FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

Semantics of RDF(S)

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Semantics of RDF(S)
Semantics of RDF(S)
 Agenda

1. Motivation and Considerations
2. Simple Entailment
3. RDF Entailment
4. RDFS Entailment
5. Downsides of RDF(S)
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1. Motivation and Considerations
2. Simple Entailment
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5. Downsides of RDF(S)
Why Formal Semantics?

- after introduction of RDF(S), criticism of tool developers: different tools were incompatible (despite the existing specification)
- e.g. triple stores:
  - same RDF document
  - same SPARQL query
  - different answers
- thus a model-theoretic formal semantics was defined for RDF(S)
How is RDF(S) Linked to a Logic?

- to start with: what are the sentences in RDF(S)?
  - basic elements (vocabulary V): IRIs, bnodes and literals (these are not sentences themselves)
  - every triple

\[(s, p, o) \in (\text{IRI} \cup \text{bnode}) \times \text{IRI} \times (\text{IRI} \cup \text{bnode} \cup \text{literal})\]

is a sentence

- every finite set of triples (denoted: graph) is a sentence
How is RDF(S) Linked to a Logic?

What is the semantics?

- consequence relation that defines when an RDF(S) graph $G'$ logically follows from an RDF(S) graph $G$, i.e. $G \models G'$
- model-theoretic semantics: we define a set of interpretations and stipulate under which conditions an interpretation is a model of a graph
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Semantics of RDF(S)

- we proceed stepwise:
  
  simple interpretations
Semantics of RDF(S)

- we proceed stepwise:

  - simple interpretations
  - RDF interpretations

the more we restrict the set of interpretations, the stronger the consequence relation becomes
Semantics of RDF(S)

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Semantics of RDF(S)

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Semantics of the Simple Entailment

Definition (Simple Interpretation)

A simple Interpretation \( \mathcal{I} \) for a vocabulary \( V \) consists of

- \( \text{IR} \), a non-empty set of resources, also referred to as domain, with
- \( \text{LV} \subseteq \text{IR} \) the set of literal values, that contains (at least) all untyped literals from \( V \), and
- \( \text{IP} \), the set of properties of \( \mathcal{I} \);
- \( \text{I}_S \), a function, mapping IRIs from \( V \) to the union of the sets \( \text{IR} \) and \( \text{IP} \), i.e., \( \text{I}_S : V \rightarrow \text{IR} \cup \text{IP} \),
- \( \text{I}_{\text{EXT}} \), a function, mapping every property to a set of pairs from \( \text{IR} \), i.e., \( \text{I}_{\text{EXT}} : \text{IP} \rightarrow 2^{\text{IR} \times \text{IR}} \), and
- \( \text{I}_L \), a function mapping typed literals from \( V \) into the set \( \text{IR} \) of resources.
Semantics of the Simple Entailment

- IR is also called domain or universe of discourse of $\mathcal{I}$
- $I_{\text{EXT}}(p)$ is also referred to as the extension of the property $p$

**Definition (interpretation function)**

Based on $I_L$ and $I_S$, we define $^\mathcal{I}$ as follows:

- every untyped literal "$a" is mapped to $a$ : $(^\mathcal{I}"a") = a$
- every untyped literal with language information "$a"@t$ is mapped to the pair $\langle a, t \rangle$, that is: $(^\mathcal{I}"a"@t) = \langle a, t \rangle$,
- every typed literal $l$ is mapped to $I_L(l)$, that is: $^\mathcal{I}l = I_L(l)$ and
- every IRI $i$ is mapped to $I_S(i)$, hence: $^\mathcal{I}i = I_S(i)$. 
Semantics of the Simple Entailment

Interpretation (schematic):
Semantics of the Simple Entailment

- Question: When is a given interpretation a model of a graph?
Semantics of the Simple Entailment

- Question: When is a given interpretation a model of a graph?
- … if it is a model for every triple of the graph!
Semantics of the Simple Entailment

- Question: When is a given interpretation a model of a graph?
- ... if it is a model for every triple of the graph!
Semantics of the Simple Entailment

- Question: When is a given interpretation a model of a triple?
Semantics of the Simple Entailment

- Question: When is a given interpretation a model of a triple?
- ... if all subject, predicate, and object are contained in V and additionally \( \langle s^\mathcal{I}, o^\mathcal{I} \rangle \in I_{\text{EXT}}(p^\mathcal{I}) \) holds
Semantics of Simple Entailment

schematically:

\[
\text{triple} \quad s \quad p \quad o
\]
Semantics of Simple Entailment

- … oops, we forgot the bnodes!
- let $A$ be a function mapping all bnodes to elements of IR
- given an interpretation $\mathcal{I}$, let $\mathcal{I} + A$ behave just like $\mathcal{I}$ on the vocabulary, and additionally for every bnode $\_:\text{label}$ let
  $$(\_:\text{label})^{\mathcal{I}+A} = A(\_:\text{label})$$
- now, an interpretation $\mathcal{I}$ is a model of an RDF graph $G$, if there exists an $A$ such that all triples are satisfied w.r.t. $\mathcal{I} + A$
Simple Interpretations: Example

given graph $G$:

and interpretation $I$:

$$
\begin{align*}
IR &= \{c, g, h, z, l, m, 1 \text{ lb}\} \\
IP &= \{h, z, m\} \\
LV &= \{1 \text{ lb}\} \\
I_{\text{EXT}} &= h \mapsto \{(c, l)\} \\
&\quad z \mapsto \{(l, g)\} \\
&\quad m \mapsto \{(l, 1 \text{ lb})\}
\end{align*}
$$

$I_S = \begin{array}{ll}
\text{ex:Chutney} & \mapsto c \\
\text{ex:greenMango} & \mapsto g \\
\text{ex:hasIngredient} & \mapsto h \\
\text{ex:ingredient} & \mapsto z \\
\text{ex:amount} & \mapsto m 
\end{array}$

$I_L$ is the “empty function”

Is $I$ a model of $G$?
Simple Interpretations: Example

\[
\begin{align*}
\text{IR} & = \{c, g, h, z, l, m, 1 \text{ lb}\} & \text{I}_S & = \text{ex:Chutney} \mapsto c \\
\text{IP} & = \{h, z, m\} & \text{ex:greenMango} \mapsto g \\
\text{LV} & = \{1 \text{ lb}\} & \text{ex:hasIngredient} \mapsto h \\
\text{I}_{\text{EXT}} & = h \mapsto \{(c, l)\} & \text{ex:ingredient} \mapsto z \\
& & \text{ex:amount} \mapsto m \\
& & \text{I}_L \text{ is the “empty function”}
\end{align*}
\]

- If we pick \(A: \_\text{:id1} \mapsto l\), then we get

\[
\begin{align*}
\langle \text{ex:Chutney}^{\mathcal{I}+A}, \_\text{:id1}^{\mathcal{I}+A} \rangle & = \langle c, l \rangle \in \text{I}_{\text{EXT}}(h) = \text{I}_{\text{EXT}}(\text{ex:hasIngredient}^{\mathcal{I}+A}) \\
\langle \_\text{:id1}^{\mathcal{I}+A}, \text{ex:greenMango}^{\mathcal{I}+A} \rangle & = \langle l, g \rangle \in \text{I}_{\text{EXT}}(z) = \text{I}_{\text{EXT}}(\text{ex:ingredient}^{\mathcal{I}+A}) \\
\langle \_\text{:id1}^{\mathcal{I}+A}, "1 \text{ lb}"^{\mathcal{I}+A} \rangle & = \langle l, 1 \text{ lb} \rangle \in \text{I}_{\text{EXT}}(m) = \text{I}_{\text{EXT}}(\text{ex:amount}^{\mathcal{I}+A})
\end{align*}
\]

- Therefore, \(\mathcal{I}\) is a model of \(G\).
Simple Entailment

- definition of simple interpretations fixes the notion of simple entailment for RDF graphs
- question: how can this (abstractly defined) semantics be turned something computable
- answer: deduction rules
Simple Entailment

deduction rules for simple entailment:

\[
\frac{u \ a \ x.}{u \ a \ \_':n.} \quad \text{se1}
\]

\[
\frac{u \ a \ x.}{\_':n \ a \ x.} \quad \text{se2}
\]

- precondition for applying this rule: the bnode has not already been associated with another IRI or literal
Simple Entailment

Theorem

A graph $G_2$ is simply entailed by a graph $G_1$ if $G_1$ can be extended to a graph $G'_1$ by applying the rules se1 and se2 such that $G_2$ is contained in $G'_1$.

Example.: the graph simply entails

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RDF interpretations

RDF interpretations are specific simple interpretations, where additional conditions are imposed on the URIs of the RDF vocabulary

\[
\text{rdf:type} \quad \text{rdf:Property} \quad \text{rdf:XMLLiteral} \quad \text{rdf:nil} \\
\text{rdf:List} \quad \text{rdf:Statement} \quad \text{rdf:subject} \quad \text{rdf:predicate} \\
\text{rdf:object} \quad \text{rdf:first} \quad \text{rdf:rest} \quad \text{rdf:Seq} \quad \text{rdf:Bag} \\
\text{rdf:Alt} \quad \text{rdf:1} \quad \text{rdf:2} \quad \ldots
\]

in order to realize their intended semantics.
Conditions for RDF Interpretations

An RDF interpretation for a vocabulary $V$ is a simple interpretation for the vocabulary $V \cup V_{RDF}$ that additionally satisfies the following conditions:

1. $x \in IP$ exactly if $\langle x, rdf:Property^I \rangle \in I_{EXT}(rdf:type^I)$. 

"For every triple predicate we can infer that it is an member of the class of all properties."
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“For every triple predicate we can infer that it is a member of the class of all properties.”

\[
\begin{array}{c}
\text{u a y} \\
\hline
\text{a rdf:type rdf:Property} \\
\end{array}
\text{ rdf1}
\]
Conditions for RDF Interpretations

2. If "s"^^rdf:XMLLiteral is contained in V and s is a well-formed XML literal, then
   - $I_L("s"^^rdf:XMLLiteral)$ is the XML value of s;
   - $I_L("s"^^rdf:XMLLiteral) \in LV$;
   - $\langle I_L("s"^^rdf:XMLLiteral), rdf:XMLLiteral^I \rangle \in I_{EXT}(rdf:type^I)$

\[ \underline{u a l} \quad \underline{r d f : t y p e \ r d f : X M L L i t e r a l} \quad ??? \quad \text{für 1 a well-formed XML literal} \]
Conditions for RDF Interpretations

2. If "s"^^\texttt{rdf:XMLLiteral} is contained in V and $s$ is a well-formed XML literal, then
   - $I_L("s"^^\texttt{rdf:XMLLiteral})$ is the XML value of $s$;
   - $I_L("s"^^\texttt{rdf:XMLLiteral}) \in LV$;
   - $\langle I_L("s"^^\texttt{rdf:XMLLiteral}), \texttt{rdf:XMLLiteral}^I \rangle \in I_{\text{EXT}}(\texttt{rdf:type}^I)$

Oops, literals must not occur in subject position!
2. If "s"^^rdf:XMLLiteral is contained in V and s is a well-formed XML literal, then
   - \( I_L("s"^^rdf:XMLLiteral) \) is the XML value of s;
   - \( I_L("s"^^rdf:XMLLiteral) \in LV; \)
   - \( \langle I_L("s"^^rdf:XMLLiteral), rdf:XMLLiteral^I \rangle \in I_{EXT}(rdf:type^I) \)
Conditions for RDF Interpretations

2. If "s"^^rdf:XMLLiteral is contained in V and s is a well-formed XML literal, then
   - \( I_L("s"^^rdf:XMLLiteral) \) is the XML value of s;
   - \( I_L("s"^^rdf:XMLLiteral) \) ∈ LV;
   - \( \langle I_L("s"^^rdf:XMLLiteral), rdf:XMLLiteral^I \rangle \) ∈ \( I_{EXT}(rdf:type^I) \)

\[\begin{array}{c}
\text{u a l} \\
\hline
\text{u a _:n}
\end{array}\]  \quad \text{lg} \quad \begin{array}{c}
\text{l a literal, _:_n} \\
\text{not bound otherwise}
\end{array}

\[\begin{array}{c}
\text{u a _:n} \\
\hline
\text{_:n rdf:type rdf:XMLLiteral}
\end{array}\]  \quad \text{rdf2} \quad \begin{array}{c}
\text{If rule lg has assigned _:_n to the XML Literal l}
\end{array}
3. If "s"^^rdf:XMLLiteral is contained in V and s is an ill-formed XML literal, then
   - $I_L("s"^^rdf:XMLLiteral) \not\in LV$ and
   - $\langle I_L("s"^^rdf:XMLLiteral), rdf:XMLLiteral^\mathcal{I} \rangle \not\in I_{EXT}(rdf:type^\mathcal{I})$. 
RDF Interpretations

- Note: $x$ is a property exactly if it is linked to the resource denoted by `rdf:Property` via the `rdf:type` property (this has the direct consequence that in every RDF interpretation holds $IP \subseteq IR$).

- The value space of the `rdf:XMLLiteral` datatype contains for every well-formed XML string exactly one so-called XML value. The RDF specs only stipulate that this value is neither an XML string itself nor a data value of any XML Schema datatype nor a Unicode string.
RDF Interpretations

- additional requirement: every RDF interpretation must be a model of the following "axiomatic" triples:

```
rdf:type               rdf:type   rdf:Property .
rdf:subject            rdf:type   rdf:Property .
rdf:.predicate         rdf:type   rdf:Property .
rdf:object             rdf:type   rdf:Property .
rdf:first              rdf:type   rdf:Property .
rdf:rest               rdf:type   rdf:Property .
rdf:value              rdf:type   rdf:Property .
rdf:_1                  rdf:type   rdf:Property .
rdf:_2                  rdf:type   rdf:Property .
...
rdf:nil                 rdf:type   rdf:List .
```

\[ u a x \]  every axiomatic triple \( u a x \) can always be derived
RDF Entailment

- Theorem: A graph $G_2$ is RDF-entailed by a graph $G_1$, if there is a graph $G'_1$, such that
  - $G'_1$ can be derived from $G_1$ via lg, rdf1, rdf2 and rdfax and
  - $G_2$ is simply entailed by $G'_1$.

- note: two-stage deduction process
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RDFS Interpretations

... RDFS interpretations are specific RDF interpretations, where additional constraints are imposed for the URIs of the RDFS vocabulary

- rdfs:domain
- rdfs:range
- rdfs:Resource
- rdfs:Literal
- rdfs:Datatype
- rdfs:Class
- rdfs:subClassOf
- rdfs:subPropertyOf
- rdfs:Container
- rdfs:member
- rdfs:ContainerMembershipProperty
- rdfs:comment
- rdfs:seeAlso
- rdfs:isDefinedBy
- rdfs:label

such that the intended semantics of these URIs is realized.
RDFS Interpretations

- for the sake of easier representation, we introduce – given an interpretation $\mathcal{I}$ – a function $I_{\text{CEXT}}$ that maps resources to sets of resources (thus: $I_{\text{CEXT}} : \mathbb{IR} \rightarrow 2^{\mathbb{IR}}$) by letting $I_{\text{CEXT}}(y)$ contain exactly those elements $x$, for which $\langle x, y \rangle$ is contained in $I_{\text{EXT}}(\text{rdf:type}^\mathcal{I})$. We call $I_{\text{CEXT}}(y)$ the (class) extension of $y$.

- moreover, we let $IC$ be the extension of the specific IRI $\text{rdfs:Class}$, hence: $IC = I_{\text{CEXT}}(\text{rdfs:Class}^\mathcal{I})$.

- note: both $I_{\text{CEXT}}$ as well as $IC$ are fully determined by $\mathcal{I}$ and $I_{\text{EXT}}$. 
RDFS Interpretations

An RDFS interpretation for a vocabulary $V$ is an RDF interpretation for the vocabulary $V \cup V_{RDFS}$, that additionally satisfies the following criteria:

- $IR = I_{CEXT}(rdfs:Resource^T)$
  Every resource is of type $rdfs:Resource$.

- $LV = I_{CEXT}(rdfs:Literal^T)$
  Every untyped and every well-formed typed literal is of type $rdfs:Literal$.

- If $\langle x, y \rangle \in I_{EXT}(rdfs:domain^T)$ and $\langle u, v \rangle \in I_{EXT}(x)$, then $u \in I_{CEXT}(y)$.
  If the property $rdfs:domain$ connects $x$ with $y$ and the property $x$ connects the resources $u$ and $v$, then $u$ is of type $y$. 
RDFS Interpretations

- If \( \langle x, y \rangle \in I_{\text{EXT}}(\text{rdfs:range}^I) \) and \( \langle u, v \rangle \in I_{\text{EXT}}(x) \), then \( v \in I_{\text{CEXT}}(y) \).

  If the property \text{rdfs:range} connects \( x \) with \( y \) and the property \( x \) connects the resources \( u \) and \( v \), then \( v \) is of type \( y \).

- \( I_{\text{EXT}}(\text{rdfs:subPropertyOf}^I) \) is reflexive and transitive on \( \mathcal{I} \).

  The \text{rdfs:subPropertyOf} property connects every property with itself. Moreover, if \text{rdfs:subPropertyOf} connects a property \( x \) with a property \( y \) and additionally \( y \) with a property \( z \), then \text{rdfs:subPropertyOf} also connects \( x \) directly with \( z \).
RDFS Interpretations

- If \( \langle x, y \rangle \in I^\text{EXT}(\text{rdfs:subPropertyOf}^I) \),
  then \( x, y \in I^P \) and \( I^\text{EXT}(x) \subseteq I^\text{EXT}(y) \).
  If \text{rdfs:subPropertyOf} connects \( x \) with \( y \), then both \( x \) and \( y \) are
  properties every pair of resources contained in the extension of \( x \) is also
  contained in the extension of \( y \).

- If \( x \in I^C \), then \( \langle x, \text{rdfs:Resource}^I \rangle \in I^\text{EXT}(\text{rdfs:subClassOf}^I) \).
  If \( x \) represents a class, then it has to be a subclass of the class of all
  resources, i.e., the pair containing \( x \) and \text{rdfs:Resource} is in the
  extension of \text{rdfs:subClassOf}.
RDFS Interpretations

- If $\langle x, y \rangle \in \text{I}_{\text{EXT}}(\text{rdfs:subClassOf}^I)$, then $x, y \in \text{IC}$ and $\text{I}_{\text{CEXT}}(x) \subseteq \text{I}_{\text{CEXT}}(y)$.

  If $x$ and $y$ are connected via the rdfs:subClassOf property, then both $x$ and $y$ are classes and the (class) extension of $x$ is a subset of the (class) extension of $y$.

- $\text{I}_{\text{EXT}}(\text{rdfs:subClassOf}^I)$ is reflexive and transitive on IC.

  The rdfs:subClassOf property connects every class to itself. Moreover, whenever this property connects a class $x$ with a class $y$ and a class $y$ with a class $z$, then it also directly connects $x$ with $z$. 
RDFS Interpretations

- If $x \in I_{\text{CEXT}}(\text{rdfs:ContainerMembershipProperty}^I)$, then $\langle x, \text{rdfs:member}^I \rangle \in I_{\text{EXT}}(\text{rdfs:subPropertyOf}^I)$. If $x$ is a property of the type $\text{rdfs:ContainerMembershipProperty}$, then it is $\text{rdfs:subPropertyOf}$-connected with the property $\text{rdfs:member}$.

- If $x \in I_{\text{CEXT}}(\text{rdfs:Datatype}^I)$, then $\langle x, \text{rdfs:Literal}^I \rangle \in I_{\text{EXT}}(\text{rdfs:subClassOf}^I)$. If some $x$ is typed as element of the class $\text{rdfs:Datatype}$, then it must be a subclass of the class of all literal values (denoted by $\text{rdfs:Literal}$).

- ... additionally we require satisfaction of the following axiomatic triples:
RDFS Interpretations

- `rdf:type` rdfs:domain rdfs:Resource.
- `rdfs:domain` rdfs:domain rdf:Property.
- `rdfs:range` rdfs:domain rdf:Property.
- `rdfs:subPropertyOf` rdfs:domain rdf:Property.
- `rdfs:subClassOf` rdfs:domain rdfs:Class.
- `rdfs:member` rdfs:domain rdfs:Resource.
- `rdf:first` rdfs:domain rdf:List.
- `rdf:rest` rdfs:domain rdf:List.
- `rdfs:seeAlso` rdfs:domain rdfs:Resource.
- `rdfs:isDefinedBy` rdfs:domain rdfs:Resource.
RDFS Interpretations

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<tr>
<th>RDF Property</th>
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</table>
RDFS Interpretations

rdfs:ContainerMembershipProperty  rdfs:subClassOf  rdf:Property .
rdf:Alt  rdfs:subClassOf  rdfs:Container .
rdf:Bag  rdfs:subClassOf  rdfs:Container .
rdf:Seq  rdfs:subClassOf  rdfs:Container .
rdfs:isDefinedBy  rdfs:subPropertyOf  rdfs:seeAlso .
rdf:XMLLiteral  rdf:type  rdfs:Datatype .
rdf:XMLLiteral  rdfs:subClassOf  rdfs:Literal .
rdfs:Datatype  rdfs:subClassOf  rdfs:Class .
rdf:_1  rdf:type  rdfs:ContainerMembershipProperty .
rdf:_1  rdfs:domain  rdfs:Resource .
rdf:_1  rdfs:range  rdfs:Resource .
rdf:_2  rdf:type  rdfs:ContainerMembershipProperty .

TU Dresden  Foundations of Semantic Web Technologies
RDFS Entailment

Automatic inference is again realized via deduction rules:

---

\[
\text{rdfsax} \quad \text{every axiomatic triple } "u \text{ a } x." \quad \text{can always be derived}
\]

\[
\text{u a } _:n. \quad \text{the converse of Rule lg: } _:n \text{ has been assigned (via Rule lg)}
\]

\[
\quad \text{to the untyped literal l}
\]

\[
\text{u a l.} \quad \text{rdfs1} \quad _:n \text{ has been assigned (via Rule lg) to the}
\]

\[
\quad \text{untyped literal l}
\]

\[
\text{a rdf:type rdfs:Literal} \quad \text{rdfs2} \quad \text{implements the semantics of}
\]

\[
\quad \text{property domains}
\]

\[
\text{a rdf:domain x. u a y.} \quad \text{rdfs3} \quad \text{implments the semantics of}
\]

\[
\quad \text{property ranges}
\]

---

\[
a, b \quad \text{IRIs}
\]

\[
u, v \quad \text{IRI or blank node}
\]

\[
x, y \quad \text{IRI, blank node or literal}
\]

\[
l \quad \text{literal}
\]

\[
_:n \quad \text{blank nodes}
\]
RDFS Entailment

\[
\begin{align*}
  & u \ a \ x . \\
  & u \ rdf:\text{type} \ rdfs:\text{Resource} . \quad \text{rdfs4a} \\
  \quad & u \ a \ v . \\
  \quad & v \ rdf:\text{type} \ rdfs:\text{Resource} . \quad \text{rdfs4b} \\
  \quad & u \ rdfs:\text{subPropertyOf} \ v . v \ rdfs:\text{subPropertyOf} \ x . \\
  \quad & u \ rdfs:\text{subPropertyOf} \ x . \\
  & u \ rdf:\text{type} \ rdf:\text{Property} . \\
  & u \ rdfs:\text{subPropertyOf} \ u . \quad \text{rdfs6} \quad \text{reflexivity} \\
  & a \ rdfs:\text{subPropertyOf} \ b . \quad u \ a \ y . \\
  & u \ b \ y . \quad \text{rdfs7} \quad \text{subproperty inferences} \\
  & u \ rdf:\text{type} \ rdfs:\text{Class} . \\
  & u \ rdf:\text{subClassOf} \ rdfs:\text{Resource} . \quad \text{rdfs8} \quad \text{classes contain only resources}
\end{align*}
\]

\hspace{1cm} \begin{align*}
  & \text{the subject of every triple is a resource} \\
  & \text{objects that are not literals are resources as well} \\
  & \text{transitivity} \\
  & \text{reflexivity} \\
  & \text{subproperty inferences for instances} \\
  & \text{classes contain only resources}
\end{align*}
RDFS Entailment

\[
\begin{align*}
\text{rdfs9} & : \quad \text{subclassen inferences for instances} \\
\text{rdfs10} & : \quad \text{reflexivity} \\
\text{rdfs11} & : \quad \text{transitivity} \\
\text{rdfs12} & : \quad \text{every datatype is a subclass of} \\
\end{align*}
\]
RDFS Entailment

- **important definition: XML clash**

```
ex:hasSmiley  rdfs:range  rdfs:Literal.
ex:evilRemark  ex:hasSmiley  ">:->"^^rdf:XMLLiteral.
```

- **occurs if a node of type `rdfs:Literal` gets assigned an ill-formed literal value**
RDFS Entailment

Theorem:
A graph $G_2$ is RDFS entailed by $G_1$, if there is a graph $G'_1$ obtained by applying the rules lg, gl, rdfax, rdf1, rdf2, rdfs1 – rdfs13 and rdfsax to $G_1$, such that

- $G_2$ is simply entailed by $G'_1$ or
- $G'_1$ contains an XML clash.
Agenda

1. Motivation and Considerations
2. Simple Entailment
3. RDF Entailment
4. RDFS Entailment
5. Downsides of RDF(S)
What RDF(S) Cannot Do

- Certain seemingly sensible consequences are not RDFS-entailed, e.g.

  \[\text{ex:talksTo rdfs:domain ex:Homo} .\]
  \[\text{ex:Homo rdfs:subClassOf ex:Primates} .\]

  should imply

  \[\text{ex:talksTo rdfs:domain ex:Primates} .\]

- possible solution: use a stronger, so-called “extensional” semantics (but this would be outside the standard)

- no possibility to express negation