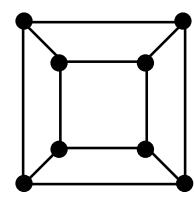
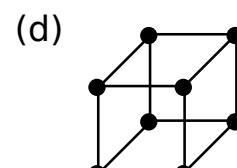
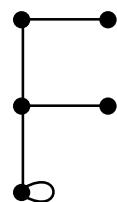
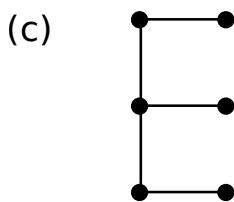
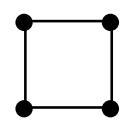
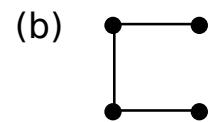
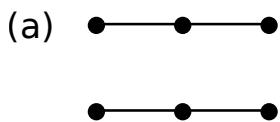


## Exercise Sheet 7: Query Optimisation and FO Query Expressivity

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**Exercise 7.1.** For the following pairs of structures, find the maximal  $r$  such that  $\mathcal{I} \sim_r \mathcal{J}$ :

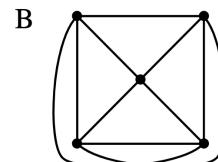
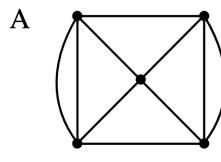


**Exercise 7.2.** A *linear order* is a relational structure with one binary relational symbol  $\leq$  that is interpreted as a reflexive, asymmetric, transitive and total relation over the domain. Up to renaming of domain elements there is exactly one linear order for every finite domain, which can be depicted as a chain of elements. We denote the linear order of size  $n$  by  $\mathcal{L}_n$ . For example:

$$\mathcal{L}_6 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \quad \text{and} \quad \mathcal{L}_7 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \leq 7$$

1. For which  $r$  are  $\mathcal{L}_6 \sim_r \mathcal{L}_7$ ?
2. More generally, for which  $r$  are  $\mathcal{L}_n \sim_r \mathcal{L}_{n+1}$ ? (\*)

**Exercise 7.3.** A graph is *planar* if it can be drawn on the plane without intersections of edges. For example, the following graph A is planar, while graph B is not:



Can the graphs A and B be distinguished by a first-order query? Show that planarity is not FO-definable by using locality.