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(based on slides by Stefan Woltran and Sarah Gaggl)

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# Abstract Argumentation

Lecture 10, 8th Jan 2024 // Foundations of Knowledge Representation, WS 2023/24

# Overview

Overall Process

Argumentation Frameworks

AFs – Semantics

AFs – Outlook

# Overall Process

# Introduction

## Argumentation:

The study of processes “concerned with how assertions are **proposed**, **discussed**, and **resolved** in the context of issues upon which several **diverging opinions** may be held”.

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]

## Formal Models of Argumentation are concerned with

- representation of an argument (i.e. an expression of opinion)
- representation of the relationship between arguments
- resolving conflicts between the arguments (“acceptability”)

# Overall Process

The overall process of using argumentation frameworks consists of the steps listed below.

Starting point: Knowledge base

1. Form arguments
2. Identify conflicts
3. Abstract from internal structure
4. Resolve conflicts
5. Draw conclusions

# Overall Process – Form Arguments

Consider the following **knowledge base**:

Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

From this, form **arguments**:

$$\langle \{w, w \rightarrow \neg s\}, \neg s \rangle$$

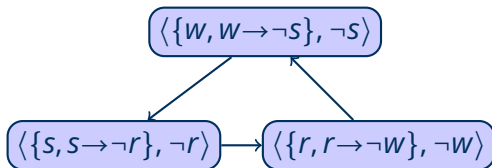
$$\langle \{s, s \rightarrow \neg r\}, \neg r \rangle$$

$$\langle \{r, r \rightarrow \neg w\}, \neg w \rangle$$

# Overall Process – Identify Conflicts

## Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

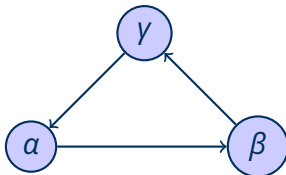


# Overall Process – Abstract from Internal Structure

Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

$F_{\Delta}$ :



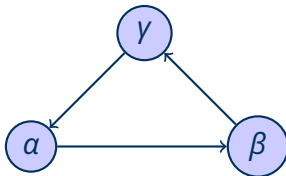


# Overall Process – Resolve Conflicts

Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

$F_\Delta$ :



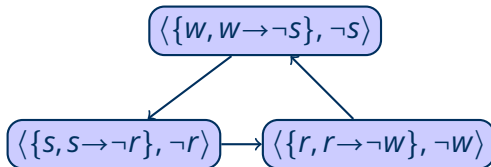
$$\text{pref}(F_\Delta) = \{\emptyset\}$$

$$\text{stage}(F_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$$

# Overall Process – Draw Conclusions

## Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$



$$Cn_{pref}(F_{\Delta}) = Cn(\top)$$

$$Cn_{stage}(F_{\Delta}) = Cn(\neg r \vee \neg w \vee \neg s)$$

# The Overall Process (ctd.)

## Some Remarks

- Main idea dates back to the seminal work of Phan Minh Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, ...)
- Separation between logical (forming arguments) and non-monotonic reasoning (“abstract argumentation frameworks”)
- Abstraction allows to compare several KR formalisms on a conceptual level (“calculus of conflict”)

## Main Challenge

- All steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

# Approaches to Form Arguments

## Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions)  $\Delta$
- An **argument** is a pair  $(\Phi, \alpha)$ , such that  $\Phi \subseteq \Delta$  is consistent,  $\Phi \models \alpha$  and for no  $\Psi \subsetneq \Phi$ ,  $\Psi \models \alpha$
- An argument  $(\Phi, \alpha)$  **attacks** argument  $(\Phi', \alpha')$  iff  $\Phi' \cup \{\alpha\}$  is inconsistent

## Example



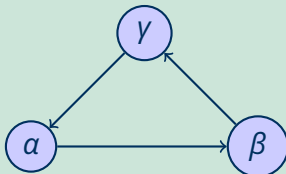
## Other Approaches:

- arguments are trees (or directed acyclic graphs) of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.

# Argumentation Frameworks

# Dung's Abstract Argumentation Frameworks

## Example



## Main Properties

- Abstract from the concrete content of arguments; only consider the **relation** between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful formalism
- Most active research area in the field of argumentation

# Dung's Abstract Argumentation Frameworks

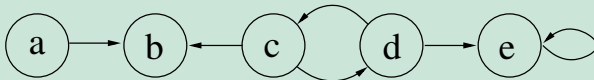
## Definition

An **argumentation framework** (AF) is a pair  $F = (A, R)$  where

- $A$  is a set of arguments,
- $R \subseteq A \times A$  is a relation representing the conflicts ("attacks").

## Example

$$F = ( \{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\} )$$



# Basic Properties (1)

## Definition (Conflict-Free Sets)

Let  $F = (A, R)$  be an AF.

A set  $S \subseteq A$  is **conflict-free** in  $F$  iff for each  $a, b \in S$  we have  $(a, b) \notin R$ .



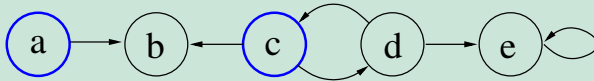
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## Example



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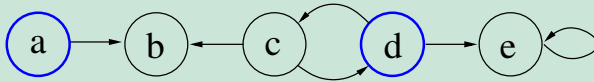
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## Example



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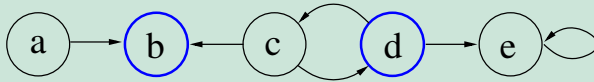
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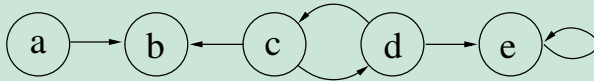
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## Example



$$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$$

# Basic Properties (2)

Definition (Admissible Sets [Dung, 1995])

Let  $F = (A, R)$  be an AF.

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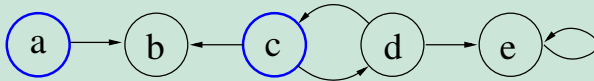
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Example



$adm(F) = \{\{a, c\},$

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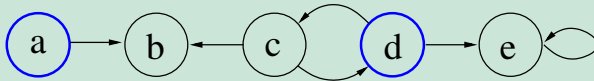
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Example



$adm(F) = \{\{a, c\}, \{a, d\},$

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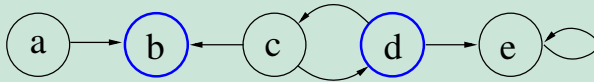
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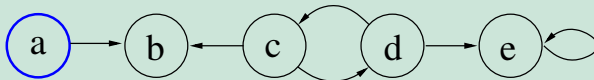
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Example



$$\text{adm}(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$$

# Basic Properties (3)

## Dung's Fundamental Lemma

Let  $S$  be admissible in an AF  $F$  and  $a, a'$  arguments in  $F$  defended by  $S$  in  $F$ . Then,

1.  $S' = S \cup \{a\}$  is admissible in  $F$ .
2.  $a'$  is defended by  $S'$  in  $F$ .

# AFs – Semantics

# Naive

## Definition (Naive Sets/Extensions)

Let  $F = (A, R)$  be an AF.

A set  $S \subseteq A$  is **naive** in  $F$  iff

- $S$  is conflict-free in  $F$ ,
- there is no conflict-free  $T \subseteq A$  in  $F$  such that  $S \subsetneq T$ .

Naive sets are subset-maximally conflict-free sets.

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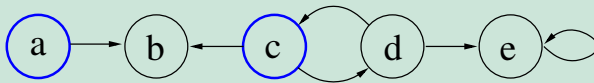
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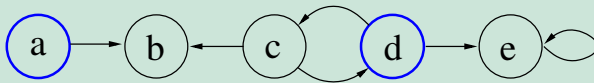
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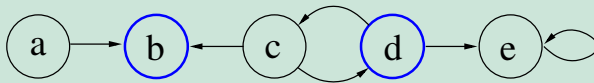
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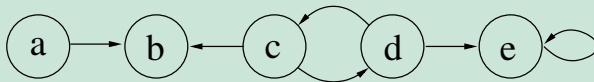
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## Example



$$\text{naive}(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$$



# Grounded

Definition (Grounded Extension [Dung, 1995])

Let  $F = (A, R)$  be an AF. The unique **grounded extension** of  $F$  is defined as the outcome  $S$  (initially empty) of the following “algorithm”:

1. put each argument  $a \in A$  that is not attacked in  $F$  into  $S$ ; if no such arguments exist, return  $S$ ;
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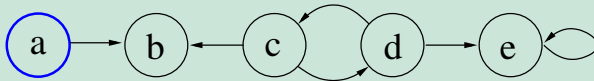
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Example



$ground(F) = \{\{a\}\}$

# Complete

Definition (Complete Extension [Dung, 1995])

Let  $(A, R)$  be an AF.

A set  $S \subseteq A$  is **complete** in  $F$  iff

- $S$  is admissible in  $F$ ,
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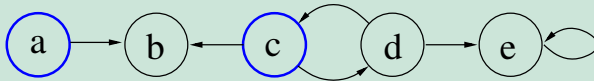
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Example



$comp(F) = \{\{a, c\},$

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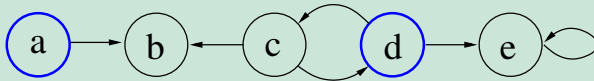
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## Example



$$\text{comp}(F) = \{\{a, c\}, \{a, d\},$$

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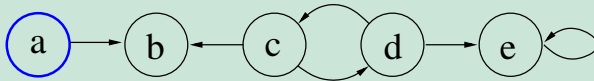
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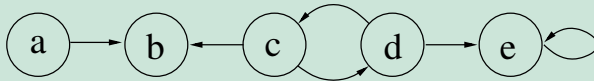
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Example



$comp(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$

# Grounded vs. Complete

## Properties of the Grounded Extension

For any AF  $F$ , the grounded extension of  $F$  is the subset-least complete extension of  $F$ .



# Grounded vs. Complete

## Properties of the Grounded Extension

For any AF  $F$ , the grounded extension of  $F$  is the subset-least complete extension of  $F$ .

## Remark

Since there exists exactly one grounded extension for each AF  $F$ , we often write  $ground(F) = S$  instead of  $ground(F) = \{S\}$ .

# Preferred

Definition (Preferred Extensions [Dung, 1995])

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A set  $S \subseteq A$  is a **preferred extension** of  $F$  iff

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Preferred extensions are subset-maximally admissible sets.

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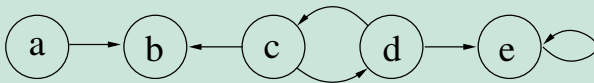
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Example



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# Stable

Definition (Stable Extensions [Dung, 1995])

Let  $F = (A, R)$  be an AF.

A set  $S \subseteq A$  is a **stable extension** of  $F$  iff

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# Stable

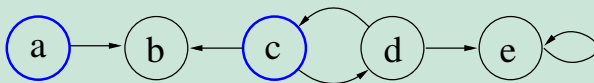
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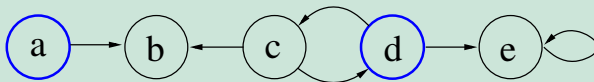
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## Example



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# Stable

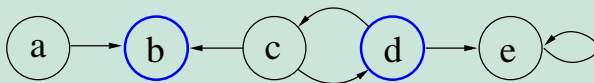
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A set  $S \subseteq A$  is a **stable extension** of  $F$  iff

- $S$  is conflict-free in  $F$ ,
- for each  $a \in A \setminus S$ , there exists a  $b \in S$  such that  $(b, a) \in R$ .

## Example



$$\text{stable}(F) = \{\{a, c\}, \{a, d\}, \{b, d\},$$

# Stable

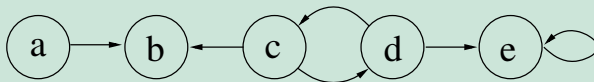
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## Example



$$\text{stable}(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset, \}$$



# Relationships Between Semantics

## Proposition

For any AF  $F$  the following relations hold:

1. Each stable extension of  $F$  is also a preferred one;
2. Each preferred extension of  $F$  is also a complete one;
3. Each complete extension of  $F$  is admissible in  $F$ .

# Semi-Stable

Definition (Semi-Stable Extensions [Caminada, 2006])

Let  $F = (A, R)$  be an AF.

A set  $S \subseteq A$  is a **semi-stable extension** of  $F$  iff

- $S$  is admissible in  $F$ ,
- there is no admissible  $T \subseteq A$  in  $F$  such that  $S^+ \subsetneq T^+$ , where
  - for  $S \subseteq A$ , define  $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$ .

Defined as **admissible stages** by Verheij [1996].

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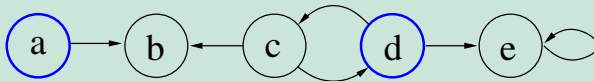
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Defined as **admissible stages** by Verheij [1996].

Example



$semi(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$

# Stage

## Definition (Stage Extensions [Verheij, 1996])

Let  $F = (A, R)$  be an AF.

A set  $S \subseteq A$  is a **stage extension** of  $F$  iff

- $S$  is conflict-free in  $F$ ,
- there is no conflict-free  $T \subseteq A$  in  $F$  such that  $S^+ \subsetneq T^+$ .
  - Recall:  $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$ .

$S^+$  is also called the **range** of  $S$ . Thus:

- Semi-stable extensions are range-maximally admissible sets.
- Stage extensions are range-maximally conflict-free sets.

# Ideal

Definition (Ideal Extension [Dung, Mancarella & Toni 2007])

Let  $F = (A, R)$  be an AF.

A set  $S \subseteq A$  is an **ideal extension** of  $F$  iff

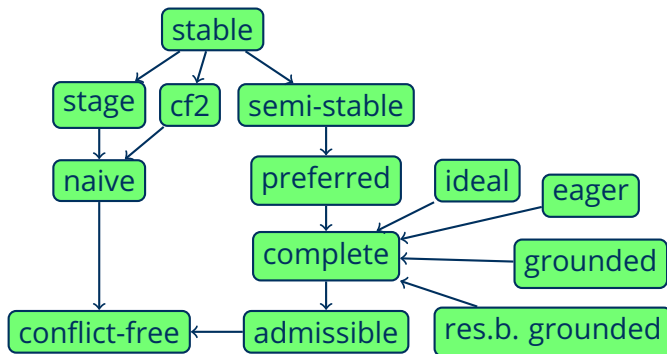
- $S$  is admissible in  $F$  and contained in each preferred extension of  $F$ ,
- there is no  $T \supsetneq S$  admissible in  $F$  and contained in each of  $\text{pref}(F)$ .

## Properties of Ideal Extensions

For any AF  $F$  the following observations hold:

1. There exists exactly one ideal extension of  $F$ .
2. The ideal extension of  $F$  is also a complete one.

# Relations Between Semantics



An arrow from semantics  $\sigma$  to semantics  $\tau$  means that each  $\sigma$ -extension is also a  $\tau$ -extension.

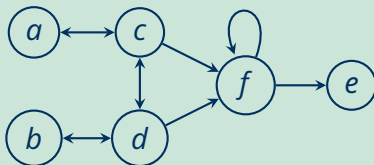
# AFs – Outlook

# Characteristics of Argumentation Semantics

## Example

$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$

$naive(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b, e\}\}$



## Natural Questions $\rightsquigarrow$ Realizability

- How to change the AF if we want  $\{a, b, e\}$  instead of  $\{a, b\}$  in  $pref(F)$ ?
- How to change the AF if we want  $\{a, b, d\}$  instead of  $\{a, b\}$  in  $pref(F)$ ?
- Can we have equivalent AFs without argument  $f$ ?



# Some Properties ...

## Theorem

For any AFs  $F$  and  $G$ , we have

- $adm(F) = adm(G)$  implies  $\sigma(F) = \sigma(G)$ , for  $\sigma \in \{pref, ideal\}$ ;
- $comp(F) = comp(G)$  implies  $\theta(F) = \theta(G)$ , for  $\theta \in \{pref, ideal, ground\}$ ;
- no other such relation between the different semantics ( $adm, pref, ideal, semi, ground, comp, stable$ ) in terms of standard equivalence holds.

# Decision Problems on AFs

## Credulous Acceptance

$\text{Cred}_\sigma$ : Given AF  $F = (A, R)$  and  $a \in A$ ;  
is  $a$  contained in **at least one**  $\sigma$ -extension of  $F$ ?

## Skeptical Acceptance

$\text{Skept}_\sigma$ : Given AF  $F = (A, R)$  and  $a \in A$ ;  
is  $a$  contained in **every**  $\sigma$ -extension of  $F$ ?

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted.<sup>1</sup>

---

<sup>1</sup>This is only relevant for stable semantics.

# Decision Problems on AFs

Hence we are also interested in the following problem:

## Skeptically and Credulously Accepted

$\text{Skept}'_{\sigma}$ : Given AF  $F = (A, R)$  and  $a \in A$ ;  
is  $a$  contained in **every** and **at least one**  $\sigma$ -extension of  $F$ ?

# Further Decision Problems

Verifying an extension

$\text{Ver}_\sigma$ : Given AF  $F = (A, R)$  and  $S \subseteq A$ ;  
is  $S$  a  $\sigma$ -extension of  $F$ ?

Does there exist an extension?

$\text{Exists}_\sigma$ : Given AF  $F = (A, R)$ ;  
Does there exist a  $\sigma$ -extension for  $F$ ?

Does there exist a nonempty extensions?

$\text{Exists}_\sigma^{-\emptyset}$ : Given AF  $F = (A, R)$ ;  
Does there exist a non-empty  $\sigma$ -extension for  $F$ ?

# Conclusion

- Abstract argumentation frameworks are constructed from KBs.
- Edges (attacks) between nodes (arguments) express (directed) conflicts.
- A variety of semantics for AFs try to make sense of acceptability:
  - Complete
  - Grounded
  - Preferred
  - Stable
  - ...
- Various inclusion relationships between the semantics hold, as they build on similar notions.