**Exercise 12.1.** Let \( \mathcal{L} \) be a fragment of first-order logic for which finite model entailment and arbitrary model entailment coincide, i.e., for every \( \mathcal{L} \)-theory \( T \) and every \( \mathcal{L} \)-formula \( \varphi \), we find that \( \varphi \) is true in all models of \( T \) if and only if \( \varphi \) is true in all finite models of \( T \).

(a) Give an example for a proper fragment of first-order logic with this property.

(b) Give an example for a proper fragment of first-order logic without this property.

(c) Show that entailment is decidable in any fragment with this property.

**Exercise 12.2.** Consider the following set of tgds \( \Sigma \):

\[
\begin{align*}
A(x) & \rightarrow \exists y. R(x, y) \land B(y) \\
B(x) & \rightarrow \exists y. S(x, y) \land A(y) \\
R(x, y) & \rightarrow S(y, x) \\
S(x, y) & \rightarrow R(y, x)
\end{align*}
\]

Does the oblivious chase universally terminate for \( \Sigma \)? What about the restricted chase?

**Exercise 12.3.** Is the following set of tgds \( \Sigma \) weakly acyclic?

\[
\begin{align*}
B(x) & \rightarrow \exists y. S(x, y) \land A(x) \\
A(x) \land C(x) & \rightarrow \exists y. R(x, y) \land B(y)
\end{align*}
\]

Does the skolem chase universally terminate for \( \Sigma \)?

**Exercise 12.4.** Termination of the oblivious (resp. restricted) chase over a set of tgds \( \Sigma \) implies the existence of a finite universal model for \( \Sigma \). Is the converse true? That is, does the existence of a finite universal model for \( \Sigma \) imply termination of the oblivious (resp. restricted) chase?

**Exercise 12.5.** Consider a set of tgds \( \Sigma \) that does not contain any constants. A term is **cyclic** if it is of the form \( f(t_1, \ldots, t_n) \) and, for some \( i \in \{1, \ldots, n\} \), the function symbol \( f \) syntactically occurs in \( t_i \). Then \( \Sigma \) is **model-faithful acyclic** (MFA) iff no cyclic term occurs in the skolem chase of \( \Sigma \cup I_* \), where \( I_* \) is the critical instance.

Show the following claims:

1. Checking MFA membership is decidable.

2. Is the set of tgds from Exercise 12.3 MFA?

3. If a set of tgds \( \Sigma \) without constants is MFA, then the skolem chase universally terminates for \( \Sigma \).