TECHNISCHE UNIVERSITÄT DRESDEN

EXISTENTIAL RULES SEMINAR

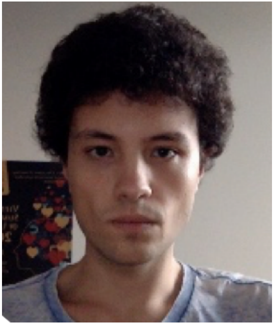
Lecture 1: Syntax and Semantics

David Carral, Markus Krötzsch

Knowledge-Based Systems

TU Dresden, April 16, 2019

Course Tutors



David Carral

Markus Krötzsch

Acknowledgements

Content

Michaël Thomazo and **Andreas Pieris**

Student Session at ESSLLI

More info at `http://esslli-stus-2015.phil.hhu.de/`

Beamer Style

Markus Krötzsch

Tips on How to Run a Seminar

Lukas Schweizer

Organisation

Lectures

Tuesdays, DS 5 (14:50–16:20), APB E005

Web Page

https://iccl.inf.tu-dresden.de/web/Seminar_Existential_Rules_(SS2019)

Lecture Notes

All slides will be available online.

Goals, Prerequisites, and Reading List

(Non-)Prerequisites

- First-order logic (syntax and semantics).
- Complexity theory (complexity classes, reductions...).

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Reading list

- Uwe Schöning: **Logic for Computer Scientists**; Birkhäuser.
- Michael Sipser: **Introduction to the Theory of Computation, International Edition**; Cengage Learning.

Structure of the Seminar and Evaluation

Lectures

- **April 2, 2019:** Introductory lecture 1
- **April 9, 2019 (i.e., today):** Introductory lecture 2
- **Afterwards:** Office hours in APB 3035 and presentations

Structure of the Seminar and Evaluation

Lectures

- **April 2, 2019:** Introductory lecture 1
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Evaluation

- **Paper summary:** self-selected research paper;[a] 10 pages
- **Presentation:** 20 minutes + discussion

[a] See the "Literature" tab at: `https://iccl.inf.tu-dresden.de/web/Seminar_Existential_Rules_(SS2019)`.

Motivation: Accessing Big Data

"Data is stored in various **heterogeneous** formats over many differently structured databases. As a result, the gathering of only relevant data spread over **disparate sources** becomes a very **time consuming task**." – Jim Crompton, W3C Workshop on Semantic Web in Oil & Gas Industry, 2008

More info at: `http://www.expertsystem.com/semantic-web-in-oil-gas-industry/`

Motivation: Accessing Big Data

Experts in geology and geophysics develop stratigraphic models of unexplored areas on the basis of data acquired from previous operations at nearby geographical locations.

Facts:

- 1000 TB of relational data
- Using diverse schemata
- Spread over 2000 tables, over multiple individual data bases

A Possible Solution

- Achieve transparency in accessing data using **logic** – e.g., **existential rules**!
- Manage data by exploiting Knowledge Representation techniques.
- Provide a conceptual, high level representation of the domain of interest of terms of an **ontology** (i.e., a logical theory).

A Simple Example

Example 1.1:

$$HasSon(x, y) \wedge HasSister(y, z) \rightarrow HasDaughter(x, z) \quad (1)$$

$$HasFather(x, y) \rightarrow HasSon(y, x) \quad (2)$$

$$Person(x) \rightarrow \exists y.HasFather(x, y) \quad (3)$$

$$Person(anakin) \quad (4)$$

$$HasFather(luke, anakin) \quad (5)$$

$$HasSister(luke, leia) \quad (6)$$

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$$HasSister(luke, leia) \quad (6)$$

Is $leia$ the daughter of $anakin$?

I.e., does $HasDaughter(anakin, leia)$ follow from (7-12)?

Syntax: Signature and Atoms

- A **signature** is a tuple $\langle \mathsf{P}, \mathsf{V}, \mathsf{C}, \mathsf{N} \rangle$ with P a set of **predicates**, V a set of **variables**, C a set of **constants**, and N a set of **nulls**.
- Every predicate $P \in \mathsf{P}$ is associated to some arity $\mathsf{ar}(P) \geq 1$.
- $\mathsf{T} = \mathsf{V} \cup \mathsf{C} \cup \mathsf{N}$ is the set of **terms**.
- An **atom** is a formula of the form $P(\vec{t})$ with $P \in \mathsf{P}$, $\mathsf{ar}(P) = |\vec{t}|$, and $t \in \mathsf{T}$ for all $t \in \vec{t}$.

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Example 1.2: Entities and atoms.

$$Person(x) \rightarrow \exists y.HasFather(x, y)$$

$$HasSister(luke, leia)$$

Syntax: Existential Rules

Definition 1.3: An **(existential) rule** is a formula of the form

$$\forall \vec{x}, \vec{z}.(\beta[\vec{x}, \vec{z}] \rightarrow \exists \vec{y}.\eta[\vec{x}, \vec{y}])$$

with β and η conjunctions of null-free atoms and $\vec{x}$, $\vec{y}$, and $\vec{z}$ mutually disjoint sequences of variables. A **fact** is a rule with an empty body that contains no occurrences of variables.

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with β and η conjunctions of null-free atoms and $\vec{x}$, $\vec{y}$, and $\vec{z}$ mutually disjoint sequences of variables. A **fact** is a rule with an empty body that contains no occurrences of variables.

Formulas (7-12) from slide 9 are existential rules.
Formulas (10-12) are also facts.

Semantics

Definition 1.4: A **homomorphism** is a partial function over the set of terms with $h(c) = c$ for all $c \in \mathsf{C}$.

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Let ϕ be a formula, h a homomorphism, and $\vec{x}$ a sequence of variables. Then,

- $h(\phi)$ is the formula that results from replacing every term t in the domain of h by $h(t)$, and
- $h_{\vec{x}} \subseteq h$ is the restriction of h over $\vec{x} \cup \mathsf{C}$.

Semantics

Definition 1.5: A pair $\langle \rho, h \rangle$ with $\rho = \beta[\vec{x}, \vec{z}] \rightarrow \exists \vec{y}.\eta[\vec{x}, \vec{y}]$ a rule and h a homomorphism is **applicable** to a set of facts F if

1. $h(\beta) \subseteq$ F, and
2. for all $h' \supseteq h_{\vec{x}}$, $h'(\eta) \not\subseteq$ F.

Alternatively, we say that $\langle \rho, h \rangle$ is **not satisfied** by F.

If $\langle \rho, h \rangle$ is applicable to F, then we define $\rho_h(\mathsf{F}) = \mathsf{F} \cup h'(\eta)$ with $h' \supseteq h$ a homomorphism mapping every $y \in \vec{y}$ to a fresh null.

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If $\langle \rho, h \rangle$ is applicable to F, then we define $\rho_h(\mathsf{F}) = \mathsf{F} \cup h'(\eta)$ with $h' \supseteq h$ a homomorphism mapping every $y \in \vec{y}$ to a fresh null.

Often, we refer to $\rho_h(\mathsf{F})$ as the **application** of $\langle \rho, h \rangle$ to F.

Semantics

Definition 1.6: An **interpretation** $\mathcal{I}$ is a set of facts. $\mathcal{I}$ **satisfies** a rule $\rho = \beta \to \exists \vec{y}.\eta$ if $\langle \rho, h \rangle$ is satisfied by $\mathcal{I}$ for every homomorphism h. $\mathcal{I}$ is a **model** of a rule set R if it satisfies every rule $\rho \in$ R.

We write $\mathcal{I} \models \rho$ to indicate that $\mathcal{I}$ satisfies ρ. Analogously, we write $\mathcal{I} \models$ R to indicate that $\mathcal{I}$ is a model of R.

Semantics

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Definition 1.7: An interpretation $\mathcal{I}$ **entails** a query $q = \exists \vec{y}.\beta$, written $\mathcal{I} \models q$, if $h(\beta) \subseteq \mathcal{I}$ for some homomorphism h. A rule set R **entails** q, written R $\models q$, if $\mathcal{M} \models q$ for all $\mathcal{M} \models$ R.

Solving CQE: The two dimensions of infinity.

To determine if a query q is entailed by a rule set R, we have to check that $\mathcal{M} \models q$ for every model $\mathcal{M}$ of R. Alas, this is not easy!

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1. R may accept an infinite number of models.
2. Each one models may be of infinite size.

To address (1), we introduce the notion of **universal models** which can be used to solve conjunctive query entailment independently.

Solving CQE with Universal Models

Definition 1.8: An interpretation $\mathcal{U}$ is a **universal model** of a rule set R if

1. $\mathcal{U}$ is a model of R, and
2. for all $\mathcal{M} \models$ R, $h(\mathcal{U}) \subseteq \mathcal{M}$ for some homomorphism h.

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Proposition 1.9: If $\mathcal{U} \models q$ with $\mathcal{U}$ a universal model of a rule set R and q a query, then R $\models q$.

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Proof:

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2. There is some h' with $h'(\beta) \subseteq \mathcal{U}$ and β the body of q.

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3. By (1) and (2): $h \circ h'(\beta) \subseteq \mathcal{M}$. Hence, $\mathcal{M} \models q$.

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3. By (1) and (2): $h \circ h'(\beta) \subseteq \mathcal{M}$. Hence, $\mathcal{M} \models q$.
4. By (1) and (3): R $\models q$.

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Definition 1.10: An interpretation $\mathcal{U}$ is a **universal model** of a rule set R if

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Theorem 1.11: Consider a rule set R, a query q, and a universal model $\mathcal{U}$ for R. Then, R $\models q$ if and only if $\mathcal{U} \models q$.

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$\Longrightarrow$ Trivial, since $\mathcal{U}$ is a model of R.

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Proof:

$\Longrightarrow$ Trivial, since $\mathcal{U}$ is a model of R.

$\Longleftarrow$ By Proposition 9.

The Chase Algorithm

Definition 1.12: A **chase sequence** of a rule set R is a (possibly infinite) sequence of sets of facts $\mathsf{F}^0, \mathsf{F}^1, \ldots$ such that

1. F^0 is the empty set,
2. for all $i \geq 1$, $\mathsf{F}^i = \mathsf{F}^{i-1} \cup \rho_h(\mathsf{F}^{i-1})$ for some $\rho \in \mathsf{R}$ and homomorphism h, and
3. if, for some $i \geq 1$, a pair $\langle \rho, h \rangle$ with $\rho \in \mathsf{R}$ is applicable to F^i; then there is some $j \geq i$ such that $\langle \rho, h \rangle$ is not applicable to F^j (**fairness**).

A **chase** for R is some (possibly infinite) set that results from the union of all sets of facts in some chase sequence.

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A **chase** for R is some (possibly infinite) set that results from the union of all sets of facts in some chase sequence.

Given a rule set R, let $\mathcal{C}(\mathsf{R})$ be some arbitrarily chosen chase of R.

The Chase: A Simple Example

Example 1.13:

$$HasSon(x, y) \land HasSister(y, z) \to HasDaughter(x, z) \tag{7}$$

$$HasFather(x, y) \to HasSon(y, x) \tag{8}$$

$$Person(x) \to \exists y. HasFather(x, y) \tag{9}$$

$$Person(anakin) \tag{10}$$

$$HasFather(luke, anakin) \tag{11}$$

$$HasSister(luke, leia) \tag{12}$$

Solving CQE with the Chase

Lemma 1.14: For a rule set R, we have that $\mathcal{C}(\mathsf{R}) \models \mathsf{R}$.

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Proof: Proof by contradiction:

1. Let us assume that $\mathcal{C}(\mathsf{R}) \not\models \mathsf{R}$.

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3. By (1): There is some pair $\langle \rho, h \rangle$ with $\rho \in \mathsf{R}$ applicable to $\mathcal{C}$ (R).

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3. By (1): There is some pair $\langle \rho, h \rangle$ with $\rho \in \mathsf{R}$ applicable to $\mathcal{C}$ (R).
4. By (2) and (3): For all $i \geq 0$, $\langle \rho, h \rangle$ is applicable to F^i.

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3. By (1): There is some pair $\langle \rho, h \rangle$ with $\rho \in \mathsf{R}$ applicable to $\mathcal{C}(\mathsf{R})$.
4. By (2) and (3): For all $i \geq 0$, $\langle \rho, h \rangle$ is applicable to F^i.
5. By (2) and (4): The sequence $\mathsf{F}^0, \mathsf{F}^1, \ldots$ does not satisfy the fairness requirement introduced in Definition 12.

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3. By (1): There is some pair $\langle \rho, h \rangle$ with $\rho \in \mathsf{R}$ applicable to $\mathcal{C}$ (R).
4. By (2) and (3): For all $i \geq 0$, $\langle \rho, h \rangle$ is applicable to F^i.
5. By (2) and (4): The sequence $\mathsf{F}^0, \mathsf{F}^1, \ldots$ does not satisfy the fairness requirement introduced in Definition 12.
6. By (5): Assumption (1) results in a contradiction.

Solving CQE with the Chase

Theorem 1.15: A rule set R entails a query q iff $\mathcal{C}(\mathsf{R}) \models q$.

Proof: The theorem follows from Theorem 11 and the fact that $\mathcal{C}$ (R) is a universal model of R (proven below).

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Theorem 1.15: A rule set R entails a query q iff $\mathcal{C}(\mathsf{R}) \models q$.

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2. Let $\mathcal{M}$ be some model of R.

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4. In the following slide, we show that there is a sequence of homomorphisms $h_0, h_1, \ldots$ such that $h_i(\mathsf{F}^i) \subseteq \mathcal{M}$ and $h_{i+1} \supseteq h_i$ for all $i \geq 0$.

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5. Let $h = \bigcup h_i$ for all $i \geq 0$. Then, $h(\mathcal{C}(\mathsf{R})) \subseteq \mathcal{M}$ by (4).

Solving CQE with the Chase

Theorem 1.15: A rule set R entails a query q iff $\mathcal{C}(\mathsf{R}) \models q$.

Proof: The theorem follows from Theorem 11 and the fact that $\mathcal{C}$ (R) is a universal model of R (proven below).

1. By Lemma 14, $\mathcal{C}(\mathsf{R}) \models \mathsf{R}$.
2. Let $\mathcal{M}$ be some model of R.
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5. Let $h = \bigcup h_i$ for all $i \geq 0$. Then, $h(\mathcal{C}(\mathsf{R})) \subseteq \mathcal{M}$ by (4).
6. By (1), (2) and (5): $\mathcal{C}(\mathsf{R})$ is a universal model.

Solving CQE with the Chase

Theorem 1.16: A rule set R entails a query q iff $\mathcal{C}(\mathsf{R}) \models q$.

Proof: We show via induction that, for every $i \geq 0$, there is a homomorphism h_i with $h_i(\mathsf{F}_i) \subseteq \mathcal{M}$ and $h_i \supseteq h_{i+1}$.

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1. Base case: $h_0 = \emptyset$.
2. Induction step: Let $i \geq 1$
   1. Let $\langle \rho, h' \rangle$ be some pair with $\rho = \beta[\vec{x}, \vec{z}] \to \exists \vec{y}.\eta[\vec{x}, \vec{y}] \in \mathsf{R}$ and $\mathsf{F}^i = \rho_{h'}(\mathsf{F}^{i-1}) \cup \mathsf{F}^{i-1}$. Note that, $h'(\beta) \subseteq \mathsf{F}^{i-1}$.
   2. By (1) and IH: $h_{i-1}(h'(\beta)) \subseteq \mathcal{M}$.
   3. By (2) and $\mathcal{M} \models \mathsf{R}$: $h''(\eta) \subseteq \mathsf{F}^{i-1}$ for some $h'' \supseteq h_{i-1} \circ h'_{\vec{x}}$.
   4. $h_i \supseteq h_{i-1}$ is the smallest function mapping $h'(y)$ to $h''(y)$ for all $y \in \vec{y}$.

Brief recap

- Syntax and semantics
- Universal models
- The chase algorithm

What's next?

- Select your own papers!