TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

DEDUCTION SYSTEMS

Lecture 5 ASP Solving II *slides adapted from Torsten Schaub [Gebser et al.(2012)]

Sarah Gaggl

Dresden, 25th June 2015

DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

Outline

Nogoods from logic programs
via loop formulas

Let P be a normal logic program and recall that:

- For $L \subseteq atom(P)$, the external supports of L for P are

$$ES_P(L) \quad = \quad \{r \in P \mid head(r) \in L, body(r)^+ \cap L = \emptyset\}$$

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- The (disjunctive) loop formula of L for P is

$$\begin{aligned} LF_P(L) \quad &= \quad \left(\bigvee_{A \in L} A\right) \rightarrow \left(\bigvee_{r \in ES_P(L)} body(r)\right) \\ &\equiv \quad \left(\bigwedge_{r \in ES_P(L)} \neg body(r)\right) \rightarrow \left(\bigwedge_{A \in L} \neg A\right) \end{aligned}$$

  – Note: The loop formula of L enforces all atoms in L to be false whenever L is not externally supported

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- The external bodies of L for P are

$$EB_P(L) \quad = \quad \{body(r) \mid r \in ES_P(L)\}$$

Nogoods from logic programs
loop nogoods

- For a logic program P and some $\emptyset \subset U \subseteq atom(P)$, define the loop nogood of an atom $a \in U$ as

$$\lambda(a, U) \quad = \quad \{\boldsymbol{T}a, \boldsymbol{F}B_1, \ldots, \boldsymbol{F}B_k\}$$

where $EB_P(U) = \{B_1, \ldots, B_k\}$

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$$\Lambda_P \quad = \quad \bigcup_{\emptyset \subset U \subseteq atom(P)} \{\lambda(a, U) \mid a \in U\}$$

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$$\Lambda_P \quad = \quad \bigcup_{\emptyset \subset U \subseteq atom(P)} \{\lambda(a, U) \mid a \in U\}$$

- The set Λ_P of loop nogoods denies cyclic support among true atoms

Example

- Consider the program

$$\left\{ \begin{array}{ll} x \leftarrow not\ y & u \leftarrow x \\ & u \leftarrow v \\ y \leftarrow not\ x & v \leftarrow u, y \end{array} \right\}$$

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- For u in the set $\{u, v\}$, we obtain the loop nogood:

$$\lambda(u, \{u, v\}) \quad = \quad \{\boldsymbol{T}u, \boldsymbol{F}\{x\}\}$$

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- For u in the set $\{u, v\}$, we obtain the loop nogood:

$$\lambda(u, \{u, v\}) \quad = \quad \{\boldsymbol{T}u, \boldsymbol{F}\{x\}\}$$

  Similarly for v in $\{u, v\}$, we get:

$$\lambda(v, \{u, v\}) \quad = \quad \{\boldsymbol{T}v, \boldsymbol{F}\{x\}\}$$

Characterization of stable models

Theorem

Let P be a logic program. Then,

$X \subseteq atom(P)$ is a stable model of P iff
$X = A^{\boldsymbol{T}} \cap atom(P)$ for a (unique) solution A for $\Delta_P \cup \Lambda_P$

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Some remarks

- Nogoods in Λ_P augment Δ_P with conditions checking for unfounded sets, in particular, those being loops
- While $|\Delta_P|$ is linear in the size of P, Λ_P may contain exponentially many (non-redundant) loop nogoods

Outline

Towards conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach
  (DPLL stands for 'Davis-Putnam-Logemann-Loveland')
  - (Unit) propagation
  - (Chronological) backtracking
  - in ASP, eg smodels
- Modern CDCL-style approach
  (CDCL stands for 'Conflict-Driven Constraint Learning')
  - (Unit) propagation
  - Conflict analysis (via resolution)
  - Learning + Backjumping + Assertion
  - in ASP, eg clasp

DPLL-style solving

loop

propagate // deterministically assign literals

if no conflict **then**

if all variables assigned **then return** solution
else decide // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable
else

backtrack // unassign literals made after last decision
flip // assign complement of last decision literal

CDCL-style solving

```
loop
    propagate                                    // deterministically assign literals
    if no conflict then
        if all variables assigned then return solution
        else decide                    // non-deterministically assign some literal
    else
        if top-level conflict then return unsatisfiable
        else
            analyze            // analyze conflict and add conflict constraint
            backjump     // unassign literals until conflict constraint is unit
```

Outline

Outline of CDNL-ASP algorithm

- Keep track of deterministic consequences by unit propagation on:
  - Program completion [Δ_P]
  - Loop nogoods, determined and recorded on demand [Λ_P]
  - Dynamic nogoods, derived from conflicts and unfounded sets [∇]

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- When a nogood in $\Delta_P \cup \nabla$ becomes violated:
  - Analyze the conflict by resolution
    (until reaching a Unique Implication Point, short: UIP)
  - Learn the derived conflict nogood δ
  - Backjump to the earliest (heuristic) choice such that the complement of the UIP is unit-resulting for δ
  - Assert the complement of the UIP and proceed
    (by unit propagation)

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  - Backjump to the earliest (heuristic) choice such that the complement of the UIP is unit-resulting for δ
  - Assert the complement of the UIP and proceed (by unit propagation)
- Terminate when either:
  - Finding a stable model (a solution for $\Delta_P \cup \Lambda_P$)
  - Deriving a conflict independently of (heuristic) choices

Algorithm 1: CDNL-ASP

Input : A normal program P
Output : A stable model of P or “no stable model”

$A := \emptyset$ // assignment over $atom(P) \cup body(P)$
$\nabla := \emptyset$ // set of recorded nogoods
$dl := 0$ // decision level

loop
  $(A, \nabla) :=$ NogoodPropagation(P, ∇, A)
  if $\varepsilon \subseteq A$ **for some** $\varepsilon \in \Delta_P \cup \nabla$ **then** // conflict
    if $\max(\{dlevel(\sigma) \mid \sigma \in \varepsilon\} \cup \{0\}) = 0$ **then return** no stable model
    $(\delta, dl) :=$ ConflictAnalysis($\varepsilon, P, \nabla, A$)
    $\nabla := \nabla \cup \{\delta\}$ // (temporarily) record conflict nogood
    $A := A \setminus \{\sigma \in A \mid dl < dlevel(\sigma)\}$ // backjumping
  else if $A^{\boldsymbol{T}} \cup A^{\boldsymbol{F}} = atom(P) \cup body(P)$ **then** // stable model
    return $A^{\boldsymbol{T}} \cap atom(P)$
  else
    $\sigma_d :=$ Select(P, ∇, A) // decision
    $dl := dl + 1$
    $dlevel(\sigma_d) := dl$
    $A := A \circ \sigma_d$

Observations

- Decision level dl, initially set to 0, is used to count the number of heuristically chosen literals in assignment A
- For a heuristically chosen literal $\sigma_d = \boldsymbol{T}a$ or $\sigma_d = \boldsymbol{F}a$, respectively, we require $a \in (atom(P) \cup body(P)) \setminus (A^{\boldsymbol{T}} \cup A^{\boldsymbol{F}})$
- For any literal $\sigma \in A$, $dl(\sigma)$ denotes the decision level of σ, viz. the value dl had when σ was assigned

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- A conflict is detected from violation of a nogood $\varepsilon \subseteq \Delta_P \cup \nabla$
- A conflict at decision level 0 (where A contains no heuristically chosen literals) indicates non-existence of stable models
- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level $k < dl$

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- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level $k < dl$
  - After learning δ and backjumping to decision level k, at least one literal is newly derivable by unit propagation
  - No explicit flipping of heuristically chosen literals !

Example: CDNL-ASP

Consider

$$P = \left\{ \begin{array}{llll} x \leftarrow not\ y & u \leftarrow x, y & v \leftarrow x & w \leftarrow not\ x, not\ y \\ y \leftarrow not\ x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d $\overline{\sigma}$	δ

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		$\vdots$	$\vdots$	
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Outline

Outline of NogoodPropagation

- Derive deterministic consequences via:
  - Unit propagation on Δ_P and ∇;
  - Unfounded sets $U \subseteq atom(P)$
- Note that U is unfounded if $EB_P(U) \subseteq A^{\boldsymbol{F}}$
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- An "interesting" unfounded set U satisfies:

$$\emptyset \subset U \subseteq (atom(P) \setminus A^{\boldsymbol{F}})$$

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  - Note: Tight programs do not yield “interesting” unfounded sets !

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  such an unfounded set contains some loop of P
  - Note: Tight programs do not yield "interesting" unfounded sets !
- Given an unfounded set U and some $a \in U$, adding $\lambda(a, U)$ to ∇ triggers a conflict or further derivations by unit propagation
  - Note: Add loop nogoods atom by atom to eventually falsify all $a \in U$

Algorithm 2: NogoodPropagation

Input : A normal program P, a set ∇ of nogoods, and an assignment A.
Output : An extended assignment and set of nogoods.

```
U := ∅                                                          // unfounded set
loop
    repeat
        if δ ⊆ A for some δ ∈ Δ_P ∪ ∇ then return (A, ∇)         // conflict
        Σ := {δ ∈ Δ_P ∪ ∇ | δ \ A = {σ̄}, σ ∉ A}                  // unit-resulting nogoods
        if Σ ≠ ∅ then let σ̄ ∈ δ \ A for some δ ∈ Σ in
            dlevel(σ) := max({dlevel(ρ) | ρ ∈ δ \ {σ̄}} ∪ {0})
            A := A ∘ σ
    until Σ = ∅
    if loop(P) = ∅ then return (A, ∇)
    U := U \ A^F
    if U = ∅ then U := UnfoundedSet(P, A)
    if U = ∅ then return (A, ∇)                 // no unfounded set ∅ ⊂ U ⊆ atom(P) \ A^F
    let a ∈ U in
        ∇ := ∇ ∪ {{Ta} ∪ {FB | B ∈ EB_P(U)}}                    // record loop nogood
```

Requirements for UnfoundedSet

- Implementations of UnfoundedSet must guarantee the following for a result U
  1. $U \subseteq (atom(P) \setminus A^{\boldsymbol{F}})$
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  3. $U = \emptyset$ iff there is no nonempty unfounded subset of $(atom(P) \setminus A^{\boldsymbol{F}})$
- Beyond that, there are various alternatives, such as:
  - Calculating the greatest unfounded set
  - Calculating unfounded sets within strongly connected components of the positive atom dependency graph of P

  - Usually, the latter option is implemented in ASP solvers

Example: NogoodPropagation

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			$\{\boldsymbol{T}u, \boldsymbol{F}\{x\}, \boldsymbol{F}\{x, y\}\} = \lambda(u, \{u, v\})$ ✘

Outline

Outline of ConflictAnalysis

- Conflict analysis is triggered whenever some nogood $\delta \in \Delta_P \cup \nabla$ becomes violated, viz. $\delta \subseteq A$, at a decision level $dl > 0$
  - Note that all but the first literal assigned at dl have been unit-resulting for nogoods $\varepsilon \in \Delta_P \cup \nabla$
  - If $\sigma \in \delta$ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$$

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- Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$
  - Iterated resolution progresses in inverse order of assignment

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  - Note that all but the first literal assigned at dl have been unit-resulting for nogoods $\varepsilon \in \Delta_P \cup \nabla$
  - If $\sigma \in \delta$ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$$

- Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$
  - Iterated resolution progresses in inverse order of assignment
- Iterated resolution stops as soon as it generates a nogood δ containing exactly one literal σ assigned at decision level dl
  - This literal σ is called First Unique Implication Point (First-UIP)
  - All literals in $(\delta \setminus \{\sigma\})$ are assigned at decision levels smaller than dl

Algorithm 3: ConflictAnalysis

Input : A non-empty violated nogood δ, a normal program P, a set ∇ of nogoods, and an assignment A.

Output : A derived nogood and a decision level.

loop

  let $\sigma \in \delta$ **such that** $\delta \setminus A[\sigma] = \{\sigma\}$ **in**

    $k := \max(\{dlevel(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\})$

    if $k = dlevel(\sigma)$ **then**

      let $\varepsilon \in \Delta_P \cup \nabla$ **such that** $\varepsilon \setminus A[\sigma] = \{\overline{\sigma}\}$ **in**

        $\delta := (\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$ // resolution

    else return (δ, k)

Example: ConflictAnalysis

Consider

$$P \quad = \quad \left\{ \begin{array}{llll} x \leftarrow not\ y & u \leftarrow x, y & v \leftarrow x & w \leftarrow not\ x, not\ y \\ y \leftarrow not\ x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ
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- Asserting nogoods direct conflict-driven search into a different region of the search space than traversed before,
  without explicitly flipping any heuristically chosen literal !

Outline

Summary

- Nogoods from loop formulas
- Conflict driven nogood learning
  - CDNL-ASP Algorithm
  - Nogood Propagation
  - Conflict Analysis

References

Martin Gebser, Benjamin Kaufmann Roland Kaminski, and Torsten Schaub.
Answer Set Solving in Practice.
Synthesis Lectures on Artificial Intelligence and Machine Learning.
Morgan and Claypool Publishers, 2012.
doi=10.2200/S00457ED1V01Y201211AIM019.

- See also: `http://potassco.sourceforge.net`