Exercise 2

SAT Solving

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Exercise 2.1 (Type of clauses)

a. Provide definitions for the following concepts:
   (a) Clause
   (b) Horn clause
   (c) Unit clause
   (d) Binary clause

b. Give 3 examples each.

Exercise 2.2 (Normal forms)

a. Give a definition for a formula in the following forms:
   (a) CNF (conjunctive normal form) and
   (b) DNF (disjunctive normal form).

b. How can we transform a formula from CNF into DNF?

c. Is it possible to transform a formula in CNF into DNF in polynomial time?
   (Provide an explanation).

Exercise 2.3 (Equivalence elimination)

a. Demonstrate the combinatorial explosion of the equivalences elimination given in the lecture.

b. Transform F_1, F_2 and F_3 into CNF with the help of the algorithm presented in the lecture.

$$F_1 = \neg p \wedge (p \rightarrow q) \wedge (\neg q \rightarrow p)$$
$$F_2 = \neg q \vee (\neg p \leftrightarrow q)$$
$$F_3 = p \leftrightarrow q$$

Exercise 2.4 (Equivalence and Equi-satisfiability)

a. An application of a rule of the form $\frac{D}{D_1|D_2}$ may lead to copies of subformulas. May this lead to a combinatorial explosion? If this is the case, then construct a sequence of examples showing the explosion. If this is not the case, then prove it.

b. (a) What is the definitional transformation (Tseitin transformation)?
   (b) Why do we use the Tseitin transformation for SAT solving?
   (c) Use the Tseitin transformation to transform F_1 and F_2 into a CNF.

$$F_1 = \neg q \vee (\neg p \rightarrow q)$$
$$F_2 = \neg p \wedge (p \rightarrow q) \wedge (\neg q \rightarrow p)$$

c. Transform your solution to Example 2.4.1 using the Tseitin transformation.

Exercise 2.5 (Reducts)

a. What is the reduct of a formula? Give an example.

b. Let $F = \langle [1,2], [-2,3], [2,3,-4], [-2], [-1,2,-5,6], [-1,3] \rangle$.
   Compute the reduct of F for the following partial interpretations:

   (a) $\{1\}$
   (b) $\{1,2,3\}$
   (c) $\{4,5,6\}$
   (d) $\{1,2,3,4,5,6\}$
   (e) $\{-2,-3,5,6\}$

Exercise 2.6

a. Given the following 4 Clauses:

$$\begin{array}{ll} C_1 = [1,2,3] & C_2 = [-1,4] \\ C_3 = [-1,2,3] & C_4 = [-2,-4] \end{array}$$

   Compute a resolvent of

   (a) C_1 and C_2,
   (b) C_1 and C_3,
   (c) C_1 and C_4,
   (d) C_3 and C_4,
   (e) C_2 and C_4,
   (f) C_2 and C_3.

b. Let F be a CNF-formula and C a resolvent of two clauses C_1 and C_2 occurring in F, prove that $F \equiv F \wedge C$.