WCS and its Applications in Human Reasoning

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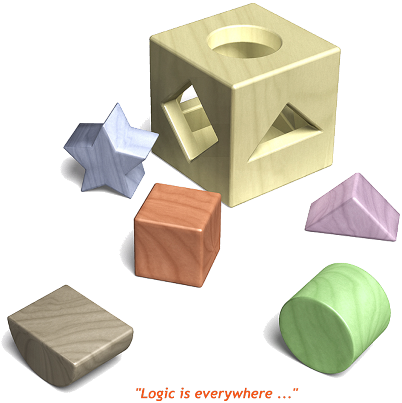
- Introduction
- Weak Completion Semantics
- Applications
- Open Questions



Human Reasoning

- Kowalski: Computational Logic and Human Life: How to be Artificially Intelligent. Cambridge University Press: 2011
- **Notice in London Underground**
  - **If there is an emergency then you press the alarm signal bottom**
    The driver will stop if any part of the train is in a station

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- **Observations**
  - **Intended meaning differs from literal meaning**
  - **Rigid adherence to classical logic is no help in modeling the examples**
  - **There seems to be a reasoning process towards more plausible meanings**

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  - **Rigid adherence to classical logic is no help in modeling the examples**
  - **There seems to be a reasoning process towards more plausible meanings**
    - **The driver will stop the train in a station**
      if the driver is alerted to an emergency
      and any part of the train is in the station

The Approach by Stenning and van Lambalgen

- Stenning, van Lambalgen: Human Reasoning and Cognititve Science
  MIT Press: 2008
- **I. Reasoning towards an appropriate representation**
- **II. Reasoning with respect to the least model of the representation**

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- **I. Reasoning towards an appropriate representation**
  - **Logic programs**
  - **Conditionals are represented as licences for implications**
- **II. Reasoning with respect to the least model of the representation**
  - **Some technical claims turned out to be false**

Logic Programs

- **Program clauses**

$$A \leftarrow B_1, \ldots, B_n \; (n > 0) \qquad A \leftarrow \top \qquad A \leftarrow \bot$$

- **Let $\mathcal{P}$ be a finite datalog program and $g\mathcal{P}$ the set of its ground instances**

Logic Programs

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$$A \leftarrow B_1, \ldots, B_n \ (n > 0) \qquad A \leftarrow \top \qquad A \leftarrow \bot$$

- **Let $\mathcal{P}$ be a finite datalog program and $g\mathcal{P}$ the set of its ground instances**
- **Let $\mathcal{S}$ be a finite set of ground literals**

$$def(\mathcal{S}, \mathcal{P}) = \{A \leftarrow body \in g\mathcal{P} \mid A \in \mathcal{S} \vee \neg A \in \mathcal{S}\}$$

Weak Completion

- For each defined atom A, replace all clauses of the form

$$A \leftarrow body_1, \ \ldots, \ A \leftarrow body_m$$

occurring in $g\mathcal{P}$ by

$$A \leftarrow body_1 \vee \ldots \vee body_m$$

- Replace all occurrences of $\leftarrow$ by $\leftrightarrow$
- The obtained program is called weak completion of $\mathcal{P}$ or $wc\mathcal{P}$

Interpretations and Models

- Let F be a formula
- An interpretation is a mapping
  from the set of formulas into the set of truth values
- A model for F is an interpretation mapping F to $\top$

Łukasiewicz Logic

- Łukasiewicz: O logice trójwartościowej. Ruch Filozoficzny 5, 169-171: 1920

$\wedge$	$\top$	**U**	$\bot$
$\top$	$\top$	**U**	$\bot$
U	**U**	**U**	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	**U**	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	**U**	**U**
$\bot$	$\top$	**U**	$\bot$

$\leftarrow$	$\top$	**U**	$\bot$
$\top$	$\top$	$\top$	$\top$
U	**U**	$\top$	$\top$
$\bot$	$\bot$	**U**	$\top$

$\leftrightarrow$	$\top$	**U**	$\bot$
$\top$	$\top$	**U**	$\bot$
U	**U**	$\top$	**U**
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$\top$	$\top$	$\top$	$\top$
U	$\top$	**U**	**U**
$\bot$	$\top$	**U**	$\bot$

$\leftarrow$	$\top$	**U**	$\bot$
$\top$	$\top$	$\top$	$\top$
U	**U**	$\top$	$\top$
$\bot$	$\bot$	**U**	$\top$

$\leftrightarrow$	$\top$	**U**	$\bot$
$\top$	$\top$	**U**	$\bot$
U	**U**	$\top$	**U**
$\bot$	$\bot$	**U**	$\top$

- **Let**

$$\langle I^\top, I^\bot \rangle \quad \cap \quad \langle J^\top, J^\bot \rangle \quad = \quad \langle I^\top \cap J^\top, I^\bot \cap J^\bot \rangle$$

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$\leftarrow$	$\top$	**U**	$\bot$
$\top$	$\top$	$\top$	$\top$
U	**U**	$\top$	$\top$
$\bot$	$\bot$	**U**	$\top$

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- **Let**

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- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics
  In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009

- **Theorem 1** **The intersection of all Ł-models of $\mathcal{P}$ is an Ł-model of $\mathcal{P}$**

A Semantic Operator

- Stenning, van Lambalgen: Human Reasoning and Cognititve Science
  MIT Press: 2008

- $\Phi_{\mathcal{P}}(I) = \langle J^\top, J^\bot \rangle$, where

$$\begin{array}{lcl} J^\top & = & \{A \mid \text{there exists } A \leftarrow body \in g\mathcal{P} \text{ with } I(body) = \top\} \\ J^\bot & = & \{A \mid \text{there exists } A \leftarrow body \in g\mathcal{P} \text{ and} \\ & & \quad \text{for all } A \leftarrow body \in g\mathcal{P} \text{ we find } I(body) = \bot\} \end{array}$$

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- **Theorem 2** The least fixed point of $\Phi_{\mathcal{P}}$ is the least Ł-model of $wc\mathcal{P}$

A Semantic Operator

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- $\Phi_{\mathcal{P}}(I) = \langle J^{\top}, J^{\perp} \rangle$, where

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  In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009
- **Theorem 2** The least fixed point of $\Phi_{\mathcal{P}}$ is the least Ł-model of $wc\mathcal{P}$
- **Notation** $\mathcal{M}_{\mathcal{P}}$ denotes the least Ł-model of $wc\mathcal{P}$
  $\mathcal{P} \models_{wcs} F$ iff $\mathcal{M}_{\mathcal{P}}(F) = \top$

Contractions

- H., Kencana Ramli: Contraction Properties of a Semantic Operator for Human Reasoning. In: Li, Yen (eds), Proc. 5th Int. Conf. on Information 228-231: 2009
- **Theorem 3** **If $\mathcal{P}$ is acyclic, then $\Phi_{\mathcal{P}}$ is a contraction**
- **Observation** **The theorem does not extend to acceptable programs**

A Connectionist Realization

- H., Kencana Ramli: Logics and Networks for Human Reasoning
  In: Alippi et.al. (eds), Artificial Neural Networks – ICANN, LNCS 5649, 464-478: 2009
- **Theorem 4** **For each $\mathcal{P}$ there exists a recurrent connectionist network which will converge to a stable state representing $\mathcal{M}_{\mathcal{P}}$ if initialized with the empty interpretation**

Relation to Well-Founded Semantics

- Dietz, H., Wernhard: Modelling the Suppression Task under Weak Completion and Well-Founded Semantics. Journal of Applied Non-Classical Logics 24, 61-85: 2014
- **Let $\mathcal{P}^+ = \mathcal{P} \setminus \{A \leftarrow \bot \mid A \leftarrow \bot \in \mathcal{P}\}$**
- **Let u be a new nullary relation symbol not occurring in $\mathcal{P}$**
- **Let $\mathcal{P}^* = \mathcal{P}^+ \cup \{B \leftarrow u \mid def(B, \mathcal{P}) = \emptyset\} \cup \{u \leftarrow \neg u\}$**
- **Theorem 5** **If $\mathcal{P}$ does not contain a positive loop then $\mathcal{M}_{\mathcal{P}}$ and the well-founded model for $\mathcal{P}^*$ coincide**

Abduction

- Consider the abductive framework $\langle \mathcal{P}, \mathcal{A}_{\mathcal{P}}, \mathcal{IC}, \models_{wcs} \rangle$, where
  - $\mathcal{A}_{\mathcal{P}} = \{A \leftarrow \top \mid def(A, \mathcal{P}) = \emptyset\} \cup \{A \leftarrow \bot \mid def(A, \mathcal{P}) = \emptyset\}$ is the set of abducibles
  - $\mathcal{IC}$ is a finite set of integrity constraints, i.e., expressions of the form $\bot \leftarrow B_1 \wedge \ldots \wedge B_n$

Abduction

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  - $\mathcal{IC}$ is a finite set of integrity constraints, i.e., expressions of the form $\bot \leftarrow B_1 \wedge \ldots \wedge B_n$
- An observation $\mathcal{O}$ is a set of ground literals
  - $\mathcal{O}$ is explainable in $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$
    iff there exists a minimal $\mathcal{E} \subseteq \mathcal{A}_\mathcal{P}$ called explanation such that $\mathcal{M}_{\mathcal{P} \cup \mathcal{E}}$ satisfies $\mathcal{IC}$ and $\mathcal{P} \cup \mathcal{E} \models_{wcs} L$ for each $L \in \mathcal{O}$

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    $\mathcal{M}_{\mathcal{P} \cup \mathcal{E}}$ satisfies $\mathcal{IC}$ and $\mathcal{P} \cup \mathcal{E} \models_{wcs} L$ for each $L \in \mathcal{O}$
  - F follows creduluously from $\mathcal{P}$ and $\mathcal{O}$
    iff there exists an explanantion $\mathcal{E}$ such that $\mathcal{P} \cup \mathcal{E} \models_{wcs} F$
  - F follows skeptically from $\mathcal{P}$ and $\mathcal{O}$
    iff for all explanantions $\mathcal{E}$ we find $\mathcal{P} \cup \mathcal{E} \models_{wcs} F$

Revision

- Dietz, H. 2015: A New Computational Logic Approach to Reason with Conditionals. In: Calimeri et.al. (eds), Logic Programming and Nonmonotonic Reasoning, LPNMR, LNAI 9345: 2015
- **Let $\mathcal{S}$ be a finite and consistent set of ground literals**

$$\boldsymbol{rev}(\mathcal{P}, \mathcal{S}) = (\mathcal{P} \setminus \boldsymbol{def}(\mathcal{S}, \mathcal{P})) \cup \{A \leftarrow \top \mid A \in \mathcal{S}\} \cup \{A \leftarrow \bot \mid \neg A \in \mathcal{S}\}$$

is called the revision of $\mathcal{P}$ with respect to $\mathcal{S}$

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- **Proposition 6**
  - ***rev* is nonmonotonic,**
    i.e., there exist $\mathcal{P}$, $\mathcal{S}$ and F such that $\mathcal{P} \models_{wcs} F$ and $rev(\mathcal{P}, \mathcal{S}) \not\models_{wcs} F$
  - **If $\mathcal{M}_{\mathcal{P}}(L) = U$ for all $L \in \mathcal{S}$, then *rev* is monotonic**
  - $\mathcal{M}_{rev(\mathcal{P},\mathcal{S})}(\mathcal{S}) = \top$

The Suppression Task – Part I

- Byrne: Suppressing Valid Inferences with Conditionals. Cognition 31, 61-83: 1989
- **Conditionals**

  LE If she has an essay to write then she will study late in the library
  LT If she has a textbook to read then she will study late in the library
  LO If the library stays open then she will study late in the library

- **Facts** E She has an essay to write
  ¬E She does not have an essay to write
- Will she study late in the library? □ yes □ no □ I don't know

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Conditionals	Facts	Yes	No
LE	E	96%	
LE & LT	E	96%	
LE & LO	E	38%	
LE	¬E		46%
LE & LT	¬E		4%
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Classical logic is inadequat!

The Suppression Task – Part II

- **Conditionals**

LE	If she has an essay to write then she will study late in the library
LT	If she has a textbook to read then she will study late in the library
LO	If the library stays open then she will study late in the library

- **Facts**

L	She will study late in the library
¬L	She will not study late in the library

- Has she an essay to write? ☐ yes ☐ no ☐ I don't know

The Suppression Task – Part II

- **Conditionals**
  LE If she has an essay to write then she will study late in the library
  LT If she has a textbook to read then she will study late in the library
  LO If the library stays open then she will study late in the library

- **Facts** L She will study late in the library
  ¬L She will not study late in the library

- Has she an essay to write? ☐ yes ☐ no ☐ I don't know

Conditionals	Facts	Yes	No
LE	L	53%	
LE & LT	L	16%	
LE & LO	L	55%	
LE	¬L		69%
LE & LT	¬L		69%
LE & LO	¬L		44%

Reasoning Towards an Appropriate Representation

- Stenning, van Lambalgen: Human Reasoning and Cognititve Science. MIT Press: 2008
- **Represent conditionals as licences for implications**

  LE & E $\{\ell \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ e \leftarrow \top\}$

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LE & LT & E	$\{\ell \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ \ell \leftarrow t \wedge \neg ab_2,\ ab_2 \leftarrow \bot,\ e \leftarrow \top\}$

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- **Reason about additional premises**

$$\text{LE \& LO \& E} \quad \{\ell \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \neg o,\ \ell \leftarrow o \wedge \neg ab_2,\ ab_2 \leftarrow \neg e,\ e \leftarrow \top\}$$

Reasoning with respect to the Least Ł-Model of $wc\mathcal{P}$

- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics. In: Hill, Warren (eds), Logic Programming, LNCS 5649, 464-478: 2009
- **LE & E**

$$\begin{aligned} & wc\{\ell \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ e \leftarrow \top\} \\ = & \{\ell \leftrightarrow e \wedge \neg ab_1,\ ab_1 \leftrightarrow \bot,\ e \leftrightarrow \top\} \end{aligned}$$

  - **Its least Ł-model $\langle\{e, \ell\}, \{ab_1\}\rangle$ assigns $\top$ to e, ℓ and $\bot$ to ab_1**
  - **It does entail ℓ**

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- **LE & LO & E**

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  - **Its least Ł-model $\langle\{e\}, \{ab_2\}\rangle$ assigns $\top$ to e, $\bot$ to ab_2, and U to ℓ, o, ab_1**
  - **It does not entail ℓ**

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- **LE & LO & E**

$$wc\{\ell \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \neg o,\ \ell \leftarrow o \wedge \neg ab_2,\ ab_2 \leftarrow \neg e,\ e \leftarrow \top\}$$
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  - **It does not entail ℓ**
- **WCS appears to be adequate!**

Abduction

- H., Philipp, Wernhard: An Abductive Model for Human Reasoning
  In: Proceedings of the 10th International Symposium on Logical Formalizations of Commonsense Reasoning (CommonSense): 2011
- Dietz, H., Ragni: A Computational Logic Approach to the Suppression Task
  In: Proceedings of the 34th Annual Conference of the Cognitive Science Society, Miyake et.al. (eds.), 1500-1505: 2012
- **Abduction is needed to solve part II of the suppression task**
  - **LE & LT & L** $\{\ell \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ \ell \leftarrow t \wedge \neg ab_2,\ ab_2 \leftarrow \bot\}$
  - **Observation** ℓ
  - **Set of abducibles** $\{e \leftarrow \top,\ e \leftarrow \bot,\ t \leftarrow \top,\ t \leftarrow \bot\}$
  - **Explanations** $\{e \leftarrow \top\}$ **and** $\{t \leftarrow \top\}$
  - **Reasoning credulously we conclude** e
  - **Reasoning skeptically we cannot conclude** e
  - Byrne 1989 **only 16% conclude** e

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- **Skeptical reasoning appears to be adequate!**

The Selection Task – Abstract Case

- Wason: Reasoning about a Rule
  The Quarterly Journal of Experimental Psychology 20, 273-281: 1968

- **Consider cards which have a letter on one side and a number on the other side**

D	**F**	**3**	**7**

- **Consider the rule:**

  if there is a D on one side, then there is a 3 on the other side

- **Which cards do you have to turn in order to show that the rule holds?**

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D	**F**	**3**	**7**

- **Consider the rule:**

  if there is a D on one side, then there is a 3 on the other side

- **Which cards do you have to turn in order to show that the rule holds?**
  - **Only 10% of the subjects give the logically correct solutions**

The Selection Task – Social Case

- Griggs, Cox: The elusive thematic materials effect in the Wason selection task British Journal of Psychology 73, 407-420: 1982
- **Consider cards which have a person's age on the one side and a drink on the other side**

beer	**coke**	**22yrs**	**16yrs**

- **Consider the rule:**

  If a person is drinking beer, then the person must be over 19 years of age

- **Which cards do you have to turn in order to show that the rule holds?**

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beer	coke	22yrs	16yrs

- **Consider the rule:**

  If a person is drinking beer, then the person must be over 19 years of age

- **Which cards do you have to turn in order to show that the rule holds?**
  - **Most people solve this variant correctly**

Formalizing the Abstract Case

- The conditional is viewed as a belief
- Let $D, F, 3, 7$ be propositional variables denoting that the corresponding symbol is on one side
- Consider $\mathcal{P}_{ac} = \{3 \leftarrow D \wedge \neg ab_1,\ ab_1 \leftarrow \bot\}$ with $\mathcal{M}_{\mathcal{P}_{ac}} = \langle \emptyset, ab_1 \rangle$
- $\mathcal{M}_{\mathcal{P}_{ac}}$ does not explain any letter on a card
- The set of abducibles is $\{D \leftarrow \top, D \leftarrow \bot, F \leftarrow \top, F \leftarrow \bot, 7 \leftarrow \top, 7 \leftarrow \bot\}$
- We obtain

$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}_{ac} \cup \mathcal{E}}$	turn
D	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, ab_1 \rangle$	yes

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$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}_{ac} \cup \mathcal{E}}$	turn
D	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, ab_1 \rangle$	*yes*
F	$\{F \leftarrow \top\}$	$\langle F, ab_1 \rangle$	*no*

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$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}_{ac} \cup \mathcal{E}}$	turn
D	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, ab_1 \rangle$	*yes*
F	$\{F \leftarrow \top\}$	$\langle F, ab_1 \rangle$	*no*
3	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, ab_1 \rangle$	*yes*

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$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}_{ac} \cup \mathcal{E}}$	turn
D	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, ab_1 \rangle$	*yes*
F	$\{F \leftarrow \top\}$	$\langle F, ab_1 \rangle$	*no*
3	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, ab_1 \rangle$	*yes*
7	$\{7 \leftarrow \top\}$	$\langle 7, ab_1 \rangle$	*no*

Formalizing the Social Case

- The conditional is viewed as a social constraint
- Let o and b be propositional variables denoting that the person is older than 19 years and is drinking beer, respectively
- The rule is encoded by $o \leftarrow b \wedge \neg ab_2 = F$
- We obtain

case	$\mathcal{P}_{sc}$	$\mathcal{M}_{\mathcal{P}_{sc}}$	$\mathcal{P}_{sc} \models_{wcs} F$	turn
beer	$\{b \leftarrow \top,\ ab_2 \leftarrow \bot\}$	$\langle b, ab_2 \rangle$	*no*	*yes*

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beer	$\{b \leftarrow \top,\ ab_2 \leftarrow \bot\}$	$\langle b, ab_2 \rangle$	*no*	*yes*
22yrs	$\{o \leftarrow \top,\ ab_2 \leftarrow \bot\}$	$\langle o, ab_2 \rangle$	*yes*	*no*

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22yrs	$\{o \leftarrow \top,\ ab_2 \leftarrow \bot\}$	$\langle o, ab_2 \rangle$	*yes*	*no*
coke	$\{b \leftarrow \bot,\ ab_2 \leftarrow \bot\}$	$\langle \emptyset, \{b, ab_2\} \rangle$	*yes*	*no*

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22yrs	$\{o \leftarrow \top,\ ab_2 \leftarrow \bot\}$	$\langle o, ab_2 \rangle$	*yes*	*no*
coke	$\{b \leftarrow \bot,\ ab_2 \leftarrow \bot\}$	$\langle \emptyset, \{b, ab_2\} \rangle$	*yes*	*no*
16yrs	$\{o \leftarrow \bot,\ ab_2 \leftarrow \bot\}$	$\langle \emptyset, \{o, ab_2\} \rangle$	*no*	*yes*

A Computational Logic Approach to the Selection Task

- **The computational logic approach to model human reasoning can be extended to adequately handle the selection task**
  - **if the social case is understood as a social constraint and**
  - **if the abstract case is understood as a belief**
- Kowalski: Computational Logic and Human Life: How to be Artificially Intelligent. Cambridge University Press: 2011
- Dietz, H., Ragni: A Computational Logic Approach to the Abstract and the Social Case of the Selection Task. In: Proceedings of the 11th International Symposium on Logic Formalizations of Commonsense Reasoning: 2013

WCS versus WFS – A Psychological Study

- **How do humans reason with positive loops?**
  - If they open the window, then they open the window
  - If they open the window, then it is cold
    If it is cold, then they wear their jackets
    If they wear their jackets, then they open the windows

WCS versus WFS – A Psychological Study

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- **A psychological study**
  - We presented conditionals with positive cycles of length one, two and three, and asked whether embedded propositions or their negations are entailed

length	yes	no (WFS)	I don't know (WCS)	response time
1	75 %	0 %	25 %	5257 msec
2	60 %	3 %	37 %	11516 msec
3	55 %	4 %	41 %	11680 msec

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  - ▶▶ Humans consider positive cycles of length one as facts
  - ▶▶ The longer the cycles, the more likely is the answer 'I don't know'
  - ▶▶ Almost nobody entailed negative propositions

Conditionals – The Firing Squad Example

- Pearl: Causality: Models, Reasoning, and Inference
  Cambridge University Press, New York, USA: 2000
- **If the court orders an execution, then the captain will give the signal upon which rifleman *A* and *B* will shoot the prisoner; consequently, the prisoner will be dead**

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- **We assume that**
  - **the court's decision is *unknown***
  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**

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  - **the prisoner is unlikely to die from any other causes**
- **Evaluate the following conditionals (*true*, *false*, *unknown*)**
  - **If the prisoner is not dead, then the captain did not signal**

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  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**

- **Evaluate the following conditionals (*true*, *false*, *unknown*)**
  - **If rifleman A shot, then rifleman B shot as well**

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  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**

- **Evaluate the following conditionals (*true*, *false*, *unknown*)**
  - **If rifleman A did not shoot, then the prisoner is not dead**

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  - **both riflemen are accurate, alert and law-abiding**
  - **the prisoner is unlikely to die from any other causes**
- **Evaluate the following conditionals (*true*, *false*, *unknown*)**
  - **If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution**

ARSC – An Abstract Reduction System for Conditionals

- **Let $\textit{cond}(\mathcal{C}, \mathcal{D})$ be a conditional, $\mathcal{P}$ a program, and $\mathcal{IC}$ a finte set of integrity constraints**

ARSC – An Abstract Reduction System for Conditionals

- Let $\mathit{cond}(\mathcal{C}, \mathcal{D})$ be a conditional, $\mathcal{P}$ a program, and $\mathcal{IC}$ a finte set of integrity constraints
- The rules
  - $\langle \mathcal{P}, \mathcal{IC}, \mathcal{C}, \mathcal{D} \rangle \longrightarrow_t \mathcal{M}_{\mathcal{P}}(\mathcal{D})$
    iff $\quad \mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$

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    iff $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$
  - $\langle \mathcal{P}, \mathcal{IC}, \mathcal{C}, \mathcal{D} \rangle \longrightarrow_c \langle rev(\mathcal{P}, \mathcal{S}), \mathcal{IC}, \mathcal{C} \setminus \mathcal{S}, \mathcal{D} \rangle$
    iff $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \bot$, where $\mathcal{S} = \{L \in \mathcal{C} \mid \mathcal{M}_\mathcal{P}(L) = \bot\}$

ARSC – An Abstract Reduction System for Conditionals

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  - $\langle \mathcal{P}, \mathcal{IC}, \mathcal{C}, \mathcal{D} \rangle \longrightarrow_c \langle rev(\mathcal{P}, \mathcal{S}), \mathcal{IC}, \mathcal{C} \setminus \mathcal{S}, \mathcal{D} \rangle$
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  - $\langle \mathcal{P}, \mathcal{IC}, \mathcal{C}, \mathcal{D} \rangle \longrightarrow_a \langle \mathcal{P} \cup \mathcal{E}, \mathcal{IC}, \mathcal{C}, \mathcal{D} \rangle$
    iff $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \mathsf{U}$, $\mathcal{O} \subseteq \mathcal{C}$, $\mathcal{O} \neq \emptyset$, for each $L \in \mathcal{O}$ we find $\mathcal{M}_{\mathcal{P}}(L) = \mathsf{U}$, and $\mathcal{E}$ explains $\mathcal{O}$ in the abductive framework $\langle \mathcal{P}, \mathcal{A}_{\mathcal{P}}, \mathcal{IC}, \models_{wcs} \rangle$

ARSC – An Abstract Reduction System for Conditionals

- Let $cond(\mathcal{C}, \mathcal{D})$ be a conditional, $\mathcal{P}$ a program, and $\mathcal{IC}$ a finte set of integrity constraints

- The rules
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    iff $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$
  - $\langle \mathcal{P}, \mathcal{IC}, \mathcal{C}, \mathcal{D} \rangle \longrightarrow_c \langle rev(\mathcal{P}, \mathcal{S}), \mathcal{IC}, \mathcal{C} \setminus \mathcal{S}, \mathcal{D} \rangle$
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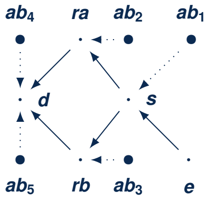
  - $\langle \mathcal{P}, \mathcal{IC}, \mathcal{C}, \mathcal{D} \rangle \longrightarrow_r \langle rev(\mathcal{P}, \mathcal{S}), \mathcal{IC}, \mathcal{C} \setminus \mathcal{S}, \mathcal{D} \rangle$
    iff $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \mathsf{U}$, $\mathcal{S} \subseteq \mathcal{C}$, $\mathcal{S} \neq \emptyset$, for each $L \in \mathcal{S}$ we find $\mathcal{M}_\mathcal{P}(L) = \mathsf{U}$, and $\mathcal{M}_{rev(\mathcal{P}, \mathcal{S})}$ satisfies $\mathcal{IC}$

ARSC – The Firing Squad Example

- If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution

ARSC – The Firing Squad Example

- If the captain gave no signal and rifleman A decides to shoot, then the court did not order an execution



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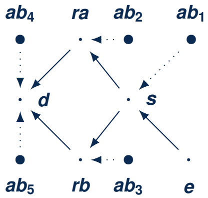
	$\rightarrow r\{\bar{s}, ra\}$	$\rightarrow r\{\bar{s}\}$ $\rightarrow c\{ra\}$	$\rightarrow r\{ra\}$ $\rightarrow a\{\bar{s}\}$	$\rightarrow a\{\bar{s}\}$ $\rightarrow c\{ra\}$	$\rightarrow a\{ra\}$ $\rightarrow c\{\bar{s}\}$ $\rightarrow c\{ra\}$
s	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
ra	$\top$	$\top$	$\top$	$\top$	$\top$
d	$\top$	$\top$	$\top$	$\top$	$\top$
rb	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
e	U	U	$\bot$	$\bot$	$\top$



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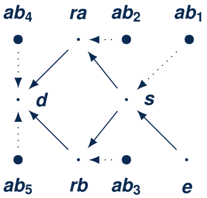
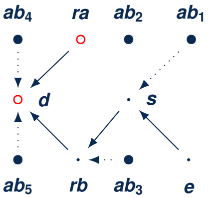
	$\overrightarrow{r}\{\overline{s}, ra\}$	$\overrightarrow{r}\{\overline{s}\}\ \overrightarrow{c}\{ra\}$	$\overrightarrow{r}\{ra\}\ \overrightarrow{a}\{\overline{s}\}$	$\overrightarrow{a}\{\overline{s}\}\ \overrightarrow{c}\{ra\}$	$\overrightarrow{a}\{ra\}\ \overrightarrow{c}\{\overline{s}\}\ \overrightarrow{c}\{ra\}$
s	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
ra	$\top$	$\top$	$\top$	$\top$	$\top$
d	$\top$	$\top$	$\top$	$\top$	$\top$
rb	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
e	U	U	$\bot$	$\bot$	$\top$





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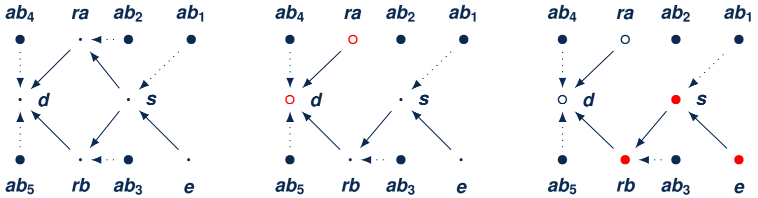
	$\rightarrow r\{\overline{s}, ra\}$	$\rightarrow r\{\overline{s}\}$ $\rightarrow c\{ra\}$	$\rightarrow r\{ra\}$ $\rightarrow a\{\overline{s}\}$	$\rightarrow a\{\overline{s}\}$ $\rightarrow c\{ra\}$	$\rightarrow a\{ra\}$ $\rightarrow c\{\overline{s}\}$ $\rightarrow c\{ra\}$
s	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
ra	$\top$	$\top$	$\top$	$\top$	$\top$
d	$\top$	$\top$	$\top$	$\top$	$\top$
rb	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
e	U	U	$\bot$	$\bot$	$\top$



Open Questions

- **Do humans reason with multi-valued logics and, if they do, which multi-valued logic are they using?**
- **Can an answer 'I don't know' be qualified as a truth value assignment or is it a meta-remark?**
- **What do we have to tell humans such that they fully understand the background information?**
- **Do humans apply abduction and/or revision if the condition of a conditional is *unknown* and, if they apply both, do they prefer one over the other?**
- **Do they prefer skeptical over creduluous abduction?**
- **Do they prefer minimal revision?**
- **How important is the order in which multiple conditions of a conditional are considered?**
- **Do humans consider abduction and/or revision steps which turn an indicative conditional into a subjunctive one?**