TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

Dr. Hannes Strass M.A. Jonas Karge

Algorithmic Game Theory

Summer Term 2024

Exercises 7

03–07/06/2024

Problem 1.
Consider the simplified version of Poker that you further simplified in the last exercise from the previous exercise sheet. Answer the following questions:

(a) Simplify this game by assuming that at an information set where a player's move is always at least as good as the other move, no matter what the other player's move or the chance move were or will be, then the player will choose that move. Draw the simplified extensive form game. A small caveat: When player I sees the middle card, you may argue by his expected payoffs, assuming Player II plays each of her strategies with equal probability.

(b) What does the simplification of (a) have to do with weakly dominated strategies?

Problem 2.
A local food processing plant, **FoodPro**, needs a small repair done. While they would normally call `Ben`'s contracting company, the repair seems small enough that Suzette, the FoodPro manager, decides to explore another contractor (possibly as an alternative for future work) by first asking `Mark`'s contracting company for a bid. `Mark`, who would like to have the work, can decide not to place a bid or to place a bid of some amount. After receiving a response from Mark, Suzette tells Ben whether Mark has submitted a bid, but not the amount of the bid if one was submitted, and asks Ben for a bid. Since the project is small, Ben does not really want the work; however, he wants to keep FoodPro as a customer over the long term and so is concerned about how his current actions will affect his relationship with FoodPro. Suzette plans to accept the lower of the two bids, but if she receives bids of similar amounts, she will choose `Ben`, her regular repair contractor, over `Mark`. Mark can choose one of three actions: not bid (`No`), bid low (`Lo`), or bid high (`Hi`). Ben only knows whether Mark bids or does not bid; if `Mark` does bid, `Ben` does not know whether he bid high or low. Since `Ben` is not interested in doing the work for a low price, we assume he chooses between two actions: not bid (`No`) or bid high (`Hi`).

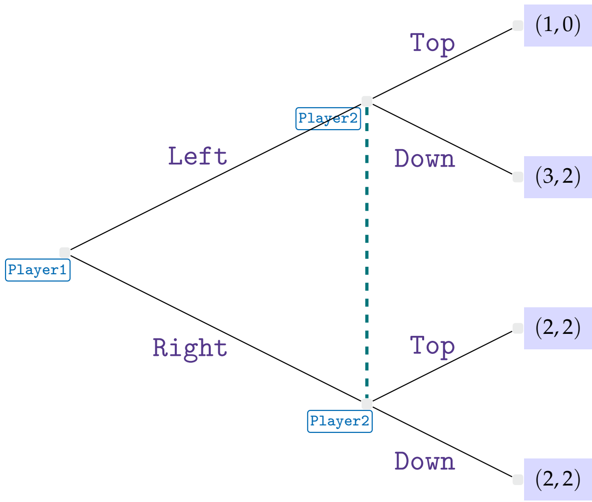
Consider the following table illustrating the payoffs for each player and each sequence of moves:

Payoff table, FoodPro.	
Terminal history z	$(u_{\texttt{Mark}}(z), u_{\texttt{Ben}}(z))$
$[\texttt{No}, \texttt{No}]$	$(1, 2)$
$[\texttt{No}, \texttt{Hi}]$	$(2, 4)$
$[\texttt{Lo}, \texttt{No}]$	$(4, 1)$
$[\texttt{Lo}, \texttt{Hi}]$	$(5, 3)$
$[\texttt{Hi}, \texttt{No}]$	$(6, 6)$
$[\texttt{Hi}, \texttt{Hi}]$	$(3, 5)$

Do the following:

- Draw the extensive form of this game with information sets.
- Let $\texttt{Ben}_1$ denote the decision node of player Ben obtained when Mark plays No, $\texttt{Ben}_2$ the one where Mark plays Lo or Hi. Decide which of the following assessments are sequentially rational and achieve consistency of beliefs:
  - (π_1, β_1):
    * $\pi_1(\texttt{Mark}) = \{\texttt{No} \mapsto 1/3, \texttt{Lo} \mapsto 1/3, \texttt{Hi} \mapsto 1/3\}$,
    * $\pi_1(\texttt{Ben}_1) = \{\texttt{No} \mapsto 1/2, \texttt{Hi} \mapsto 1/2\}$,
    * $\pi_1(\texttt{Ben}_2) = \{\texttt{No} \mapsto 1/3, \texttt{Hi} \mapsto 2/3\}$,
    * $\beta_1(\texttt{Ben}_2) = \{\texttt{Lo} \mapsto 1/2, \texttt{Hi} \mapsto 1/2\}$;
  - (π_2, β_2):
    * $\pi_2(\texttt{Mark}) = \{\texttt{No} \mapsto 0, \texttt{Lo} \mapsto 1, \texttt{Hi} \mapsto 0\}$,
    * $\pi_2(\texttt{Ben}_1) = \{\texttt{No} \mapsto 0, \texttt{Hi} \mapsto 1\}$,
    * $\pi_2(\texttt{Ben}_2) = \{\texttt{No} \mapsto 0, \texttt{Hi} \mapsto 1\}$,
    * $\beta_2(\texttt{Ben}_2) = \{\texttt{Lo} \mapsto 1/2, \texttt{Hi} \mapsto 1/2\}$;
  - (π_3, β_3):
    * $\pi_3(\texttt{Mark}) = \{\texttt{No} \mapsto 0, \texttt{Lo} \mapsto 1, \texttt{Hi} \mapsto 0\}$,
    * $\pi_3(\texttt{Ben}_1) = \{\texttt{No} \mapsto 0, \texttt{Hi} \mapsto 1\}$,
    * $\pi_3(\texttt{Ben}_2) = \{\texttt{No} \mapsto 0, \texttt{Hi} \mapsto 1\}$,
    * $\beta_3(\texttt{Ben}_2) = \{\texttt{Lo} \mapsto 1, \texttt{Hi} \mapsto 0\}$.

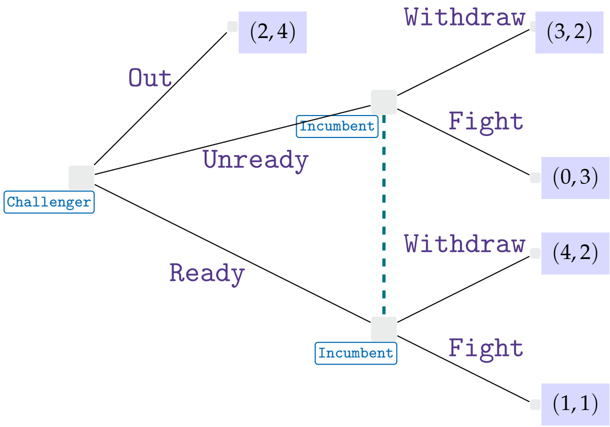
Problem 3.
Consider the following 2-player sequential game where `Player2` does not know whether `Player1` played `Left` or `Right`.



For each of the following assessments decide whether it is a weak sequential equilibrium. For this, first state which conditions have to be met for an assessment to constitute a weak sequential equilibrium. Then, if an assessment is a weak sequential equilibrium, show why it is the case. If not, show why it is not the case.

- Assessment 1: (π_1, β_1):
  - $\pi_1(\texttt{Player1}) = \{\texttt{Left} \mapsto \frac{1}{2}, \texttt{Right} \mapsto \frac{1}{2}\}$,
  - $\pi_1(\texttt{Player2}) = \{\texttt{Top} \mapsto \frac{1}{2}, \texttt{Down} \mapsto \frac{1}{2}\}$,
  - $\beta_1(\texttt{Player2}) = \{\texttt{Left} \mapsto \frac{1}{2}, \texttt{Right} \mapsto \frac{1}{2}\}$.
- Assessment 2: (π_1, β_1):
  - $\pi_1(\texttt{Player1}) = \{\texttt{Left} \mapsto 1, \texttt{Right} \mapsto 0\}$,
  - $\pi_1(\texttt{Player2}) = \{\texttt{Top} \mapsto 0, \texttt{Down} \mapsto 1\}$,
  - $\beta_1(\texttt{Player2}) = \{\texttt{Left} \mapsto 1, \texttt{Right} \mapsto 0\}$.

Problem 4.
Consider the following game description where a political `Challenger` decides whether or not to enter a race against a long time `Incumbent`. We assume the `Challenger` will spend a certain amount of time, money, and effort before announcing whether she will enter or stay out of the campaign. Afterwards, the `Incumbent` can choose to retire from office or fight for reelection with some level of effort. The `Incumbent` does not know how much preparation went into the `Challenger`'s announcement; however, it is reasonable to speculate that the more preparation that went into an announcement to enter the campaign, the more likely it is that the `Challenger` can defeat the `Incumbent` in the election. This can be represented in a game tree as follows:



One weak sequential equilibrium is the assessment in which the `Challenger` selects the strategy `Out` and the `Incumbent` selects the strategy `Fight` given the belief that the `Challenger` "has selected" (i.e., is) `Unready`. Answer the following questions without calculations:

- In that assessment, what belief can the `Incumbent` assign to the histories in his information set?
- Can this assessment happen in play?