Exercise 5.1. Transform the following concepts into negation normal form:

(a) \( \neg(A \cap \forall r.B) \)

(b) \( \neg\forall r.\exists s.(-B \sqcup \exists r.A) \)

(c) \( \neg((\neg A \cap \exists r.\top) \sqcup \geq 3 s.(A \sqcup \neg B)) \)

Exercise 5.2. Apply the tableau algorithm in order to check if the axiom \( A \sqsubseteq B \) is a logical consequence of the TBox \{\( \neg C \sqsubseteq B, A \cap C \sqsubseteq \bot \)\}.

Exercise 5.3. Aply the tableau algorithm in order to check satisfiability of the concept \( A \cap \forall r.B \) w.r.t. the TBox \{\( A \sqsubseteq \exists r.A, B \sqsubseteq \exists r\neg .C, C \sqsubseteq \forall r.\forall r.B \)\}.

Exercise 5.4. Markus wants to apply the tableau algorithm for checking the satisfiability of the concept \( B \cap \exists r\neg .A \) w.r.t. the TBox \{\( A \sqsubseteq \exists r\neg .A \cap \exists r.B, \top \sqsubseteq 1 r \)\}. He arrives at the situation depicted below and concludes that no further rules are applicable, since \( v_2 \) is blocked by \( v_1 \). What is Markus’ error? Continue the algorithm until its termination. (You don’t have to illustrate all intermediate steps, just provide the final state.)

\[
\begin{array}{c}
\text{\( v_0 \)} \\
\text{\( v_1 \)} \\
\text{\( v_2 \)} \\
\end{array}
\]

\[
L(v_0) = \{ B \cap \exists r\neg .A, B, \exists r\neg .A, C_T, \neg A, \leq 1 r \}
\]

\[
L(v_1) = \{ A, C_T, \exists r\neg .A, \exists r.B, \leq 1 r \}
\]

\[
L(v_2) = \{ A, C_T, \exists r\neg .A, \exists r.B, \leq 1 r \}
\]

Exercise 5.5. Extend the \( \leq 1 \) rule in a way that also qualified functionality axioms of the form \( \top \sqsubseteq \leq 1 r.A \) can be treated correctly, where \( A \) is an atomic concept. Can you also treat arbitrary axioms of the form \( C \sqsubseteq \leq 1 r.D \)?