TECHNISCHE
UNIVERSITÄT
DRESDEN

DATABASE THEORY

Lecture 6: Tree-like Conjunctive Queries

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Overview

See course homepage [⇒ link] for more information and materials

Review

Conjunctive queries (CQs) are simpler than FO-queries:

- NP combined and query complexity (instead of $\mathrm{PS{\small PACE}}$)
- data complexity remains in AC^0

CQs become even simpler if they are tree-shaped:

- GYO algorithm defines acyclic hypergraphs
- acyclic hypergraphs have join trees
- join trees can be evaluated in P with Yannakakis' Algorithm

This time:

- Find more general conditions that make CQs tractable
  $\leadsto$ "tree-like" queries that that are not really trees
- Play some games

Is Yannakakis' Algorithm Optimal?

We saw that tree queries can be evaluated in polynomial time, but we know that there are much simpler complexity classes:

$$\mathrm{NC}^0 \subset \mathrm{AC}^0 \subset \mathrm{NC}^1 \subseteq \mathrm{L} \subseteq \mathrm{NL} \subseteq \mathrm{AC}^1 \subseteq \ldots \subseteq \mathrm{NC} \subseteq \mathrm{P}$$

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Indeed, tighter bounds have been shown:

Theorem (Gottlob, Leone, Scarcello: J. ACM 2001)

Answering tree BCQs is complete for LOGCFL.

LOGCFL: the class of problems $\textsc{LogSpace}$-reducible to the word problem of a context-free language:

$$\mathrm{NC}^0 \subset \mathrm{AC}^0 \subset \mathrm{NC}^1 \subseteq \mathrm{L} \subseteq \mathrm{NL} \subseteq \mathrm{LOGCFL} \subseteq \mathrm{AC}^1 \subseteq \ldots \subseteq \mathrm{NC} \subseteq \mathrm{P}$$

$\leadsto$ highly parallelisable

Generalising Tree Queries

In practice, many queries are tree queries,
but even more queries are “almost” tree queries, but not quite . . .

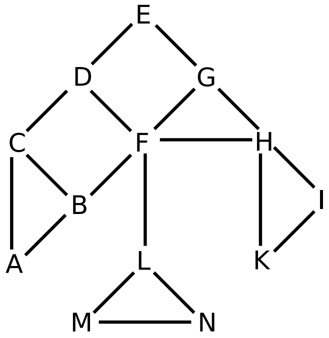
How can we formalise this idea?

Several attempts to define “tree-like” queries:

- Treewidth: a way to measure tree-likeness of graphs
- Query width: towards tree-like query graphs
- Hypertree width: adoption of treewidth to hypergraphs

How to recognise trees . . .

. . . from quite a long way away:



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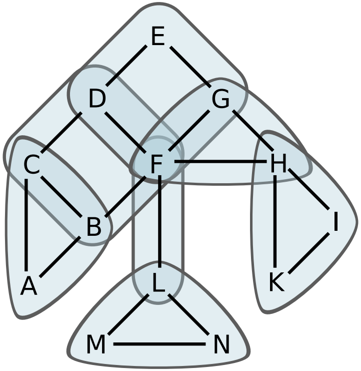


Tree Decompositions

Idea: if we can group the edges of a graph into bigger pieces, these pieces might form a tree structure

Definition

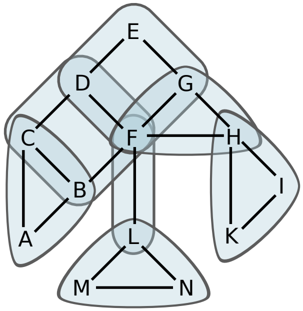
Consider a graph $G = \langle V, E\rangle$. A tree decomposition of G is a tree structure T where each node of T is a subset of V, such that:

- The union of all nodes of T is V.
- For each edge $(v_1 \to v_2) \in E$, there is a node N in T such that $v_1, v_2 \in N$.
- For every vertex $v \in V$, the set of nodes of T that contain v form a subtree of T; equivalently: if two nodes contain v, then all nodes on the path between them also contain v (connectedness condition).

Nodes of a tree decomposition are often called bags

(not related to the common use of “bag” as a synonym for “multiset”)

Tree Decompositions: Example



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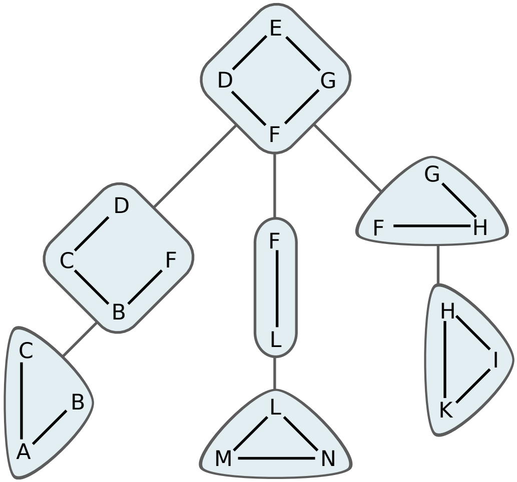


Treewidth

The treewidth of a graph defines how “tree-like” it is:

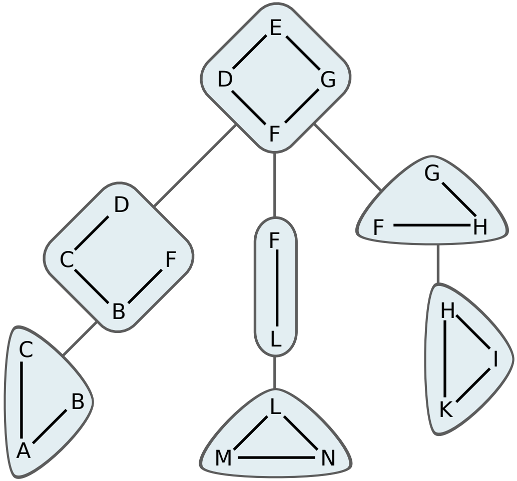
Definition

The width of a tree decomposition is the size of its largest bag minus one.
The treewidth of a graph G, denoted $\mathrm{tw}(G)$, is the smallest width of any of its tree decompositions.

Simple observations:

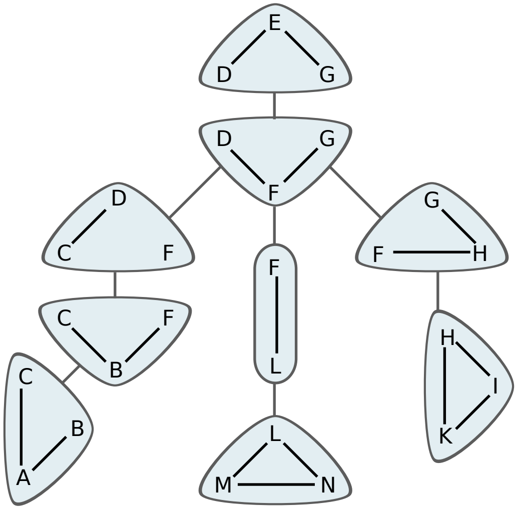
- If G is a tree, then we can decompose it into bags that contain only one edge $\leadsto$ trees have treewidth 1
- Every graph has at least one tree decomposition where all vertices are in one bag $\leadsto$ max. treewidth = number of vertices

Treewidth: Example



$\rightsquigarrow$ tree decomposition of width 3

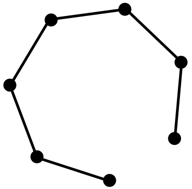
Treewidth: Example

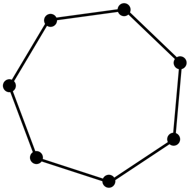


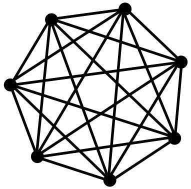
$\rightsquigarrow$ tree decomposition of width 2 = treewidth of the example graph

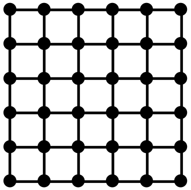
More Examples

What is the treewidth of the following graphs?









Treewidth and Conjunctive Queries

Treewidth is based on graphs, not hypergraphs

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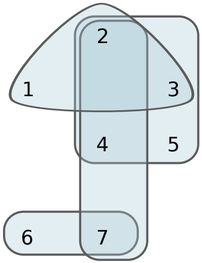
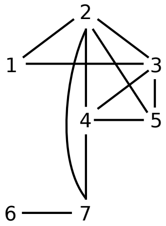
$\rightsquigarrow$ treewidth of CQ = treewidth of primal graph of query hypergraph

Query graph and corresponding primal graph:





$\rightsquigarrow$ Treewidth 3

Observation: acyclic hypergraphs can have unbounded treewidth!

Exploiting Treewidth in CQ Answering

Queries of low treewidth can be answered efficiently:

Theorem (Dechter/Chekuri+Rajamaran ’97/Kolaitis+Vardi ’98/Gottlob & al. ’98)

Answering BCQs of treewidth k is possible in time $O(n^k \log n)$, and thus in polynomial time if k is fixed.
The problem is also complete for LOGCFL.

Checking for low treewidths can also be done efficiently:

Theorem (Bodlaender ’96)

Given a graph G and a fixed number k, one can check in linear time if tw$(G) \leq k$, and the corresponding tree decomposition can also be found in linear time.

Warning: neither CQ answering nor tree decomposition might be practically feasible if k is big

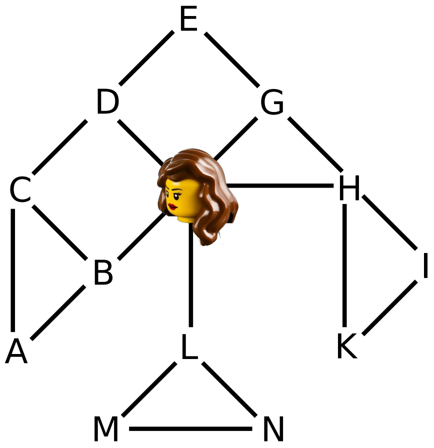
Treewidth via Games

Seymour and Thomas [1993] gave
an alternative characterisation of treewidth:

The Cops-and-Robber Game

- The game is played on a graph G
- There are k cops and one robber, each located at one vertex
- In each turn:
  - the cops can fly to an arbitrary vertex in the graph
  - the robber can run along the edges of the graph, as far as she likes, as long a she does not pass through any vertex that was occupied by a cop before or after the turn
    (the robber can run to a place where a cop was before the turn, but not pass through such a place)
- The goal of the cops is to catch the robber;
  the goal of the robber is never to be caught

Cops and Robbers: Example



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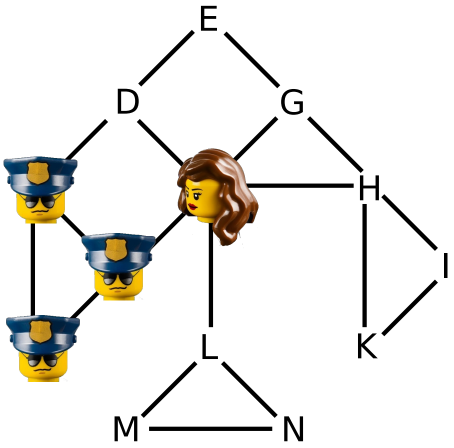

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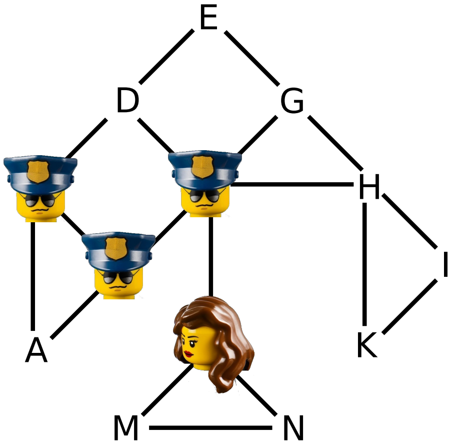

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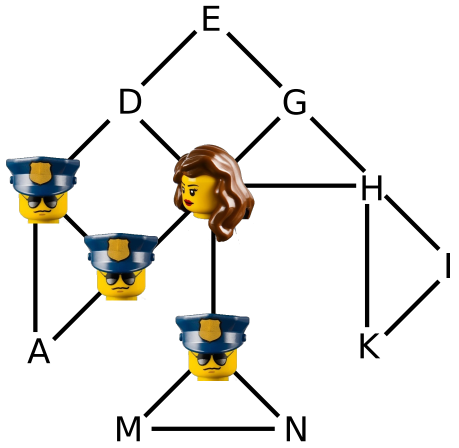

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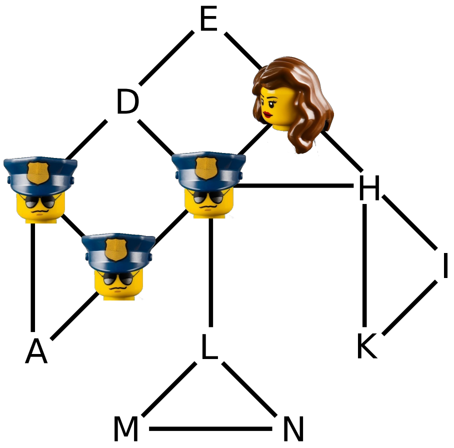

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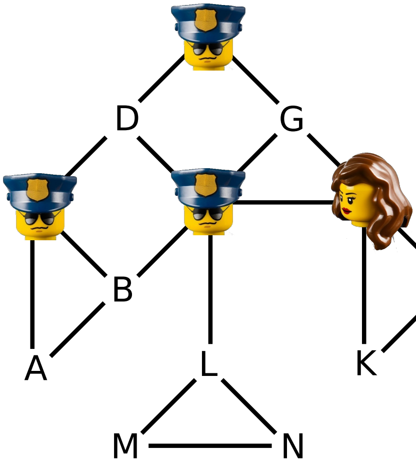

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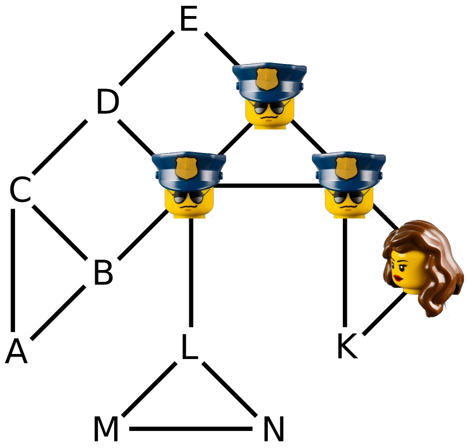

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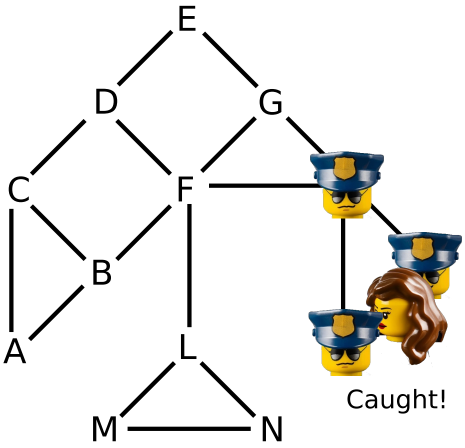

Cops & Robbers and Treewidth

Theorem (Seymour and Thomas)

A graph G is of treewidth $\leq k-1$ if and only if k cops have a winning strategy in the cops & robber game on G.

Intuition: the cops together can block even the widest branch and still move in on the robber

Treewidth via Logic

Kolaitis and Vardi [1998] gave a logical characterisation of treewidth

Bounded treewidth CQs correspond to certain FO-queries:

- We allow FO-queries with $\exists$ and $\wedge$ as only operators
- But operators can be nested in arbitrary ways (unlike in CQs)
- Theorem: A query can be expressed with a CQ of treewidth k if and only if it can be expressed in this logic using a query with at most $k + 1$ distinct variables

Intuition: variables can be reused by binding them in more than one $\exists$
$\leadsto$ Apply a kind of "inverted prenex-normal-form transformation"
$\leadsto$ Variables that occur in the same atom or in a "tightly connected" atom must use different names
$\leadsto$ minimum number of variables $\Leftrightarrow$ treewidth (+1)

Treewidth: Pros and Cons

Advantages:

- Bounded treewidth is easy to check
- Bounded treewidth CQs are easy to answer

Disadvantages:

- Even families of acyclic graphs may have unbounded treewidth
- Loss of information when using primal graph
  (cliques might be single hyperedges – linear! –
  or complex query patterns – exponential!)

$\leadsto$ Are there better ways to capture “tree-like” queries?

Query Width

Idea of Chekuri and Rajamaran [1997]:

- Create tree structure similar to tree decomposition
- But consider bags of query atoms instead of bags of variables
- Two connectedness conditions:
  1. Bags that refer to a certain variable must be connected
  2. Bags that refer to a certain query atom must be connected

Query width: least number of atoms needed in bags of a query decomposition

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Query width: least number of atoms needed in bags of a query decomposition

Theorem

Given a query decomposition for a BCQ, the query answering problem can be decided in time polynomial in the query width.

Problems with Query Width

Theorem (Gottlob et al. 1999)

Deciding if a query has query width at most k is NP-complete.

In particular, it is also hard to find a query decomposition

$\leadsto$ Query answering complexity drops from NP to P …
…but we need to solve another NP-hard problem first!

Generalised Hypertree Width

Gottlob, Leone, and Scarcello had another idea on defining tree-like hypergraphs:

Intuition:

- Combine key ideas of tree decomposition and query decomposition
- Start by looking at a tree decomposition
- But define the width based on query atoms:
  How many atoms do we need to cover all variables in a bag?

$\leadsto$ Generalised hypertree width
$\leadsto$ A technical condition is needed to get a simpler-to-check notion

Hypertree Width

Definition

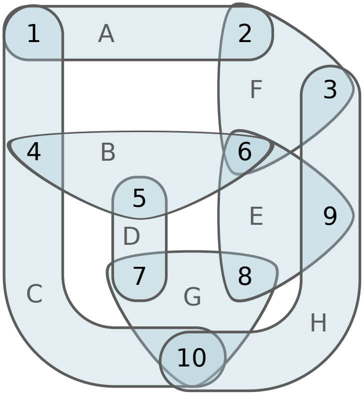
Consider a hypergraph $G = \langle V, E \rangle$. A hypertree decomposition of G is a tree structure T where each node n of T associated with a bag of variables $B_n \subseteq V$ and with a set of edges $G_n \subseteq E$, such that:

- T with B_n yields a tree decomposition of the primal graph of G.
- For each node n of T:
  (1) the vertices used in the edges G_n are a superset of B_n,
  (2) if a vertex v occurs in an edge of G_n and this vertex also occurs in B_m for some node m below n in T, then $v \in B_n$.

The width to T is the largest number of edges in a set G_n.
The hypertree width of G, hw(G), is the least width of its hypertree decompositions.

((2) is the "special condition": without it we get the generalised hypertree width)

Hypertree Width: Example



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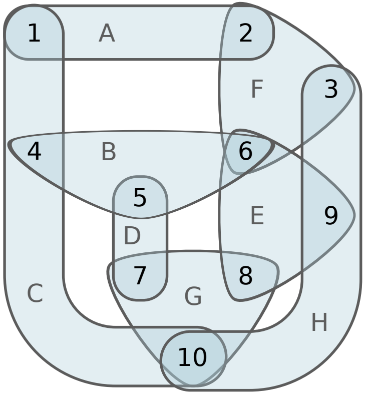

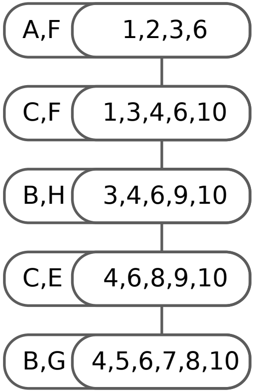

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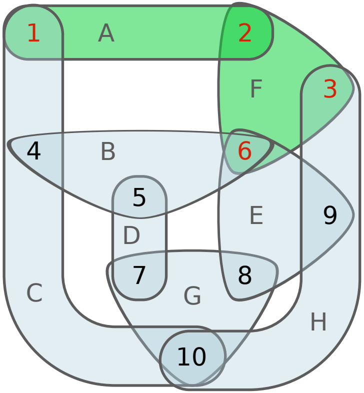

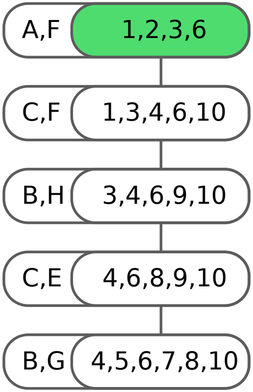

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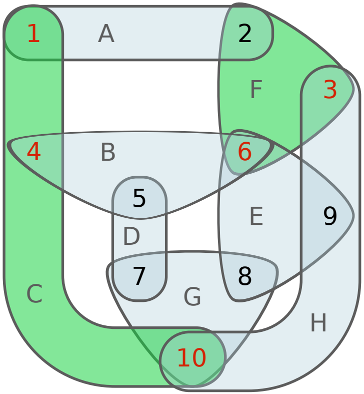

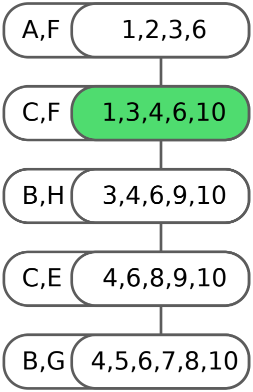

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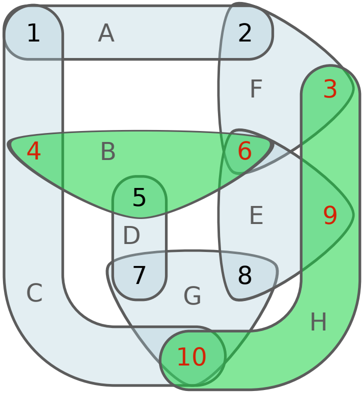

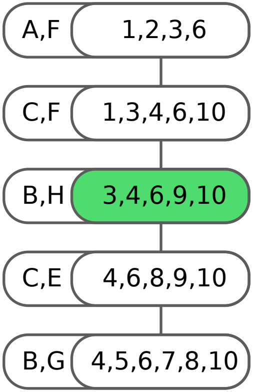

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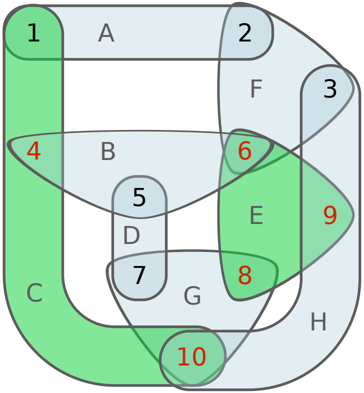

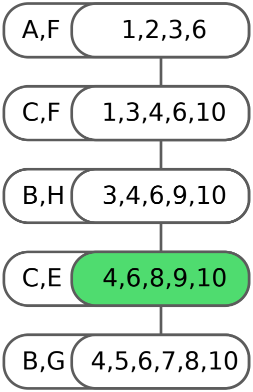

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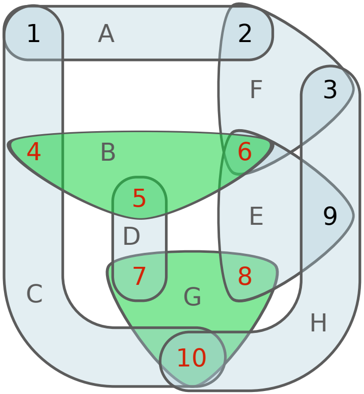

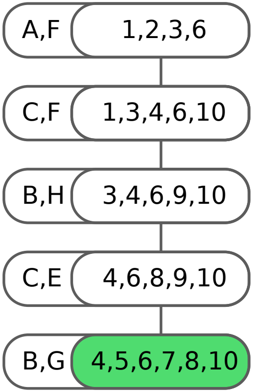

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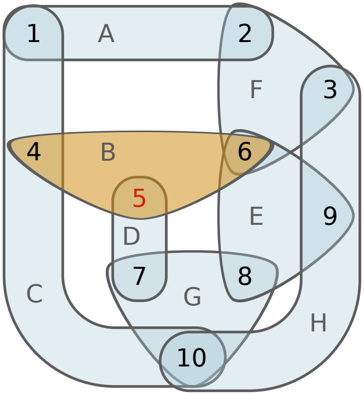
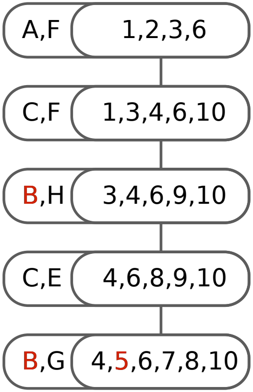
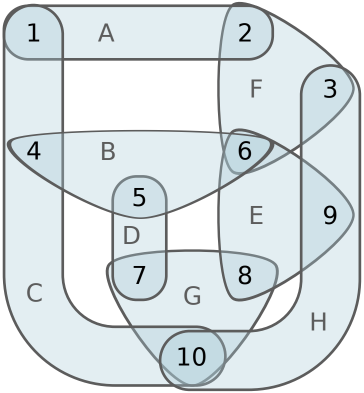




Special condition violated $\rightsquigarrow$ no hypertree decomposition
$\rightsquigarrow$ But generalised hypertree decomposition of width 2

Hypertree Width: Example



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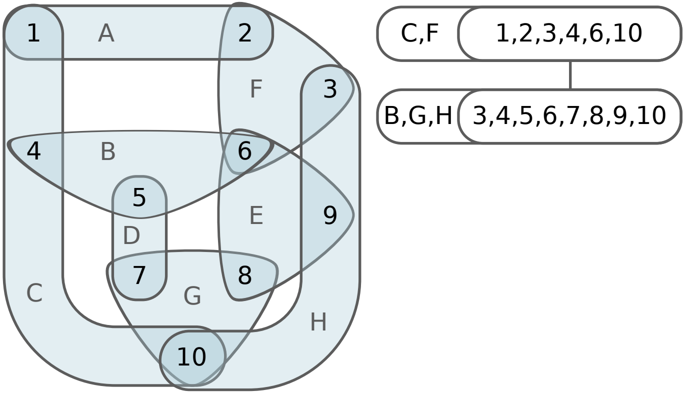

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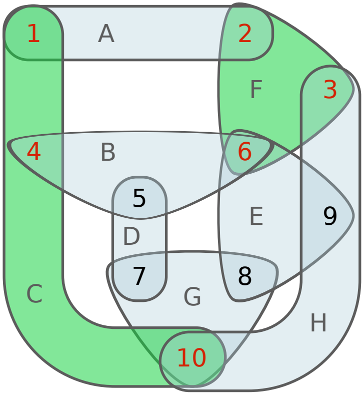

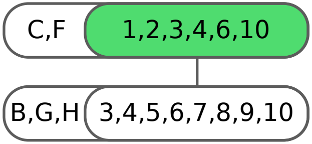

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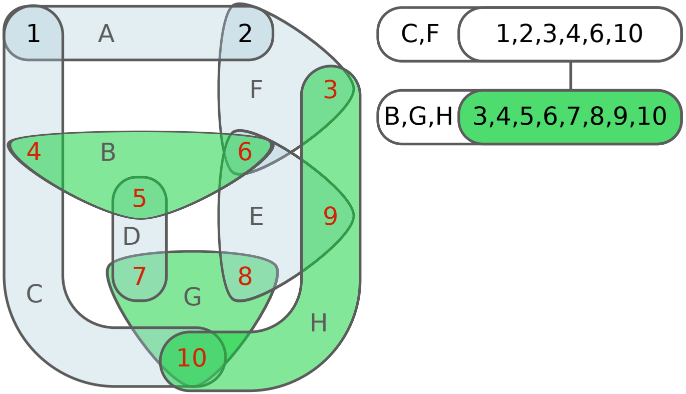

Special condition satisfied $\rightsquigarrow$ hypertree decomposition of width 3

Hypertree Width: Results

- Relationships of hypergraph tree-likeness measures:
  generalised hypertree width $\leq$ hypertree width $\leq$ query width
  (both inequalities might be $<$ in some cases)

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- Deciding "query width < k?" is NP-complete
- Deciding "generalised hypertree width < 4?" is NP-complete
- Deciding "hypertree width < k?" is polynomial (LOGCFL)
- Hypertree decompositions can be computed in polynomial time if k is fixed

Theorem

For a BCQ of (generalised) hypertree width k, query answering can be decided in polynomial time, and is complete for LOGCFL.

. . . but the degree of the polynomial time bound is greater than k

Hypertree Width via Games

There is also a game characterisation of (generalised) hypertree width.

The Marshals-and-Robber Game

- The game is played on a hypergraph
- There are k marshals, each controlling one hyperedge, and one robber located at a vertex
- Otherwise similar to cops-and-robber game
- Special condition: Marshals must shrink the space that is left for the robber in every turn!

Hypertree width $\leq k$ if and only if k marshals have a winning strategy
$\leadsto$ hypergraph is acyclic iff 1 marshal has a winning strategy

Hypertree Width via Logic

There is also a logical characterisation of hypertree width.

Loosely k-Guarded Logic

- Fragment of FO with $\exists$ and $\wedge$
- Special form for all $\exists$ subexpressions:

$$\exists x_1, \ldots, x_n.(G_1 \wedge \ldots \wedge G_k \wedge \varphi)$$

  where G_i are atoms ("guards") and every variable that is free in φ occurs in one such atom G_i.

A query has hypertree width $\leq k$ if and only if it can be expressed as a loosely k-guarded formula
$\leadsto$ tree queries correspond to loosely 1-guarded formulae
("loosely 1-guarded" logic is better known as guarded logic and widely studied)

Summary and Outlook

Besides tree queries, there are other important classes of CQs that can be answered in polynomial time:

- Bounded treewidth queries
- Bounded hypertree width queries

General idea: decompose the query in a tree structure

Other possible characterisations via games and logic

Next topics:

- What else is there besides query answering? $\leadsto$ optimisation
- Measure expressivity rather than just complexity