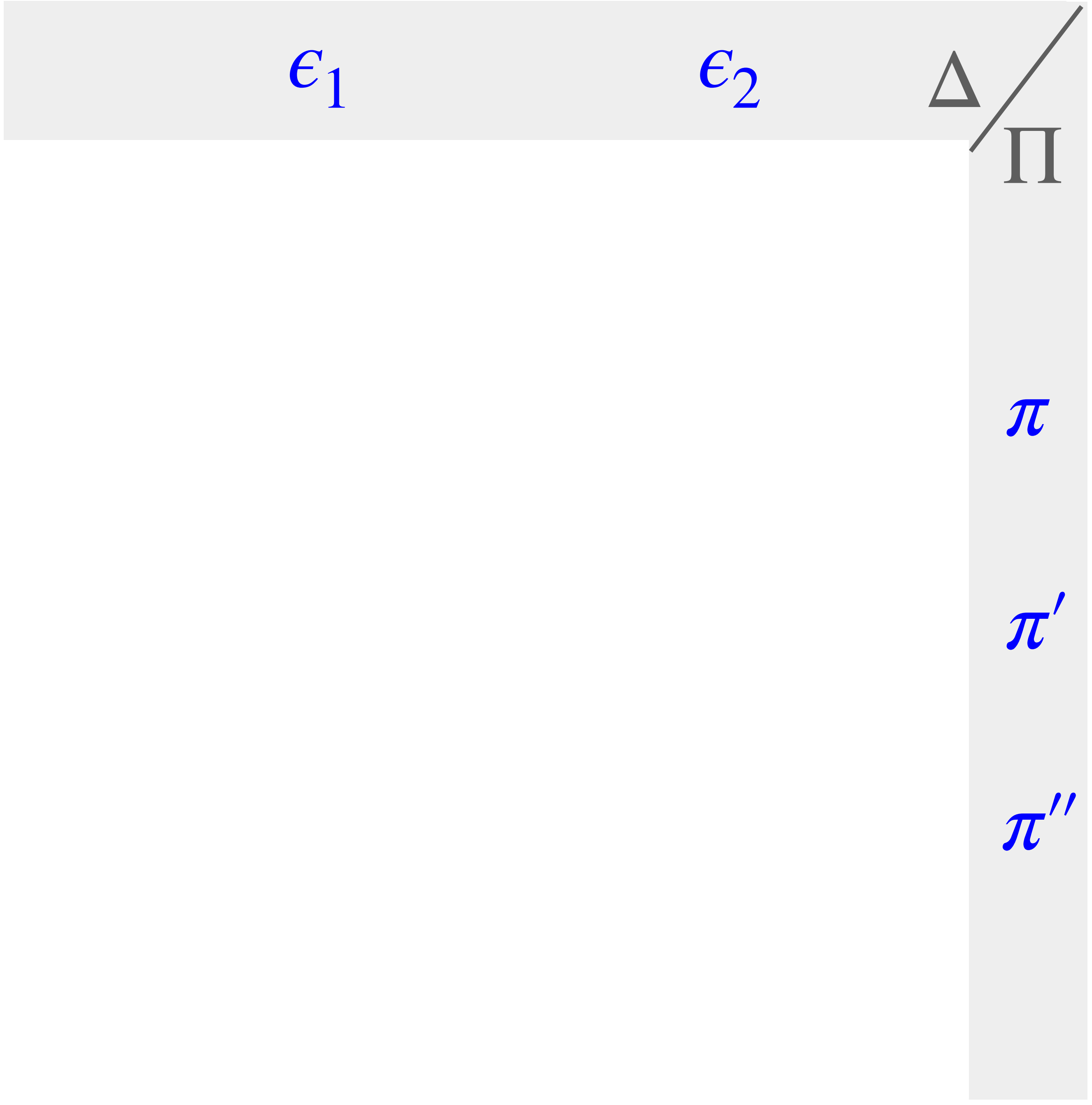

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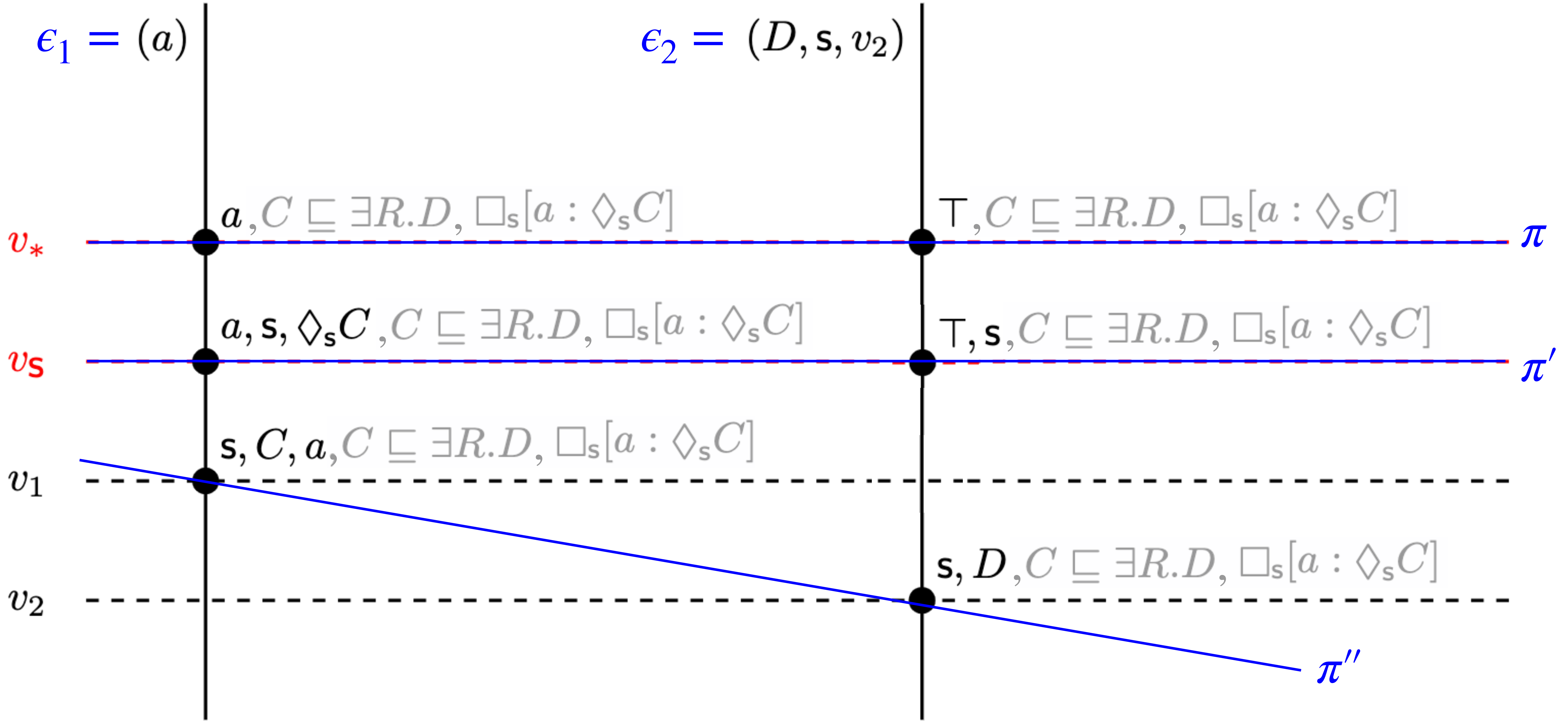

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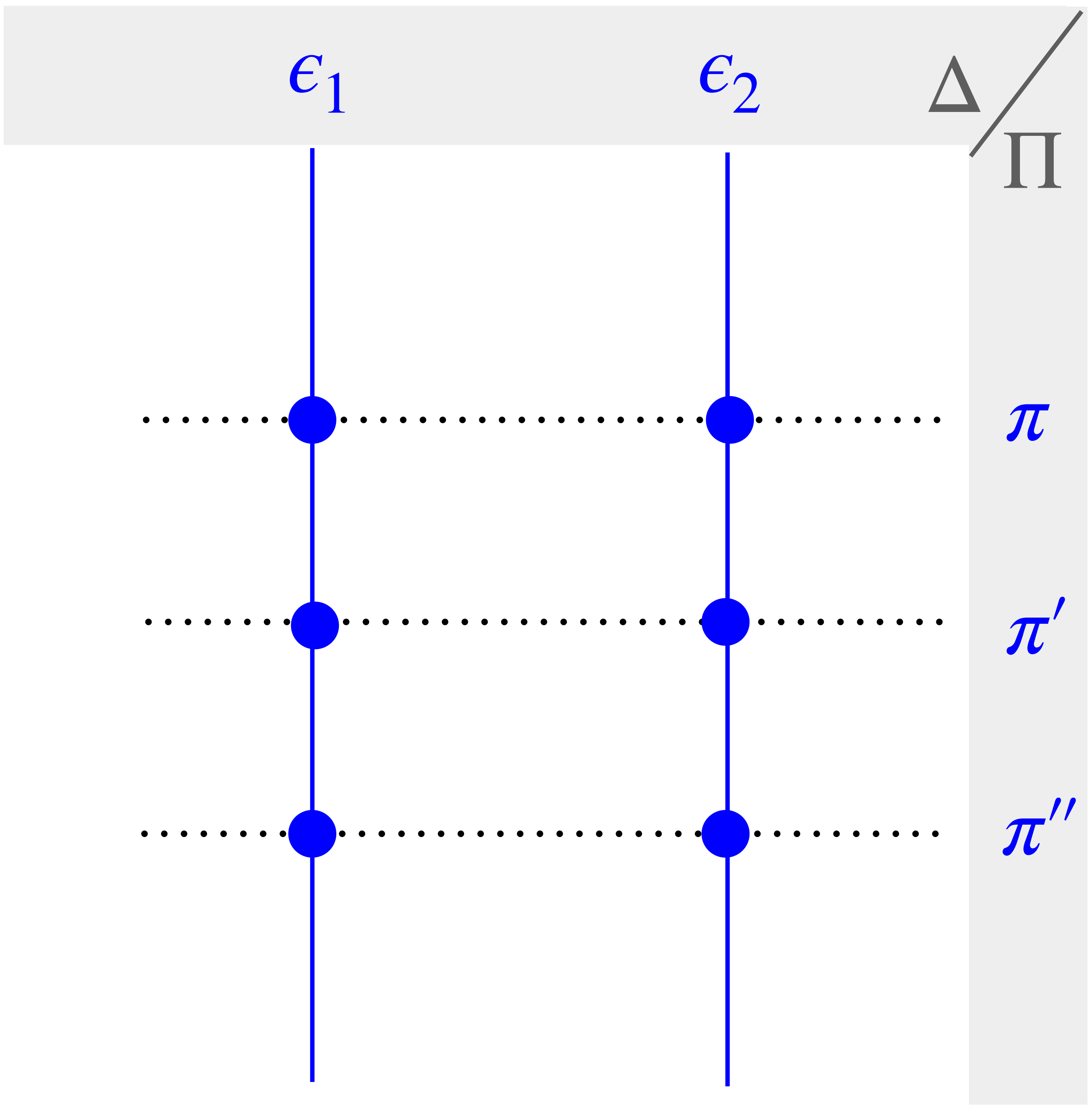

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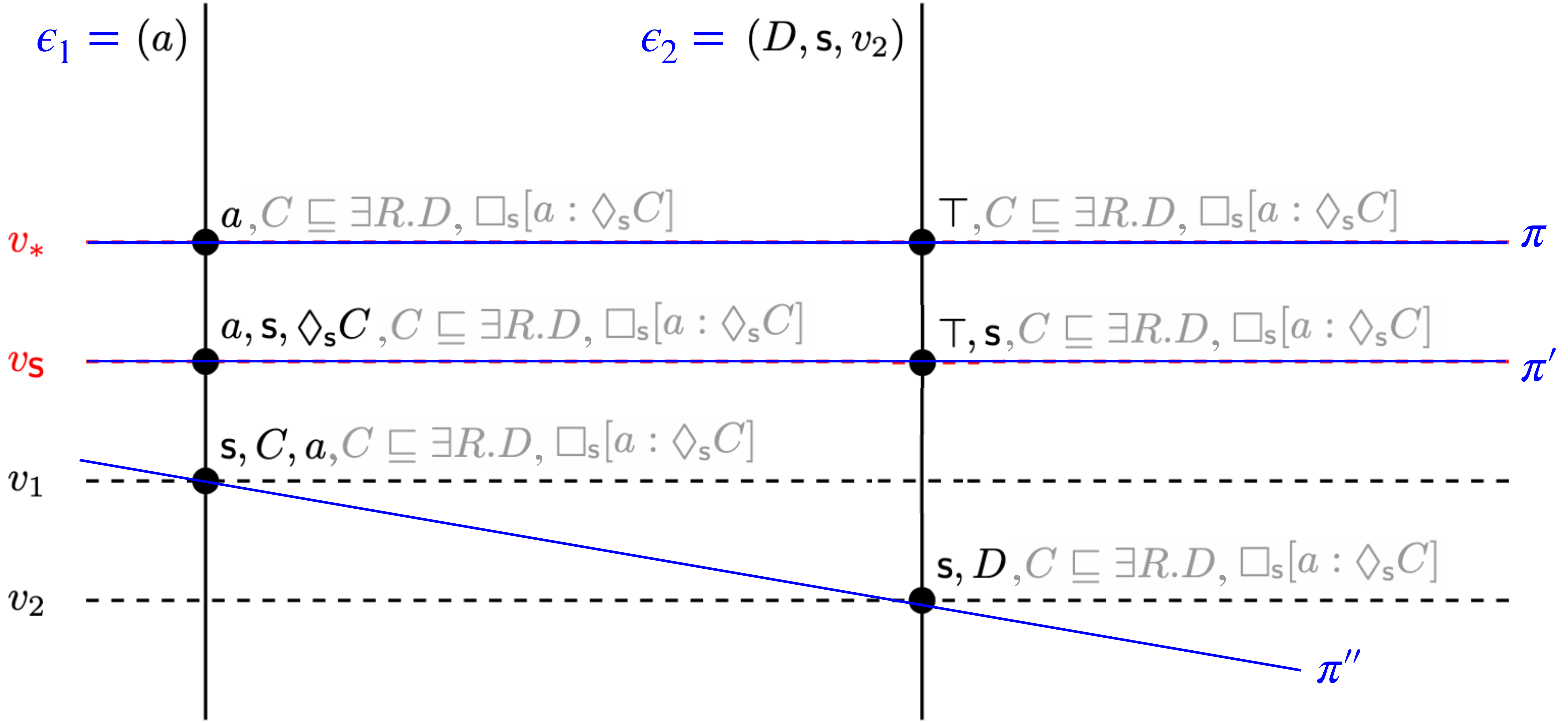

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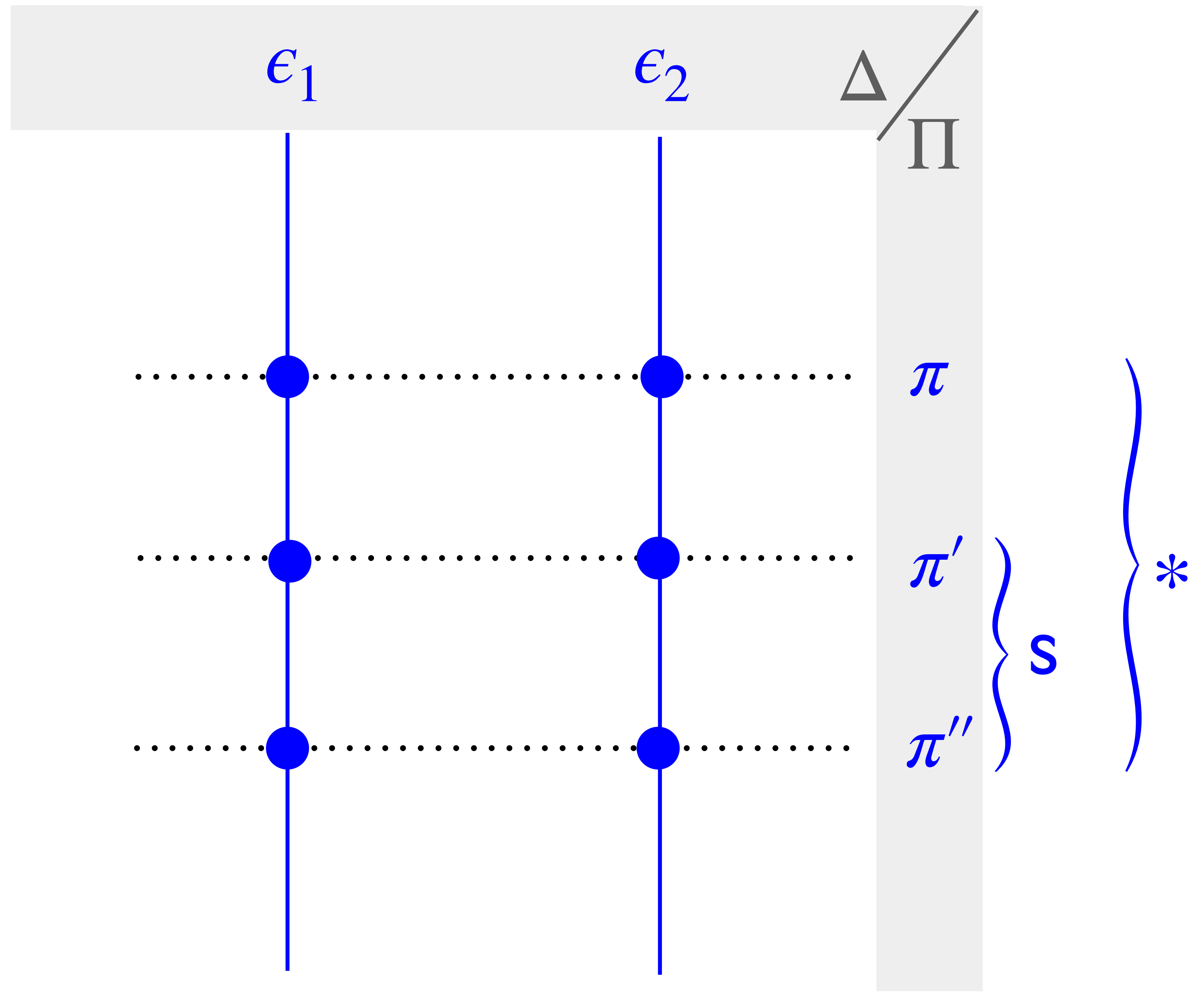

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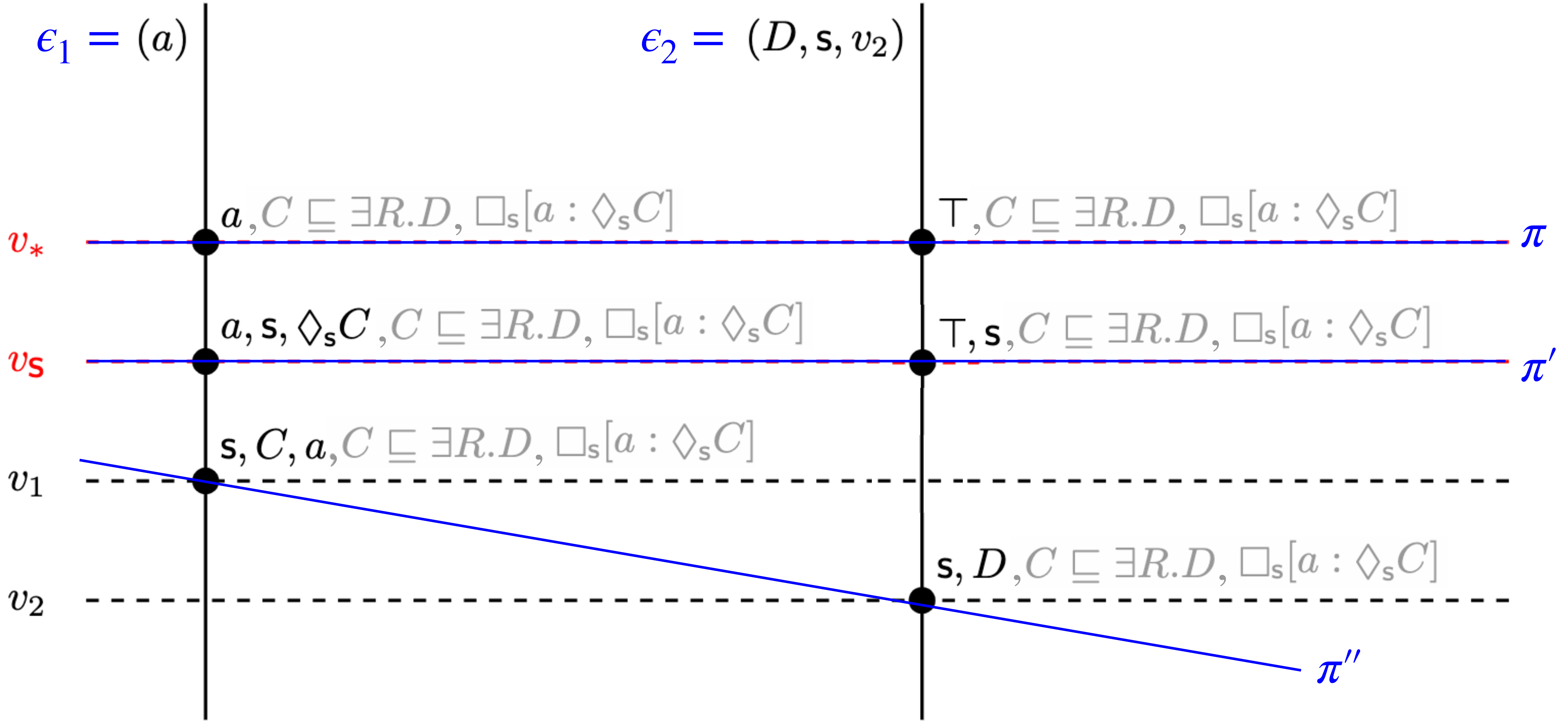

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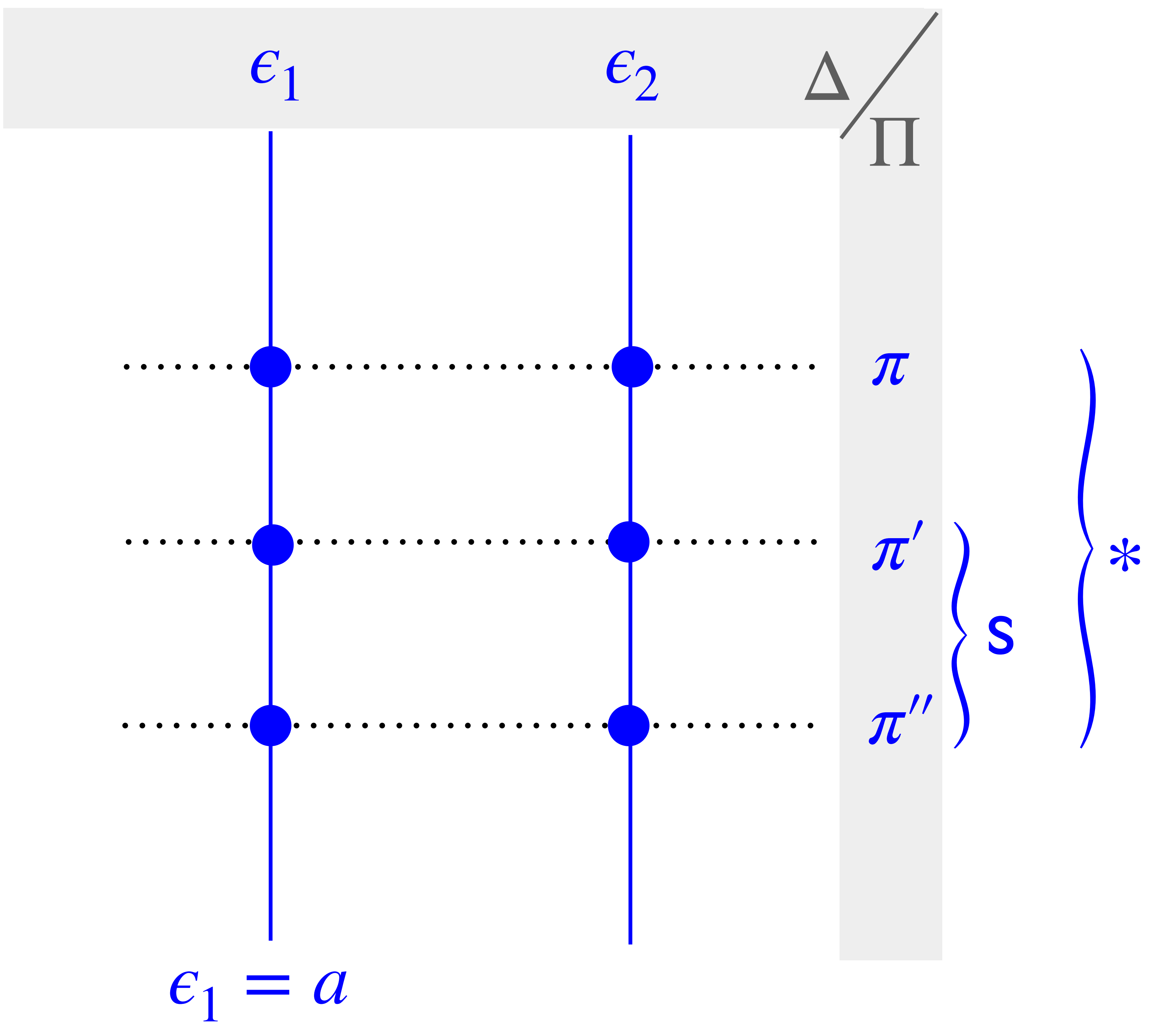

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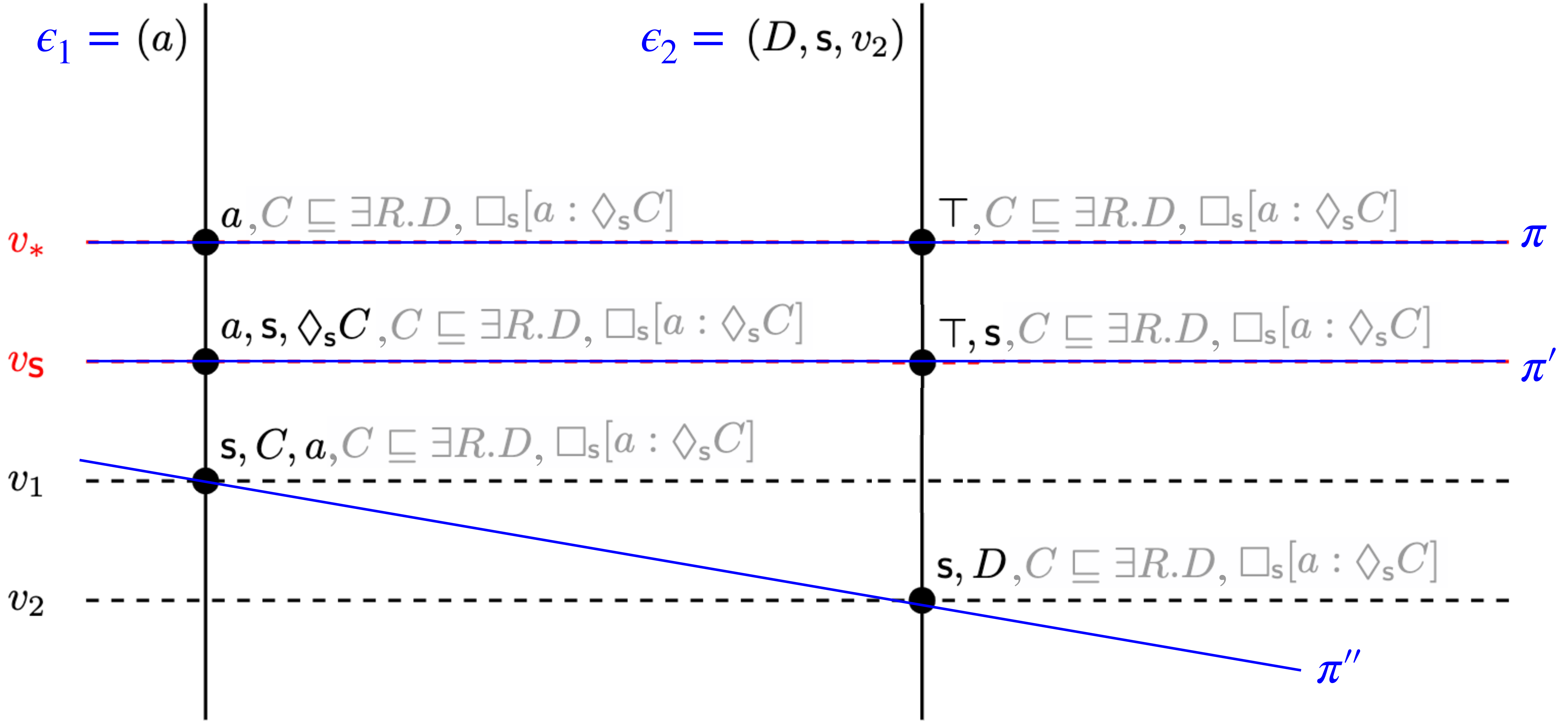

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