EXERCISE 2

Human Reasoning and Computational Logic

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winter semester 2019/20

Problem 2.1

Given a program $\mathcal{P}$. Show that $\Phi_{\mathcal{P}}$ is monotonic.

Problem 2.2

Given a program $\mathcal{P}$. Show that if $I \models \mathsf{wc}\,\mathcal{P}$ then $I \models \mathcal{P}$.
Does the other direction also hold?

Problem 2.3

Given a program $\mathcal{P}$. Show that if I is a fixed point of $\Phi_{\mathcal{P}}$ then I is a model of $\mathsf{wc}\,\mathcal{P}$.
Does the other direction also hold?

Problem 2.4

Show that the following proposition holds:

Proposition 19 Let $\mathcal{P}$ be a program, ℓ a (total) level mapping for $\mathcal{P}$, $\mathcal{I}$ the set of (three-valued) interpretations for $\mathcal{P}$, and $I, J \in \mathcal{I}$. The function $d_\ell : \mathcal{I} \times \mathcal{I} \mapsto \mathbb{R}$ defined as

$$d_\ell(I,J) = \begin{cases} (\frac{1}{2})^n & I \neq J \text{ and } I(A) = J(A) \neq \mathsf{U} \text{ for all A with } \ell(A) < n \text{ and,} \\ & \text{for some } A \text{ with } \ell(A) = n, I(A) \neq J(A) \text{ or } I(A) = J(A) = \mathsf{U} \\ 0 & \text{otherwise.} \end{cases}$$

is a metric.

Problem 2.5

Show that the deduction theorem is not satisfied under Łukasiewicz and Kleene logic.